ECE 5390/MSE 5472, Fall Semester 2015
Quantum Transport in Electron Devices & Novel Materials

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Assignment 1

Policy on assignments: Please turn them in by 5pm of the due date. The due date for this assignment is **Wednesday, Sept 9th, 2015**.

General notes: Present your solutions *neatly*. Do not turn in rough unreadable worksheets - learn to **take pride in your presentation**. Show the relevant steps, so that partial points can be awarded. BOX your final answers. Draw figures wherever necessary. Please print out this question sheet and staple to the top of your homework. Write your name and email address on the cover. Some problems may lead to publishable results - be on the lookout!

Problem 1.1) Ballistic vs diffusive space-charge transport

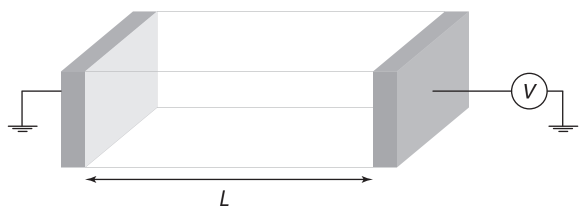

Figure 1: Setup for ballistic and diffusive space-charge transport.

Figure 1 shows what may be the 'simplest' transport problem: a voltage V is applied across two metal plates separated by a distance L in a sealed enclosure. We will find the electron current that flows. All symbols have their 'usual' meanings: q is the electron charge, m_0 is the rest mass of electrons, and ϵ_0 the permittivity of vacuum.

(a) If the enclosure is vacuum, any current flow will be due to the *ballistic* transport of electrons from one electrode to the other, because there is nothing to scatter electrons. Show that the ballistic space-charge current density is $\boxed{J^{sc}_{ballistic} = \frac{4}{9}\sqrt{\frac{2q}{m_0}}\frac{\epsilon_0}{L^2}V^{\frac{3}{2}}}$. This is the Child-Langmuir law, which was experimentally measured and theoretically understood *well before quantum mechanics*!

(b) Suppose in the enclosure there is an 'insulator' of dielectric constant ϵ_s that scatters electrons. The electron mobility μ leads to a velocity $v(x) = \mu F(x)$, where $F(x)$ is the local electric field. Show then the diffusive space-charge current density is $\boxed{J^{sc}_{diff} = \frac{9}{8}\frac{\epsilon_s \mu}{L^3}V^2}$, the Mott-Gurney law.

(c) If the field is large enough for the velocity to saturate $v = v_{sat}$, show that the current density is $\boxed{J^{sc}_{sat} = 2\frac{\epsilon_s v_{sat}}{L^2}V}$.

(d) Argue that *none* of the above three transport results are Ohm's law, which states $\boxed{J = \frac{qn\mu}{L}V}$. Contrast and explain the differences. Explain the conspicuous 'absence' of $\hbar$ in all of the above.

Problem 1.2) Quantum Mechanics Recap

We derived in class that the allowed wavefunctions representing an electron on a circular ring of circumference L is $\psi_n(x) = \frac{1}{\sqrt{L}} e^{ik_n x}$, where $k_n = \frac{2\pi}{L} n$ are *quantized* because $n = 0, \pm 1, \pm 2, ...$ The *angular momentum* of a particle is defined as $\mathcal{L} = \mathbf{r} \times \mathbf{p}$, where $\mathbf{r}$ is the 'radius' of the circle, and $\mathbf{p}$ is the linear momentum.

(a) Show that the angular momentum of an electron in state $\psi_n(x)$ is $\mathcal{L}_n = n\hbar$, where $\hbar = \frac{h}{2\pi}$ is the 'reduced' Planck's constant. This implies that the angular momentum is *quantized* to values $0, \pm\hbar, \pm 2\hbar, ...$ Compare the quantized angular momentum $\mathcal{L}_1$ for $n = +1$ with the classical angular momentum $\mathcal{L}_{cl}$ of a mass $m = 1$ kg being spun by a string of length $R = 1$ m with tangential velocity $v = 1$ m/s to appreciate how 'nano' is the quantum of angular momentum.

(b) By balancing the classical centrifugal force and the electromagnetic Lorentz force, show that for an electron to be in the quantum state $\psi_n(x)$ on the ring, we need a *magnetic* field B_n such that the magnetic flux is $\Phi_n = B_n \cdot A = n \times \frac{h}{2e}$. Here A is the area of the ring, e is the electron charge and $h = 2\pi\hbar$. $\Phi_0 = \frac{h}{2e}$ is known as the *quantum of magnetic flux*, and has been measured experimentally in nanostructured rings.

(c) Consider the quantum state obtained by the superposition $\psi(x) = a[\psi_{n=1}(x) + \psi_{n=-1}(x)]$ from the eigenstates of the electron on the ring. Normalize the state to find the constant a. You may need the result $\int_0^L \cos^2(\frac{2\pi}{L} x) dx = \frac{L}{2}$. Does this superposition state have a definite momentum?

(d) We derived that the quantum expression for current flux is $\mathbf{j} = \frac{1}{2m}(\psi^\star \hat{\mathbf{p}} \psi - \psi \hat{\mathbf{p}} \psi^\star)$, where $\hat{\mathbf{p}} = -i\hbar\nabla$ is the momentum operator, which takes the form $\hat{p}_x = -i\hbar \frac{\partial}{\partial x}$ for the particle on the ring. Show that even though the states $\psi_{n=1}(x)$ and $\psi_{n=-1}(x)$ carry *net* currents, their superposition state of part (c) does not. Explain.

Problem 1.3) Tunneling and transmission

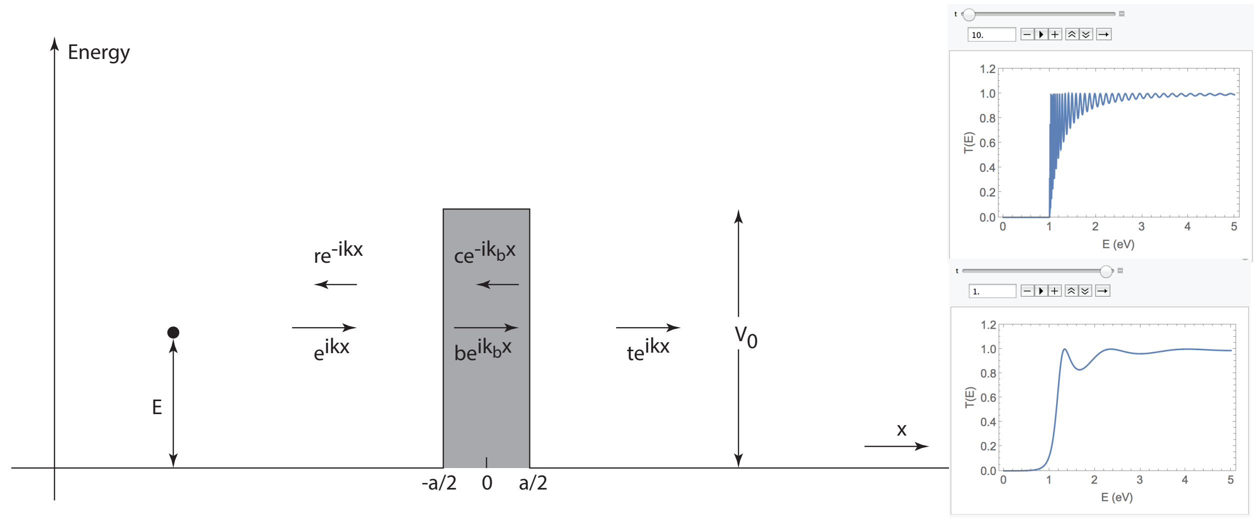

Figure 2: Tunneling and transmission of electrons through a single potential barrier.

Here you solve the problem of quantum mechanical tunneling of an electron through a barrier (see figure 2). For an electron of mass m and kinetic energy E, write an expression for k. Also write an expression for k_b for a barrier height V_0. Explain when k_b is real and when it is *imaginary*.

(a) There are 4 unknowns: the coefficients r, b, c, and t. Use boundary conditions that the wavefunction and its derivative are continuous at $x = +a/2$ and at $x = -a/2$ to get 4 equations. Solving these 4 equations will yield the 4 unknown coefficients, solving the problem completely.

(b) Since we are interested in the tunneling probability, explain why this probability is $T(E) = |t|^2$. Show then that it is given by the expression

$$T(E) = \frac{1}{1 + \frac{V_0^2}{4E(E-V_0)} \sin^2(k_b a)} \tag{1}$$

(c) Note that when the kinetic energy of the electron is lower than the barrier, k_b is imaginary, and $\sin(iy) = i \sinh(y)$. Make a plot of the tunneling probability as a function of the electron energy E for a fixed barrier height $V_0 = 1$ eV. Use barrier thicknesses of $a = 10$ nm and $a = 1$ nm. Compare with my calculation shown in figure 2. Note how sensitive tunneling is to thickness, and the resonances when $E > V_0$. These values are routinely encountered in semiconductor devices and nanostructures.