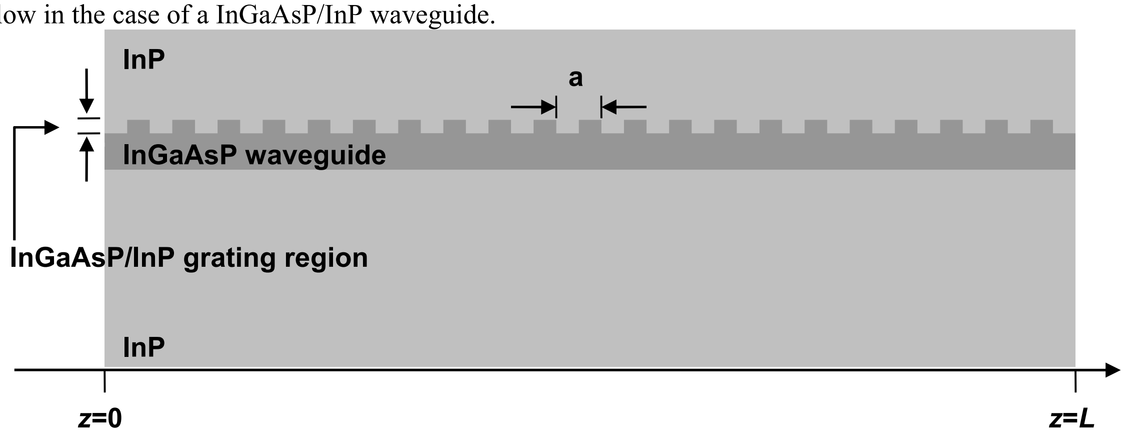
Chapter 13

Distributed Feedback (DFB) Structures and Semiconductor DFB Lasers

13.1 Distributed Feedback (DFB) Gratings in Waveguides

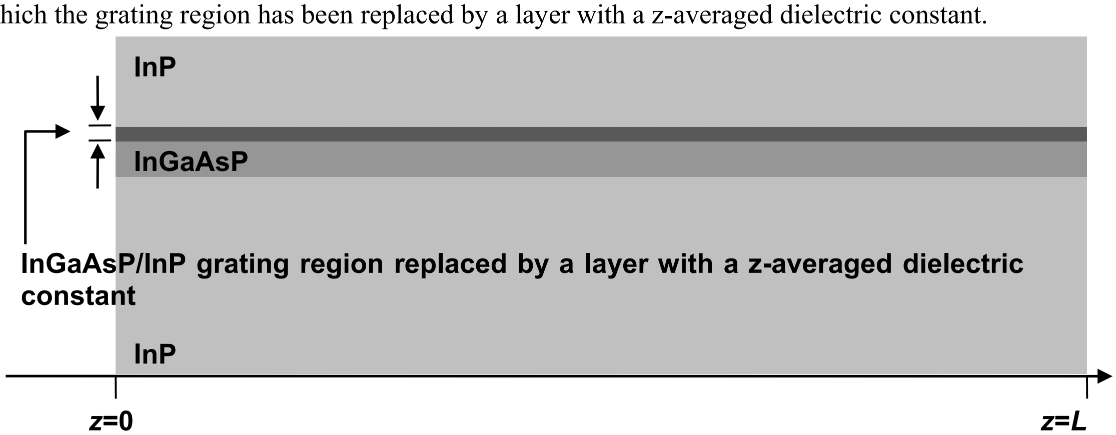
13.1.1 Introduction:
Periodic structures, like the DBR mirrors in VCSELs, can be also realized in a waveguide, as shown below in the case of a InGaAsP/InP waveguide.



In the waveguide shown above, periodic grooves have been etched in the top surface of the InGaAsP waveguide before the growth of the top InP layer. Such periodic grating structures are examples of one dimensional photonic bandgap materials. The relative dielectric constant is a function of the z-coordinate and can be written as,

$$\varepsilon(x,y,z) = \varepsilon_{avg}(x,y) + \Delta\varepsilon(x,y,z)$$

The average dielectric constant $\varepsilon_{avg}(x,y)$ corresponds to the waveguide structure shown below in which the grating region has been replaced by a layer with a z-averaged dielectric constant.



The z-average of the part $\Delta\varepsilon(x,y,z)$ is therefore zero. If the period of the grating is a , then one may expend $\Delta\varepsilon(x,y,z)$ in terms of a Fourier series,

$$\Delta\varepsilon(x,y,z) = f(x,y)\sum_{p\neq 0} d_p e^{ipGz}$$

where the reciprocal lattice vector (also called the grating vector) G equals $2\pi/a$. If $\Delta\varepsilon(x,y,z)$ is real then, $d_{-p} = d_p^*$. $f(x,y)$ equals one in the grating region and equals zero everywhere else. In the above Fourier series for $\Delta\varepsilon(x,y,z)$, usually the fundamental harmonic dominates and therefore we will assume that,

$$\Delta\varepsilon(x,y,z) = f(x,y)\left[d_1 e^{iGz} + d_{-1}e^{-iGz}\right]$$

A wave travelling in the waveguide with a wavevector β can Bragg scatter from the periodic grating provided the conditions for Bragg scattering are satisfied,

$$\beta \pm G = \beta_{\text{final}}$$
$$\omega(\beta) = \omega(\beta_{\text{final}})$$

The only way these conditions can be satisfied in one dimension is when $\beta_{\text{final}} = -\beta$, i.e. the wave is reflected in the opposite direction,

$$\beta - G = -\beta$$
$$\Rightarrow \beta = \frac{G}{2}$$
$$\Rightarrow a = \frac{\lambda}{2n_{eff}}$$

So a forward traveling wave will be Bragg reflected if its wavevetor is close to $G/2 = \pi/a$. If we call this special wavevector β_o then $\beta_o = G/2 = \pi/a$. We can write $\Delta\varepsilon(x,y,z)$ as,

$$\Delta\varepsilon(x,y,z) = f(x,y)\left[d_1 e^{i2\beta_o z} + d_{-1}e^{-i2\beta_o z}\right]$$

13.1.2 Wave propagation in a DFB Grating Waveguide – Coupled Mode Technique:
One can analyze wave propagation in a DFB grating waveguide in two steps discussed below.

Step 1:
First consider the waveguide corresponding to $\varepsilon_{\text{avg}}(x,y)$ shown in the Figure above and solve for the eigemodes and the propagation vectors (eigenvalues) for all frequencies of interest. The eignemodes, $\vec{E}(x,y)e^{i\beta z}$ and $\vec{H}(x,y)e^{i\beta z}$ satisfy Maxwell's equations,

$$\nabla\times\vec{E}(x,y)e^{i\beta z} = i\omega\mu_o\vec{H}(x,y)e^{i\beta z}$$
$$\nabla\times\vec{H}(x,y)e^{i\beta z} = -i\omega\varepsilon_o\varepsilon_{\text{avg}}(x,y)\vec{E}(x,y)e^{i\beta z}$$

or,

$$(\nabla_t + i\beta\hat{z})\times\vec{E}(x,y) = i\omega\mu_o\vec{H}(x,y)$$
$$(\nabla_t + i\beta\hat{z})\times\vec{H}(x,y) = -i\omega\varepsilon_o\varepsilon_{\text{avg}}(x,y)\vec{E}(x,y)$$

The above equations can be solved to give the mode effective index $n_{eff}(\omega)$. Given a grating structure, we can now find the frequency ω_o that will Bragg scatter from the relation,

$$\beta(\omega_o) = \frac{\omega_o}{c}n_{eff}(\omega_o) = \beta_o = \frac{\pi}{a}$$

If the wavevector β is very different from β_o then the grating structure will likely not affect the solution much (there will be not much scattering). The interesting case is when $\beta \approx \beta_o$. This case is discussed below.

Step 2:
We treat the part $\Delta\varepsilon(x,y,z)$ as a perturbation. The perturbation will have the strongest affect when $\beta \approx \beta_o$. For $\beta \approx \beta_o$, we write the solution as,

$$\vec{E}(x,y,z) = A_+(z)\,\vec{E}(x,y)e^{i\beta z} + A_-(z)\,\vec{E}^*(x,y)e^{-i\beta z}$$
$$\vec{H}(x,y,z) = A_+(z)\,\vec{H}(x,y)e^{i\beta z} - A_-(z)\,\vec{H}^*(x,y)e^{-i\beta z}$$

Here, the functions $A_+(z)$ and $A_-(z)$ are assumed to be slowly varying in space. The form of the solution allows for coupling between the forward and backward going waves because of Bragg scattering from the grating. The technique described below is called coupled mode theory. Plugging the assumed form of the solution in Maxwell's equations gives,

$$\nabla\times\left[A_+(z)\,\vec{E}(x,y)e^{i\beta z} + A_-(z)\,\vec{E}^*(x,y)e^{-i\beta z}\right] = i\omega\mu_o\left[A_+(z)\,\vec{H}(x,y)e^{i\beta z} - A_-(z)\,\vec{H}^*(x,y)e^{-i\beta z}\right]$$
$$\nabla\times\left[A_+(z)\,\vec{H}(x,y)e^{i\beta z} - A_-(z)\,\vec{H}^*(x,y)e^{-i\beta z}\right] = -i\omega\varepsilon_o\left[\varepsilon_{avg}(x,y) + f(x,y)\left(d_1e^{i2\beta_o z} + d_{-1}e^{-i2\beta_o z}\right)\right]$$
$$\times\left[A_+(z)\,\vec{E}(x,y)e^{i\beta z} + A_-(z)\,\vec{E}^*(x,y)e^{-i\beta z}\right]$$

Using the Maxwell's equation satisfied by the eigenmode we get,

$$\hat{z}\times\vec{E}(x,y)\frac{dA_+(z)}{dz}e^{i\beta z} + \hat{z}\times\vec{E}^*(x,y)\frac{dA_-(z)}{dz}e^{-i\beta z} = 0$$
$$\hat{z}\times\vec{H}(x,y)\frac{dA_+(z)}{dz}e^{i\beta z} - \hat{z}\times\vec{H}^*(x,y)\frac{dA_-(z)}{dz}e^{-i\beta z} = -i\omega\varepsilon_o f(x,y)\left(d_1e^{i2\beta_o z} + d_{-1}e^{-i2\beta_o z}\right)$$
$$\times\left[A_+(z)\,\vec{E}(x,y)e^{i\beta z} + A_-(z)\,\vec{E}^*(x,y)e^{-i\beta z}\right]$$

We multiply the first equation above by $\vec{H}^*(x,y)$ and multiply the second equation above by $\vec{E}^*(x,y)$ and then subtract the two equations, and keep only the terms that are approximately phase matched to get on left and right hand sides to get,

$$\frac{dA_+(z)}{dz} = i\omega\varepsilon_o d_1 e^{-i2(\beta-\beta_o)z}\,\frac{\iint f(x,y)\vec{E}^*(x,y)\,.\,\vec{E}^*(x,y)\,dxdy}{\iint\left[\vec{E}(x,y)\times\vec{H}^*(x,y) + \vec{E}^*(x,y)\times\vec{H}(x,y)\right].\,\hat{z}\,dxdy}\,A_-(z)$$

If instead of subtracting, we add the two equations then we obtain,

$$\frac{dA_-(z)}{dz} = -i\omega\varepsilon_o d_{-1} e^{+i2(\beta-\beta_o)z}\,\frac{\iint f(x,y)\vec{E}(x,y)\,.\,\vec{E}(x,y)\,dxdy}{\iint\left[\vec{E}(x,y)\times\vec{H}^*(x,y) + \vec{E}^*(x,y)\times\vec{H}(x,y)\right].\,\hat{z}\,dxdy}\,A_+(z)$$

If (and only if) $\Delta\varepsilon(x,y,z)$ is real and $d_{-1} = d_1^*$, then the above two equations can be written as,

$$\frac{d}{dz}\begin{bmatrix}A_+(z)\\A_-(z)\end{bmatrix} = \begin{bmatrix}0 & i\kappa\,e^{-i2(\beta-\beta_o)z}\\ -i\kappa^*\,e^{i2(\beta-\beta_o)z} & 0\end{bmatrix}\begin{bmatrix}A_+(z)\\A_-(z)\end{bmatrix}$$

where the coupling constant κ is,

$$\kappa = \frac{\omega\varepsilon_o d_1}{2nn_g^M}\,\frac{\iint\limits_{\text{Grating region}} nn_g^M\vec{E}^*(x,y)\,.\,\vec{E}^*(x,y)\,dxdy}{\iint nn_g^M\vec{E}^*(x,y)\,.\,\vec{E}(x,y)\,dxdy}\,\frac{\iint nn_g^M\vec{E}^*(x,y)\,.\,\vec{E}(x,y)\,dxdy}{\mathrm{Re}\iint\left[\vec{E}(x,y)\times\vec{H}^*(x,y)\right].\,\hat{z}\,dxdy}$$
$$\approx \frac{\omega\,d_1}{2nn_g^M v_g}\Gamma_G$$

Here, nn_g^M is the product of the index and the (material) group index of the grating region, v_g is the group velocity of the mode, and the overlap integral Γ_G is the usual mode confinement factor for the grating region provided the mode electric field is real (for example, the mode electric field will be real if $\varepsilon_{avg}(x,y)$ is real and the z-component of the field is negligible). The coupling constant couples the forward and the backward propagating waves. To solve the above set of equations, we assume,

$$B_+(z) = A_+(z)e^{i(\beta-\beta_o)z}$$

$$B_-(z) = A_-(z)e^{-i(\beta-\beta_o)z}$$

$B_+(z)$ and $B_-(z)$ satisfy,

$$\frac{d}{dz}\begin{bmatrix} B_+(z) \\ B_-(z) \end{bmatrix} = \begin{bmatrix} i(\beta-\beta_o) & i\kappa \\ -i\kappa^* & -i(\beta-\beta_o) \end{bmatrix}\begin{bmatrix} B_+(z) \\ B_-(z) \end{bmatrix}$$

$$= \begin{bmatrix} i\Delta\beta & i\kappa \\ -i\kappa^* & -i\Delta\beta \end{bmatrix}\begin{bmatrix} B_+(z) \\ B_-(z) \end{bmatrix}$$

We have a 2x2 linear system of equations. The eigenvalues of the matrix on the right hand sided are $\pm s$ where,

$$s = \sqrt{|\kappa|^2 - \Delta\beta^2} = -iq \qquad -s = -\sqrt{|\kappa|^2 - \Delta\beta^2} = iq$$

The corresponding eigenvectors are,

$$-s \Leftrightarrow \begin{bmatrix} -\kappa \\ \Delta\beta - is \end{bmatrix} \qquad s \Leftrightarrow \begin{bmatrix} -\kappa \\ \Delta\beta + is \end{bmatrix}$$

The most general form of the solution is,

$$\begin{bmatrix} B_+(z) \\ B_-(z) \end{bmatrix} = C_1\begin{bmatrix} -\kappa \\ \Delta\beta - is \end{bmatrix}e^{iqz} + C_2\begin{bmatrix} -\kappa \\ \Delta\beta + is \end{bmatrix}e^{-iqz} \qquad \left\{\begin{matrix} iq = -s \\ -iq = s \end{matrix}\right\}$$

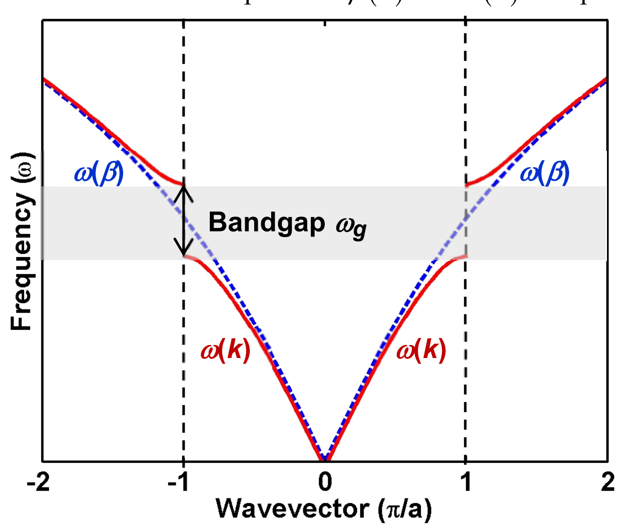
The constants C_1 are C_2 are determined by the boundary conditions. Note that in terms of $B_+(z)$ and $B_-(z)$ the electric field can be written as,

$$\vec{E}(x,y,z) = \vec{E}(x,y)B_+(z)e^{i\beta_o z} + \vec{E}^*(x,y)B_-(z)e^{-i\beta_o z}$$

From the expression above, the effective propagation vector of, say the forward going wave, at frequency ω is not $\beta(\omega)$ anymore but is $k(\omega)$ where,

$$k(\omega) = \beta_o \pm q(\omega) = \beta_o \pm \sqrt{\Delta\beta^2 - |\kappa|^2} = \beta_o \pm \sqrt{(\beta(\omega) - \beta_o)^2 - |\kappa|^2}$$

The difference between the modal dispersions $\beta(\omega)$ and $k(\omega)$ is depicted in the Figure below.



Note that a frequency gap (or a bandgap) opens in the dispersion relation of magnitude given by,

$$\omega_g = 2v_g|\kappa|$$

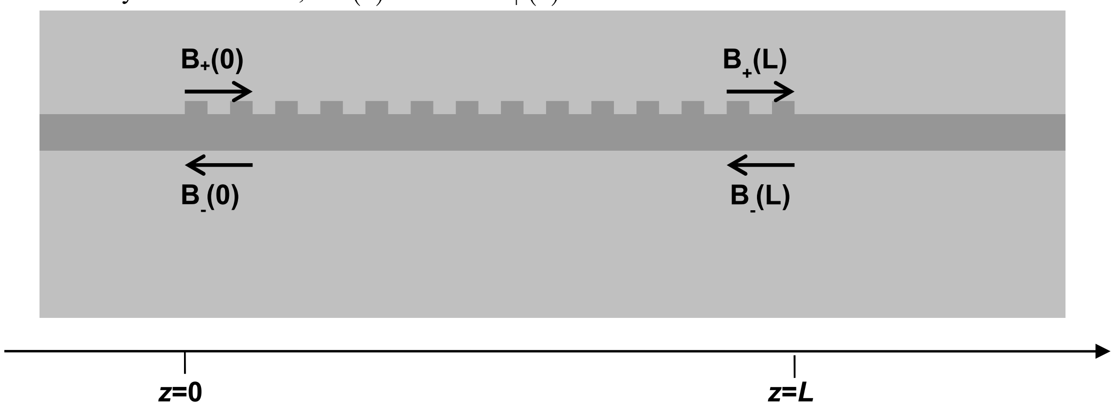
For values of frequency that fall in this bandgap, no real value of the propagation vector $k(\omega)$ satisfies the dispersion relation given above.

13.1.3 DFB Waveguide Mirror (or a Distributed Bragg Reflector (DBR)):

Consider a DFB structure as shown in the Figure below. We need to calculate the reflectivity of the mirror for a wave coming in inside the waveguide from the left side. The reflection and transmission coefficients are,

$$r = \frac{B_-(0)}{B_+(0)} \qquad\qquad t = \frac{B_+(L)}{B_+(0)} e^{i\beta_o L}$$

The boundary conditions are, $B_-(L) = 0$ and $B_+(0) \neq 0$.



We need to find the constants C_1 are C_2 in,

$$\begin{bmatrix} B_+(z) \\ B_-(z) \end{bmatrix} = C_1 \begin{bmatrix} -\kappa \\ \Delta\beta - is \end{bmatrix} e^{iqz} + C_2 \begin{bmatrix} -\kappa \\ \Delta\beta + is \end{bmatrix} e^{-iqz} \qquad \begin{Bmatrix} iq = -s \\ -iq = s \end{Bmatrix}$$

in order to satisfy the above boundary conditions. The final result is,

$$B_+(z) = \frac{-\Delta\beta \sinh[s(z-L)] + is\cosh[s(z-L)]}{\Delta\beta \sinh(sL) + is\cosh(sL)} B_+(0)$$

$$B_-(z) = \frac{\kappa^* \sinh[s(z-L)]}{\Delta\beta \sinh(sL) + is\cosh(sL)} B_+(0)$$

The reflection coefficient is,

$$r = \frac{B_-(0)}{B_+(0)} = \frac{-\kappa^* \sinh(sL)}{\Delta\beta \sinh(sL) + is\cosh(sL)}$$

The transmission coefficient is,

$$t = \frac{B_+(L)}{B_+(0)} e^{i\beta_o L} = \frac{is}{\Delta\beta \sinh(sL) + is\cosh(sL)} e^{i\beta_o L}$$

The magnitude of the reflection coefficient is maximum when the wavevector β of the incident wave is equal to β_o and $\Delta\beta = 0$,

$$r\big|_{\Delta\beta=0} = i\frac{\kappa^*}{|\kappa|} \tanh(|\kappa|L)$$

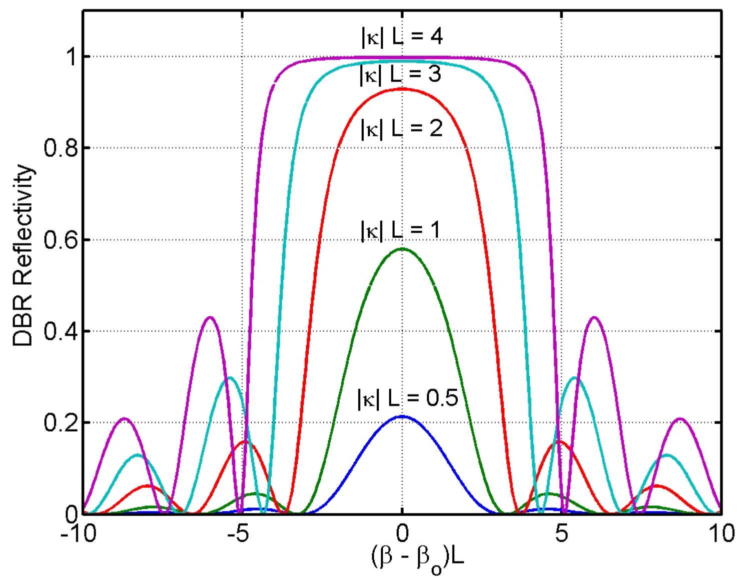
$$\Rightarrow R_{max} = \tanh^2(|\kappa|L)$$

The Figure below plots the reflectivity of a DBR mirror as a function of the wavelength (or wavevector) for different values of the coupling constant. Note that the reflection coefficient r goes to zero when,

$$sL = in\pi \qquad\qquad \{\, n = \text{nonzero integer}$$

$$\Rightarrow (\Delta\beta)^2 - |\kappa|^2 = \frac{n^2\pi^2}{L^2}$$

$$\Rightarrow (\beta - \beta_o)^2 = |\kappa|^2 + n^2\left(\frac{\pi}{L}\right)^2$$



The first zero in the reflection on either side of β_o determines the bandwidth over which the DBR mirror is an effective reflector. This bandwidth $\Delta\beta_{\text{DBR}}$ is,

$$\Delta\beta_{\text{DBR}} = 2\sqrt{|\kappa|^2 + \left(\frac{\pi}{L}\right)^2} \qquad \Rightarrow \qquad \Delta\omega_{\text{DBR}} = 2v_g\sqrt{|\kappa|^2 + \left(\frac{\pi}{L}\right)^2}$$

An infinitely long DFB structure is a one dimensional photonic bandgap material. The stopband or the bandgap ω_g of this material is,

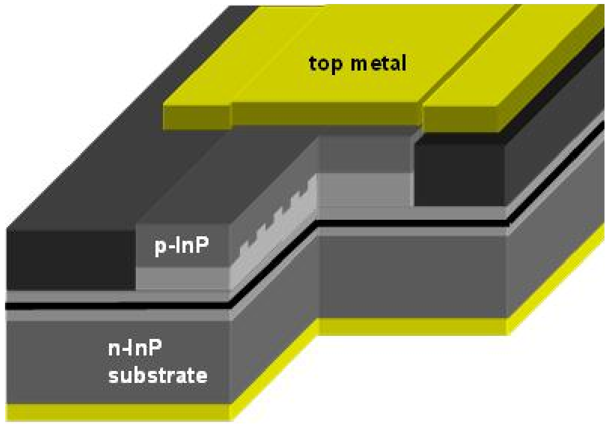
$$\omega_g = \Delta\omega_{\text{DBR}}\big|_{L\to\infty} = 2v_g|\kappa|$$

In crystals, the bandgap in the electron energy spectrum comes about as a result of the Bragg scattering of electrons from the periodic atomic potential and the magnitude of the bandgap is proportional to the strength of the scattering potential. In DFB structures, the photonic bandgap also results from the Bragg scattering of electromagnetic waves from the periodic index of the medium, and the strength of the bandgap also depends on the strength of the index variations as captured by the coupling constant κ .

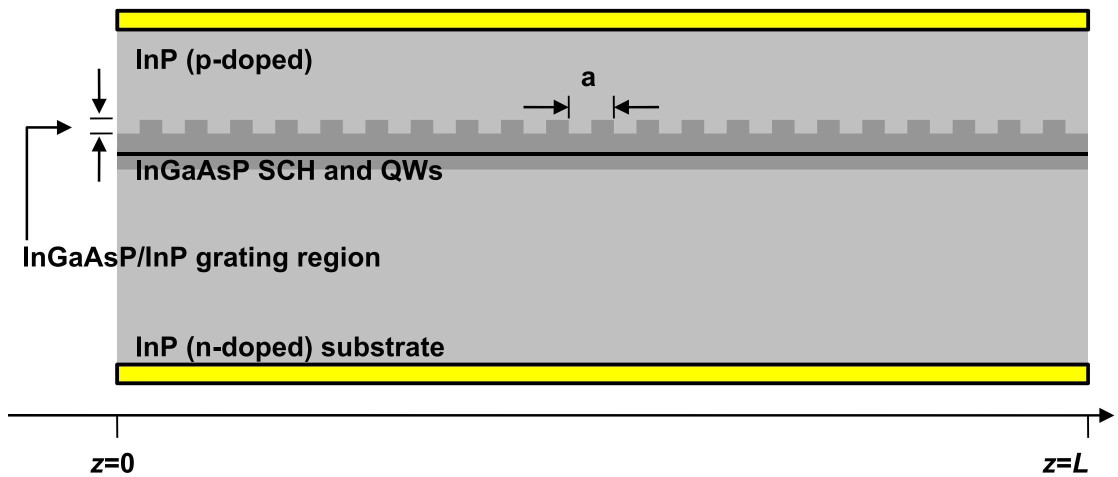
13.2 Distributed Feedback (DFB) Lasers (1D Photonic Crystal Lasers)

13.2.1 Introduction:

The structure of a DFB laser is shown in the Figures below. The laser cavity is not like any we have seen before. There is no distinction between the optical cavity and the mirrors. The DFB grating provides back reflection that keeps the photons from escaping from the two end facets. The facets are assumed to be perfectly AR coated and provide no reflection. The laser cavity "minors" are "distributed" along the entire length of the cavity. The techniques developed in the last section are adequate to analyze lasing in DFB lasers. Analyzing a laser involves at least: (i) finding the frequencies of the lasing modes, (ii) finding the threshold gain $\tilde{g}_{th}$ and the photon lifetime of each mode, and (iii) finding the output coupling efficiency η_o .



A DFB Laser



13.2.2 DFB Laser Analysis:

For the waveguide cavity shown above, photon lifetime is related to the threshold gain $\tilde{g}_{th}$ by the familiar relation:

$$\Gamma_a v_g \tilde{g}_{th} = \frac{1}{\tau_p}$$

Photon lifetime is related to the two different kinds of losses; mirror or external losses, and cavity internal losses,

$$\frac{1}{\tau_p} = v_g(\tilde{\alpha}_m + \tilde{\alpha})$$

To analyze the DFB laser shown above, we first assume $\tilde{\alpha} = 0$ (i.e. no material losses in any region) and calculate the threshold gain, $\tilde{g}_{th}$. From the previous Section, the electric field and the magnetic field are,

$$\vec{E}(x,y,z) = A_+(z)\,\vec{E}(x,y)e^{i\beta z} + A_-(z)\,\vec{E}^*(x,y)e^{-i\beta z}$$

$$\vec{H}(x,y,z) = A_+(z)\,\vec{H}(x,y)e^{i\beta z} - A_-(z)\,\vec{H}^*(x,y)e^{-i\beta z}$$

The propagation vector β now includes the modal gain,

$$\beta = \beta' + i\beta'' = \frac{\omega}{c} n_{eff} - i\,\Gamma_a \frac{\tilde{g}}{2}$$

The solution obtained in the previous Section for the boundary conditions, $B_-(L) = 0$ and $B_+(0) \neq 0$, was,

$$B_+(z) = \frac{-\Delta\beta \sinh[s(z-L)] + is\cosh[s(z-L)]}{\Delta\beta \sinh(sL) + is\cosh(sL)} B_+(0)$$
$$B_-(z) = \frac{\kappa^* \sinh[s(z-L)]}{\Delta\beta \sinh(sL) + is\cosh(sL)} B_+(0)$$

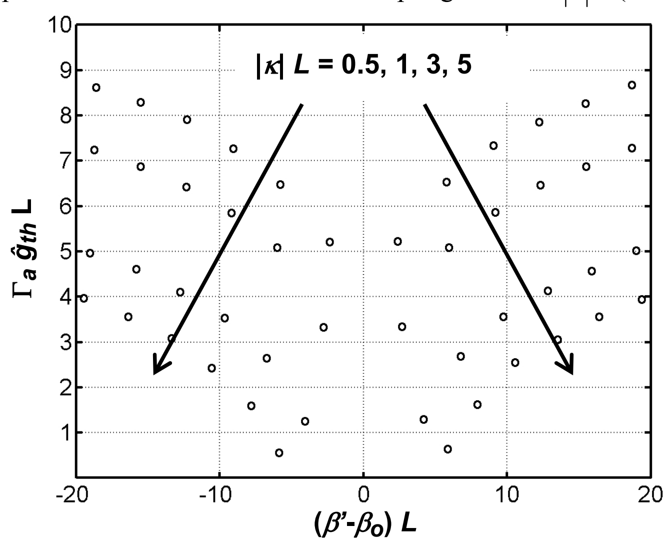
Here,

$$\Delta\beta = \beta - \beta_o = \beta' - i\Gamma_a \frac{\tilde{g}}{2} - \beta_o$$
$$s = \sqrt{|\kappa|^2 - \Delta\beta^2}.$$

Therefore, $\Delta\beta$ is now complex. Recall from Chapter 12 that the condition for lasing is that light comes out of the device when no light goes into the device. This can happen if $B_+(L) \neq 0$ and $B_-(0) \neq 0$ when both $B_+(0) = 0$ and $B_-(L) = 0$. Using the expressions given above, it is not difficult to see that lasing implies,

$$\Delta\beta \sinh(sL) + is\cosh(sL) = 0$$

This a complex equation. The real and imaginary parts of the expression on the left hand side must separately equal zero. This gives us two equations. We have two unknowns; the threshold gain $\tilde{g}_{th}$ and the frequency (or the value of β') of the lasing mode. Solution of the above equation gives multiple pairs. A pair is a value for the lasing mode propagation vector β' and a corresponding value for the threshold gain $\tilde{g}_{th}$. The solutions $(\beta', \tilde{g}_{th})$ are shown in the Figure below as circles in a $(\beta' - \beta_o) - \Gamma_a \tilde{g}_{th} L$ plane for different values of the coupling constant $|\kappa| L$ (assuming $\tilde{\alpha} = 0$).



As is the case in all lasers, the modes with the lowest threshold gain lase, and the other modes do not lase. For any given value of $|\kappa| L$, the two modes that have the lowest threshold gain are the ones whose β' values are located closest to the $(\beta' - \beta_o)L = 0$ axis. (i.e. those modes whose β' values are closest to β_o). Also note that there are no lasing modes with frequencies (or β' values) within the bandgap of the DFB structure. The two modes with the lowest threshold gain are symmetrically located on the edges of the photonic bandgap. Also note that the threshold gain goes down with the increase in the value of the coupling constant (i.e. with the increase in the grating strength).

Once we have determined $\tilde{g}_{th}$, the photon lifetime τ_p and the mirror loss $\tilde{\alpha}_m$ can be found from the relation,

$$\frac{1}{\tau_p} = v_g \tilde{\alpha}_m = \Gamma_a v_g \tilde{g}_{th}$$

Once $\tilde{\alpha}_m$ has been determined, we can introduce internal cavity loss in the following way. The complex propagation vector in the presence of loss is,

$$\beta = \beta' + i\beta'' = \frac{\omega}{c} n_{eff} - i\,\Gamma_a \frac{\tilde{g}}{2} + i\,\frac{\tilde{\alpha}}{2}$$

The photon lifetime with material loss is,

$$\frac{1}{\tau_p} = v_g (\tilde{\alpha}_m + \tilde{\alpha})$$

And the threshold gain in the presence of material loss is then,

$$\Gamma_a v_g \tilde{g}_{th} = \frac{1}{\tau_p} = v_g (\tilde{\alpha}_m + \tilde{\alpha})$$

The output coupling efficiency η_o is,

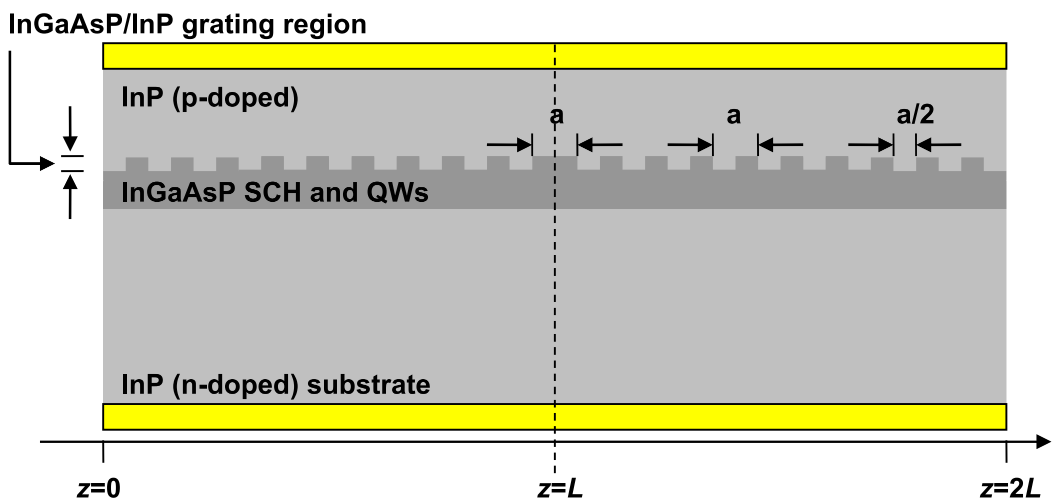
$$\eta_o = \frac{\tilde{\alpha}_m}{\tilde{\alpha}_m + \tilde{\alpha}}$$

From the analysis so far it seems that DFB lasers should have two modes lasing at the same time. In real DFB lasers, only one of these two modes ends up lasing. Effects such as structural imperfections, spatial hole burning, and imperfect and unequal AR coatings end up decreasing the threshold gain of one of these two modes compared to that of the other one.

13.3 Quarter-Wave Shifted (QWS) Distributed Feedback (DFB) Lasers (1D Photonic Crystal Defect Lasers)

13.3.1 Introduction:

A QWS DFB laser is realized by shifting the grating on the left half of the device by one-quarter of a wavelength with respect to the grating on the right half of the device, as shown in the Figure below, thereby creating a "crystal defect".



We know from solid state physics that crystal defects can have energies within the bandgap. The frequency of the lasing mode in a QWS DFB laser is also within the bandgap of the DFB structure and the intensity of the lasing mode is localized near the defect. One can also think of the QWS DFB laser as being similar to a VCSEL in which a half-wavelength long cavity is sandwiched between two DBR mirrors. A quarter-wave shifted (QWS) DFB laser has the advantage that it has a unique mode, the defect mode, that has the lowest threshold gain compared to all the other modes, and this mode always ends up being the lasing mode. Therefore, the lasing frequency can be designed with high accuracy without being at the mercy of structural imperfections.

13.3.2 QWS DFB Laser Analysis:

One can obtain the threshold gain of the QWS DFB laser by the same method as used in the previous section for ordinary DFB lasers. Here, we will use a different technique. We assume that the laser facets have perfect AR coatings. If one stands right in the middle of the laser cavity at $z = L$ and calculates the reflection coefficient r_R looking towards the right half of the cavity the answer, obtained earlier, is,

$$r_R = \frac{-\kappa^* \sinh(sL)}{\Delta\beta \sinh(sL) + is\cosh(sL)}$$

Similarly, the reflection coefficient looking towards the left will be exactly the same,

$$r_L = \frac{-\kappa^* \sinh(sL)}{\Delta\beta \sinh(sL) + is\cosh(sL)}$$

Now imagine there is a cavity of zero length located right at $z = L$. The roundtrip condition for lasing for this cavity will be simply,

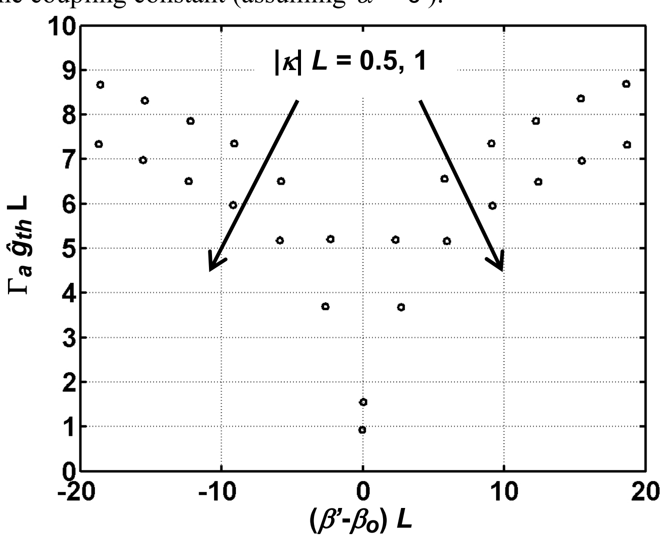
$$r_L r_R = 1$$

This gives,

$$\left[\frac{-\kappa^* \sinh(sL)}{\Delta\beta \sinh(sL) + is\cosh(sL)}\right]^2 = 1$$

$$\Rightarrow \frac{-\kappa^* \sinh(sL)}{\Delta\beta \sinh(sL) + is\cosh(sL)} \pm 1 = 0$$

This is again a complex equation. Depending on the phase of the coupling constant, only one choice of the sign will give physically correct answers. The real and imaginary parts of the expression on the left hand side must separately be equal to zero. This gives us two equations. We have two unknowns; the threshold gain $\tilde{g}_{th}$ and the frequency (or the value of β') of the lasing mode. Solution of the above equation gives multiple pairs. A pair is a value for the lasing mode propagation vector β' and a corresponding value for the threshold gain $\tilde{g}_{th}$. The results are depicted in the Figure below for two different values of the coupling constant (assuming $\tilde{\alpha} = 0$).



Note that in addition to the usual modes that appear on either side of the bandgap, there is a single mode with frequency right in the middle of the bandgap. This is the mode localized at the defect and has a threshold gain much smaller than all the other modes and is therefore the only mode that lases.

Once we have found $\tilde{g}_{th}$, the photon lifetime τ_p and the mirror loss $\tilde{\alpha}_m$ can be found from the relation,

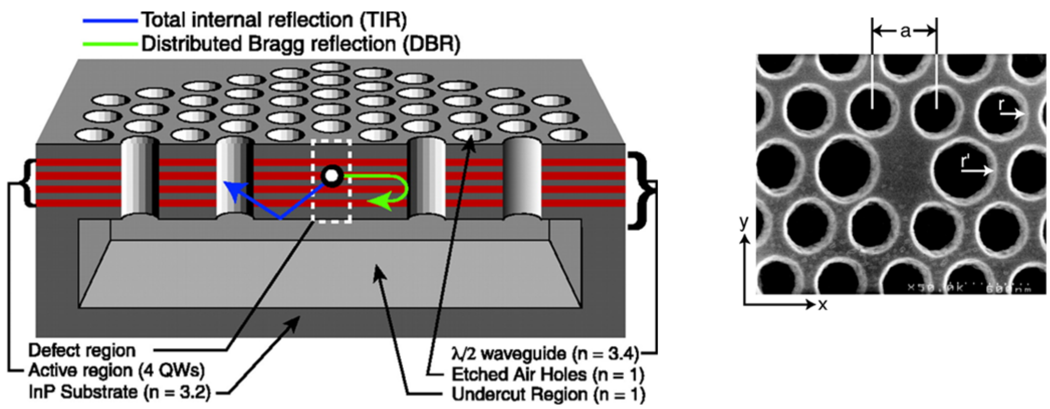
$$\frac{1}{\tau_p} = v_g \tilde{\alpha}_m = \Gamma_a v_g \tilde{g}_{th}$$

And once $\tilde{\alpha}_m$ has been determined, we can introduce internal cavity loss in the same way as was done in the case of DFB lasers without a quarter–wave shift. QWS DFB lasers have become important in modern wave division multiplexed (WDM) fiber optic communication systems that use dense wavelength packing which requires very precise control on the wavelengths of the lasers.

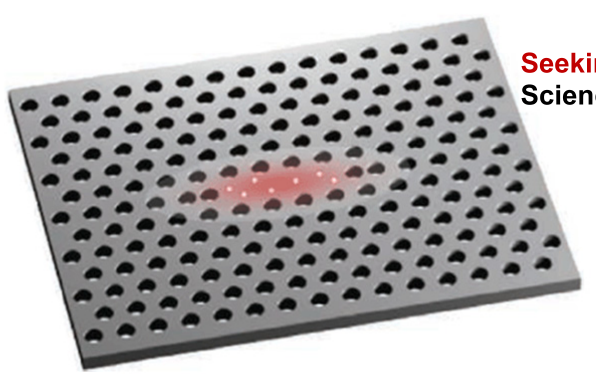
13.4 2D Photonic Crystal Defect Lasers

13.4.1 Introduction:

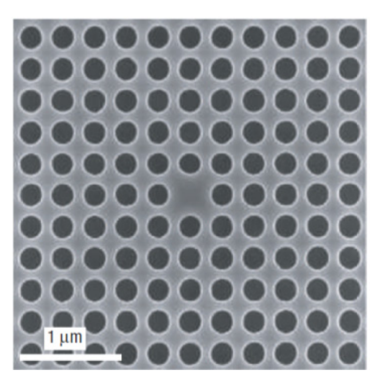
Two dimensional photonic crystal (2D PC) lasers have also been realized in the last decade. Although, 2D PC lasers require numerical tools for their analysis, the basic principles are the same as in the case of the 1D PC lasers (DFB lasers). Some examples of 2D PC lasers with defects are shown in the Figures below. In two dimensions, one has more freedom to tailor defects which localize the lasing mode.



Two-Dimensional Photonic Bandgap Defect Mode Laser
Science, 11 June (1999).



Seeking the Ultimate Nanolaser
Science, 13 October (2006)



Ultrafast Photonic Crystal Nanocavity Laser
Nature Physics, July (2006)

DBR mirror:

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Suppose an incident wave is coming from the left side. $\Rightarrow B^{+}(0) \neq 0$ and $B^{-}(L) = 0$. These are the two boundary conditions needed to determine c_1 and c_2 in (2). The result is (note: $iq = -s$):

$$B^{+}(z) = \frac{-\Delta\beta \sinh[s(z-L)] + is\cosh[s(z-l)]}{\Delta\beta \sinh(SL) + is\cosh(SL)} B^{+}(0)$$

$$B^{-}(z) = \frac{k^{*} \sinh[s(z-L)]}{\Delta\beta \sinh(SL) + is\cosh(SL)} B^{+}(0)$$

The minor reflection coefficient r is

$$r = \frac{B^{-}(0)}{B^{+}(0)} = \frac{-k^{*} \sinh s(zL)}{\Delta\beta \sinh(SL) + is\cosh(SL)}$$

and the reflectivity R is $|r|^2$.

$t = \dfrac{B^{+}(L)e^{i\beta_0 L}}{B^{+}(0)}$ note the $ei^{\beta_0 L}$ factor – since the reflection + transmission coefficients are defined for the actual electric fields. The reflectivity R is maximum when $\Delta\beta = \beta - \beta_0 = 0$ (i.e. when the input wave's β corresponds to the Bragg vector β_0 of the grating).

$R_{max} = \tanh^2(|k|L) \Rightarrow R_{max}$ increases with the value of the product $|k|L$. On the next page R is plotted as a function of $\Delta\beta L$ for different values of $|k|L$. Note that R goes to zero when $SL = in\pi \Rightarrow (\Delta\beta)^2 - |k|^2 = \dfrac{n^2\pi^2}{L^2} \Rightarrow (\beta-\beta_0)^2 = |k|^2 + n^2\left(\dfrac{\pi}{L}\right)^2$, and the first null occurs when $(\beta-\beta_0) = \sqrt{|k|^2 + \left(\dfrac{\pi}{L}\right)^2} \Rightarrow$ Reflection bandwidth increases with $|k|$.

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DBR Lasers

DBR lasers use DBR mirrors in a waveguide geometry, as shown below:

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The length of the active region is L (there are no quantum wells in the DBR mirrors). We shall assume that the grating is shallow and presents a small perturbation to the waveguide defined by $\varepsilon_{avg}(x,y)$. The lasting condition is:

$$\Gamma_{xy}\,\tilde{g}_{th} - \alpha_i = \frac{1}{L}\ln\left(\frac{1}{\sqrt{R_1 R_2}}\right)$$

where α_i = Cavity intrinsic loss V_g = group velocity of mode in the optical cavity L = length of the cavity and the photon lifetime τ_p is:

$$\frac{1}{\tau_p} = \sqrt{xy}V_g\,\tilde{g}_{th} = \frac{V_g}{L}\ln\left(\frac{1}{\sqrt{R_1 R_2}}\right) + V_g\,\alpha_i = V_g\,(\alpha_i + \alpha_m)$$

Note that the reflectivities R_1 and R_2 depend on the value of β (or frequency ω since $\beta = \frac{\omega}{c}n_{eff}(\omega)$). Consequently, the threshold gain $\tilde{g}_{th}$ will also depend on frequency ω.
Longitudinal modes of the cavity whose frequencies fall within the reflection bandwidth of the DBR mirrors see high reflectivity mirrors while those whose frequencies fall outside the reflection bandwidth of the mirrors see low reflectivity mirrors.
The questions then are: what are the frequencies of the longitudinal modes? How many longitudinal modes are present within the reflection bandwidth of the minus? What is the frequency spacing of the longitudinal modes?
Let the DBR mirror reflection coefficients be written as

$$r_1 = \sqrt{R_1}e^{i\phi_1} \qquad r_2 = \sqrt{R_2}e^{i\phi_2}$$

The roundtrip phase condition is then:

$$2\beta L + \phi_1 + \phi_2 = 2n\pi \qquad \{n = 1, 2, 3\cdots\}$$

But note that ϕ_1 and ϕ_2 depend on β for DBR mirrors, i.e.

$$2\beta L + \phi_1(\beta) + \phi_2(\beta) = 2n\pi$$

So its not that trivial to use the above equation to determine the β values (or the frequencies) of the cavity longitudinal modes.
But we know that for a DBR mirror that is AR coated on the other end the mirror reflection coefficient is:

$$r_2 = \frac{-k^*\sinh(SL_2)}{\Delta\beta\sinh(SL_2) + iS\cosh(SL_2)} \qquad where \begin{cases} \Delta\beta = \beta - \beta_0 \\ S = \sqrt{|k|^2 - \Delta\beta^2} \end{cases}$$

Since r_2 has a small bandwidth and is significant for only those values of β that are close to β_0, we may expand the phase of r_2, i.e. ϕ_2, around β_0.

$$\phi_2(\beta) = \phi_2(\beta_0) + \left.\frac{\partial\phi_2}{\partial\beta}\right|_{\beta_0}(\beta - \beta_0)$$

But

$$\phi_2(\beta) = \pi - \phi_{k2} - \tan^{-1}\left\{\frac{S\cosh(SL_2)}{\Delta\beta\sinh(SL_2)}\right\}$$

where $k = |k|e^{i\phi k_2}$, $k^* = |k|e^{-i\phi k_2}$, and the grating phase ϕ_k is some important constant phase.

$$\Rightarrow \phi_2(\beta_0) = \pi - \phi_{k_1} - \frac{\pi}{2} = \frac{\pi}{2} - \phi_{k_1}$$

$$\left.\frac{\partial\phi_2}{\partial\beta}\right|_{\beta_0} = \frac{\tanh(|k|L_2)}{|K|}$$

Define an effective grating length L_{eff} as:

$$L_{eff\,2} = \left.\frac{1}{2}\frac{\partial\phi_2}{\partial\beta}\right|_{\beta_0} = \frac{\tanh|k|L_2}{2|k|}$$

The roundtrip phase condition becomes:

$$2\beta L + \phi_1(\beta) + \phi_2(\beta) = 2n\pi$$

$$2\beta L + \pi - (\phi_{k_1} + \phi_{k_2}) + 2(\beta - \beta_0)L_{eff\,1} + 2(\beta - \beta_0))L_{eff\,2} = 2n\pi.$$

The above equation can be solved for the allowed values of β (that are near β_0).

Mode Spacing & Mode Selection:

The spacing δ_β between the cavity modes follows from the above equation:

$$\delta_\beta [2L + 2L_{eff\,1} + 2L_{eff\,2}] = 2\pi$$

$$\delta_\beta = \frac{\pi}{L + L_{eff\,1} + L_{eff\,2}}$$

From earlier analysis, the reflectivity bandwidths of the DBR mirrors are:

$$\delta_{\beta\, bandwidth,2} = 2\sqrt{|k|^2 + \left(\frac{\pi}{L_2}\right)^2}, \delta_{\beta\, bandwidth,1} = 2\sqrt{|k|^2 + \left(\frac{\pi}{L_1}\right)^2}$$

There can be many longitudinal modes lasing at the same time if the cavity mode spacing δ_β is much smaller than the DBR mirrors' reflectivity bandwidths. Due can have only a single longitudinal mode lasing in the cavity if δ_β is equal to a larger than the reflectivity bandwidths of the DBR mirrors. We have,

$$\delta_\beta = \frac{\pi}{L + L_{eff\,1} + L_{eff\,2}} \underbrace{<<}_{<< \left\{\delta_{\beta\, bandwidth,2}, \delta_{\beta\, bandwidth,1}\right\}} \left\{2\sqrt{|k|^2 + \left(\frac{\pi}{L_2}\right)^2}, 2\sqrt{|k|^2 + \left(\frac{\pi}{L_1}\right)^2}\right\}$$

$\Rightarrow$ Multiple longitudinal mode lasing. And,

$$\delta_\beta = \frac{\pi}{L + L_{eff\,1} + L_{eff\,2}} \geq \delta_{\beta\, bandwidth,2}, \delta_{\beta\, bandwidth,1}$$

$\Rightarrow$ Single longitudinal mode lasing.

The spacing in frequency between the cavity longitudinal modes can be obtained as follows:

$$\delta_\beta = \frac{\pi}{L + L_{eff\,1} + L_{eff\,2}}$$

$$\frac{\delta_\beta}{\delta\omega}\delta\omega = \frac{\pi}{L + L_{eff\,1} + L_{eff\,2}}$$

$$\Rightarrow \delta\omega = V_g \frac{\pi}{L + L_{eff\,1} + L_{eff\,2}}$$

In the same way, the reflectivity band widths of the DBR mirrors in frequency are:

$$\delta_{\omega bandwidth,2} = 2V_g\sqrt{|k|^2 + \left(\frac{\pi}{L_2}\right)^2}$$

$$\delta_{\omega bandwidth,1} = 2V_g\sqrt{|k|^2 + \left(\frac{\pi}{L_1}\right)^2}$$

Single Mode Operation:

Longitudinal mode selection (for lasing) in single mode DBR lasers can be understood as follows: Suppose there are three longitudinal modes within the reflectivity bandwidth of the DBR mirrors (as shown on the next page).

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The longitudinal mode in the center sees the highest reflectivity from the DBR mirrors and so has the smallest value of the threshold gain $\tilde{g}_{th}$. The two modes on either side of the center mode see lower mirror reflectivities and, consequently, have higher threshold gains compared to the center mode. When the laser is electrically pumped and the gain $\tilde{g}$ becomes equal to the threshold gain $\tilde{g}_{th}$ of the center mode, the center mode starts lasing. When the center mode starts lasing, the carrier density in the gain region gets clamped to the value N_{th} corresponding to the threshold gain $\tilde{g}_{th}$ of the center mode. If the current is increased, further, the photon density in the center mode increases but the carrier density remains fixed at N_{th}. The gain of the active region also, therefore, remains fixed at the threshold gain $\tilde{g}_{th}$ of the center mode. The gain of the active religion, therefore, never becomes large enough so that the side modes can also large. In short, the clamping of the carrier density (and gain) at the onset of the lasing of the center mode prevents the side modes (which have higher threshold gains) from lasing.