INSTRUCTIONS:

- You must show all the relevant work
- Correct answers with wrong reasoning will not get points
- Correct answers with no reasoning will also not get points
- All questions do not carry equal points
- All questions do not have the same level of difficulty
- You are only allowed to consult the course material (no other texts and material is allowed)

Problem 1: 20 points
Problem 2: 20 points
Problem 3: 20 points
Problem 4: 40 points
Problem 5: 20 points

Total: 120 points
Problem 1: (Semiconductor lasers)

a) Consider a semiconductor Fabry-Perot shown in the Figure below.

Suppose the laser is lasing in a particular longitudinal mode and the lasing wavelength is \( \lambda \) (and equal to 1550 nm). Suppose the refractive index of the active region changes by \( \Delta n \) because of temperature change. Find the magnitude and sign of the change in the lasing wavelength \( \Delta \lambda \) in terms of \( \Delta n \). Define all variables that you use in your answer. (5 points)

Now for parts (b) and (c), consider a two-section semiconductor laser shown below.

The active region (i.e. the quantum wells) have been removed from the SCH region of section 2. All quantities belonging to section 1 can be labeled by a subscript 1 and those of section 2 by the subscript 2. For example, the confinement factor of the SCH region of the waveguide in section 1 and section 2 are \( \Gamma_{s1} \) and \( \Gamma_{s2} \), respectively. The confinement factor for the active (gain) region in section 1 is \( \Gamma_{a1} \). The roundtrip condition for lasing can be written as,

\[
\Gamma_{a1}\tilde{g}_1L_1 - \tilde{\alpha}_1L_1 - \tilde{\alpha}_2L_2 = \log\left(\frac{1}{R}\right)
\]

The advantage of the two-section device is that carriers can be injected separately into Section 2 pn-junction via an applied current thereby increasing the carrier density in only section 2 SCH region without disturbing section 1.

b) Suppose the laser is lasing in a particular longitudinal mode and the lasing wavelength is \( \lambda \) (and equal to 1550 nm). Suppose the refractive index of the SCH region in section 2 changes by \( \Delta n_{s2} \) because of carrier injection via current. Find the magnitude and sign of the change in the lasing wavelength \( \Delta \lambda \) in terms of \( \Delta n_{s2} \). Define all variables that you use in your answer. (5 points)
c) The $\alpha_h$ parameter (the subscript “h” is introduced in order to avoid confusion with loss, and stands for Charles Henry who first introduced this alpha-parameter) relates the changes in the real and the imaginary parts of the refractive index,

$$\alpha_h = \frac{\Delta n'_r}{\Delta n''}$$

If the real part of the refractive index in the SCH region in section 2 changes by $\Delta n_{s2}$ because of carrier injection, then the corresponding change in the material loss (related to the imaginary part of the index) would be,

$$\Delta \alpha_{s2} = 2 \frac{\omega}{c} \frac{\Delta n_{s2}}{\alpha_h}$$

And the change in the waveguide modal loss in section 2 would be,

$$\Delta \tilde{\alpha}_{s2} = \Gamma_{s2} \Delta \tilde{\alpha}_{s2}$$

The change in the loss would spoil the laser roundtrip condition (would change the laser threshold gain, in fact) and the carrier density in the gain region would have to adjust to compensate. After all this has happened, find the magnitude and sign of the change in the lasing wavelength $\Delta \lambda$ in terms of $\Delta n_{s2}$. Define all the variables that you use in your answer. (5 points)

d) Consider now a semiconductor DFB laser shown in the Figure below and identical to the one in homework problem 8.4.

Suppose the refractive index of the active region changes by $\Delta n$ because of temperature change. Find the magnitude and sign of the change in the lasing wavelength $\Delta \lambda$ in terms of $\Delta n$. Define all variables that you use in your answer. Refer to homework problem 8.4 if you need details of the laser structure. (5 points)
Problem 2: (Optical transitions in semiconductor quantum wires)

Quantum wires are like quantum wells except that electrons are confined in two dimensions and free in just one dimension. A quantum wire is shown below.

Consider a quantum wire as shown above. The wire has widths $L_x$ and $L_y$ in x- and y-directions, respectively, and runs infinitely long the z-direction. The wire material is 2 and the barrier material is 1. The bulk energy dispersion for each material is,

$$E_{c1}(k) = E_{c1} + \frac{\hbar^2}{2m_{e1}} \left( k_x^2 + k_y^2 + k_z^2 \right)$$

$$E_{c2}(k) = E_{c2} + \frac{\hbar^2}{2m_{e2}} \left( k_x^2 + k_y^2 + k_z^2 \right)$$

$$E_{v1}(k) = E_{v1} - \frac{\hbar^2}{2m_{h1}} \left( k_x^2 + k_y^2 + k_z^2 \right)$$

$$E_{v2}(k) = E_{v2} - \frac{\hbar^2}{2m_{h2}} \left( k_x^2 + k_y^2 + k_z^2 \right)$$

The valence band is assumed to be the heavy-hole band. Electrons in the conduction and valence bands are free in the z-direction and quantum confined in the x- and y-directions. The wavefunctions in the effective mass approximation for the conduction and valence band states near the wavevector $k_0 = 0$ can be written as,

$$\psi_{c,s,k_z}(\vec{r}) = \frac{e^{ik_zz}}{\sqrt{L_z}} \phi^c_{s}(x,y) \sqrt{V} \psi_{c,k_o}(\vec{r})$$

$\{ s = 1, 2, 3, ... \}$
\[ \psi_{v,p,k} \equiv \frac{e^{ik_x z}}{\sqrt{L_z}} \phi_p(x,y) \sqrt{\psi_{v,k_0}}(\hat{r}) \{ p = 1, 2, 3, \ldots \} \]

The envelope functions \( \phi^c(x,y) \) and \( \phi^v(x,y) \) satisfy the two dimensional effective mass Schrödinger equation. Suppose the effective mass equation has been solved and the quantized energy levels and the envelope functions have been obtained and the result is,

\[ E_s(s,k_z) = E_s^{c2} + E_s^{c} + \frac{\hbar^2 k_z^2}{2m^*_2} \{ s = 1, 2, 3 \ldots \} \]
\[ E_p(p,k_z) = E_p^{v2} - E_p^{v} - \frac{\hbar^2 k_z^2}{2m^*_2} \{ p = 1, 2, 3 \ldots \} \]

The band gap is \( E_g \). The optical momentum matrix element is (assuming the light polarization unit vector to be \( \hat{n} \)),

\[ \langle \hat{p}_{cv}(k_0, k_0) \cdot \hat{n} \rangle^2 = \frac{m E_p}{4} \left( \frac{n_x^2 + n_y^2}{2} + n_z^2 \right) \]

The material index of both the materials is assumed to be \( n \). The group index is \( n_g^M \). A plane wave polarized in the z-direction and propagating in the x-direction and with photon density \( n_p \) and optical frequency \( \omega \) is incident on the quantum wire. The conduction band is empty and the valence band is completely full of electrons.

a) Calculate an analytical expression for the photon absorption rate per unit volume in the quantum wire (i.e. the number of stimulated absorption transitions per unit volume per second). There should be no undetermined constants or unevaluated integrals in your expression. State all the selection rules that come out of your analysis. Make sure your final answer has the correct units. (15 points)

b) Sketch the photon absorption rate per unit volume as a function of the photon energy \( h\omega \) and label all the important energies. (5 points)
Problem 3: (Semiconductor DFB lasers)

Consider a semiconductor DFB laser shown in the Figure below.

![DFB Laser Diagram](image)

The front facet of the DFB laser is AR coated to have zero reflectivity and the rear facet is HR coated to have a reflection coefficient of $r$ for a guided wave incident on the facet from the right, and a reflectivity of $R = r^2 = 0.8$. The rear facet is HR coated so that most of the light comes out from the front facet. The grating coupling constant is,

$$\kappa = |\kappa| e^{i\phi}$$

where $|\kappa|L = 2.0$. The active region confinement factor is $\Gamma_a$. The waveguide has no internal loss.

a) Find an analytic condition whose solution would give the frequency (or the propagation vector) of the lasing mode and the threshold gain. (10 points)

b) Generate scatter plots of $\Gamma_a g L$ vs $(\beta - \beta_0)L$ for two different values of the grating phase $\phi$; phase equal to 0-degrees and -90-degrees. (5 points)

c) Assuming the grating phase is zero degrees, and the laser is lasing in the mode with the lowest threshold gain, find the output coupling efficiency of the laser (numerical value needed) for output taken only from the front facet when the laser. (5 points)
Problem 4: (gain-grating DFB lasers)

In this problem you will look at a gain-grating DFB laser operating at wavelengths close to 1550 nm. The waveguide for the laser is shown below.

The thicknesses of various layers in the core region (SCH layers plus quantum wells) are shown in the figure below. The quantum well barriers and SCH layers have the same composition. The core region is undoped, and shown in detail below. The refractive indices of InP, InGaAs SCH layer, InGaAsP quantum well barrier, InGaAsP quantum well, and the insulator are 3.170, 3.386, 3.386, 3.550, and 1.70, respectively, at wavelengths close to 1550 nm.

The length of the laser cavity be \( L = 1000 \ \mu m \). The optical confinement factor \( \Gamma_{xy} \) for the quantum wells is 0.07. The effective index of the mode is 3.2, and the group index is 3.6. The two facets have perfect AR coatings.

The gain-gratings are formed by etching out the quantum wells in a periodic way (see the figure below) and substituting them with some material that has the same (real part of the) refractive index as the quantum wells (which is 3.55) but no gain. The net result is that the waveguide modal gain is not constant but a periodic function of distance along the z-direction. However, the (real part of the) refractive index is constant along the z-direction. The optical mode experiences Bragg reflection from this gain-grating.

The side view of the device is as shown below (in the y-z plane going through the center of the device). For this problem you should use the following approximate relation between the complex dielectric constant and the complex refractive index: \( \varepsilon_r + i\varepsilon_i = (n_r + in_i)^2 \approx n_r^2 + 2i n_r n_i \).
The period of the gain-grating is \( \Lambda = 2d \) and \( \beta_o = \pi/\Lambda \). The electric field in the waveguide is written as: \( E = \phi(x,y) \left[ B^+ (z) e^{i\beta_o z} + B^- (z) e^{-i\beta_o z} \right] \). The material gain in the quantum wells is \( g \). The modal gain in the sections of the waveguide where the quantum wells are present is \( \Gamma_{xy} \tilde{g} \), and it is zero in the sections where the quantum wells are removed. The intrinsic loss in the waveguide is assumed to be zero.

a) In the quantum wells, what is imaginary part \( \varepsilon_i \) of the dielectric constant in terms of the material gain \( g \)? (5 points)

b) What should be the period \( \Lambda \) of the gain-grating for maximum feedback at 1550 nm? (5 points)

c) Carefully derive the coupled mode equations for the slowly varying amplitudes \( B^+(z) \) and \( B^-(z) \) and write them in a matrix form? There should be no undetermined constants in your final result? (10 points)

d) Find the analytical condition whose solutions would give the values of the allowed longitudinal mode frequencies and the corresponding threshold gains. (10 points)

e) Generate a scatter plot of \( \Gamma_{xy} \tilde{g}_{th} L \) versus \( (\beta - \beta_o) L \) like the one in the lecture notes, and calculate the threshold gain \( \tilde{g}_{th} \) for the lasing mode (i.e. the longitudinal mode with the lowest threshold gain). (5 points)
Problem 5: (Plasmonic Nanowires)

Consider a metal nanowire. Assume it is infinitely long in the z-direction and has a radius equal to $R$.

The wire is illuminated by electromagnetic radiation polarized in the x-direction and propagating in the y-direction. The radiation excites the surface plasmon resonance frequency of the gold nanowire. Your goal is to find this surface plasmon resonance frequency. Assume that the bulk dielectric constant of the metal is:

$$\varepsilon(\omega) = \varepsilon_0 \left(1 - \frac{\omega_p^2}{\omega^2}\right)$$

You may or may not have noticed in the lecture handouts that for small metal objects (much smaller than a wavelength) field solutions for the plasmon mode inside and outside the metal appear as if they are derived from an electrostatic potential that satisfies Laplace's equation of electrostatics. For example, in the case of the small spherical metal particle, the solutions inside and outside correspond to the dipole-like solutions of Laplace's equation in spherical coordinates.

a) Write down trial expressions for the electric field of the desired plasmon mode inside and outside the gold nanowire with unknown constants. Your field expressions must have zero divergence in order to satisfy Laplace's equation. (10 points)

**Hint:** Work in cylindrical coordinates. You may consult your favorite undergraduate E&M book for this question.

b) Match appropriate boundary conditions and then derive an expression that gives the surface plasmon resonance frequency $\omega_{sp}$ in terms of the bulk plasmon frequency $\omega_p$. (10 points)