Polaritons

Polaritons consist of electromagnetic waves coupled with some material wave or material excitation

It is the name given to the phenomena where electromagnetic energy becomes strongly coupled with material degrees of freedom

Some common examples of polaritons are:

1) Phonon-Polaritons

Electromagnetic waves become strongly coupled with the optical phonons of a polar medium

2) Plasmon-Polaritons

Electromagnetic waves become strongly coupled with the plasma waves of a conducting medium

3) Exciton-Polaritons

Electromagnetic waves become strongly coupled with excitons (bound electron-hole pairs)

Maxwell's Equations for Polarizable Media

For any medium, Maxwell's equations are:

$$\vec{D} = \varepsilon_o \vec{E} + \vec{P} = \varepsilon_{eff}(\omega)\vec{E} \qquad \rho_p = -\nabla.\vec{P}$$

$$\nabla.\vec{E} = \frac{\rho_u + \rho_p}{\varepsilon_o} \qquad \nabla.\vec{D} = \frac{\rho_u}{\varepsilon_o}$$

$\rho_p =$ Charge density due to material polarization (paired charge density)

$\rho_u =$ Charge density due to free unpaired charge

When a medium is subjected to an electric field, it can polarize and charge dipoles are created and the charge density associated with these dipoles is described by ρ_p

External charge placed inside a medium is described by ρ_u

Longitudinal and Transverse Polaritons

Longitudinal Polaritons:

In longitudinal polaritons, the E-field has a non-zero divergence but the D-field has a zero divergence:

$$\nabla.\vec{E} = \frac{\rho_u + \rho_p}{\varepsilon_o} = \frac{\rho_p}{\varepsilon_o} \neq 0 \qquad \nabla.\vec{D} = \frac{\rho_u}{\varepsilon_o} = 0 \qquad \Rightarrow \qquad \rho_p = -\nabla.\vec{P} \neq 0$$

If the E-field has a wave-like form: $\vec{E} = \hat{n} E_o e^{i\vec{q}.\vec{r} - i\omega t}$

Then: $\nabla.\vec{E} \neq 0 \qquad \Rightarrow \qquad \vec{q}.\hat{n} \neq 0$

$\Rightarrow$ E-field has a non-zero component in the direction of wave propagation

Transverse Polaritons:

In transverse polaritons, the E-field and the D-field both have a zero divergence:

$$\nabla.\vec{E} = 0 \qquad \nabla.\vec{D} = 0 \qquad \Rightarrow \qquad \rho_p = 0$$

If the E-field has a wave-like form: $\vec{E} = \hat{n} E_o e^{i\vec{q}.\vec{r} - i\omega t}$

Then: $\nabla.\vec{E} = 0 \qquad \Rightarrow \qquad \vec{q}.\hat{n} = 0$

$\Rightarrow$ E-field has no component in the direction of wave propagation

Longitudinal Polaritons

Suppose the E-field has a wave-like form:

$$\vec{E} = \hat{n} E_o e^{i\vec{q}.\vec{r} - i\omega t}$$

The D-field is given as:

$$\vec{D} = \varepsilon_o \vec{E} + \vec{P} = \varepsilon_{eff}(\omega) \vec{E}$$

For longitudinal polaritons we must have:

$$\nabla.\vec{E} \neq 0$$
$$\nabla.\vec{D} = \varepsilon_{eff}(\omega) \nabla.\vec{E} = 0$$

The only way that both these equations can hold is if the frequency of the longitudinal polaritons is such that at that frequency:

$$\varepsilon_{eff}(\omega) = 0$$

The above equation gives the frequency of the longitudinal polaritons

Longitudinal Polaritons

Longitudinal Phonon-Polaritons: Consider a non-conducting polar medium (polar semiconductor or a polar insulator) whose dielectric constant at frequencies much smaller than the material bandgap energies is approximately,

$$\varepsilon_{eff}(\omega) = \varepsilon(\infty) - \frac{nf^2/M_r}{\omega^2 - \omega_{TO}^2} = \varepsilon(\infty) - \omega_{TO}^2 \frac{(\varepsilon(0) - \varepsilon(\infty))}{\omega^2 - \omega_{TO}^2} = \varepsilon(\infty)\left[\frac{\omega^2 - \omega_{LO}^2}{\omega^2 - \omega_{TO}^2}\right]$$

The condition, $\varepsilon_{eff}(\omega) = 0$ gives:

$$\omega = \omega_{LO}$$

The longitudinal phonon-polaritons are just the polar longitudinal optical phonons!

Longitudinal Plasmon-Polaritons: Consider a conducting medium (like gold, silver) whose dielectric constant at frequencies much larger than the phonon frequencies but much smaller than the material bandgap energies is approximately,

$$\varepsilon_{eff}(\omega) = \varepsilon(\infty) + i\frac{\sigma(\omega)}{\omega} = \varepsilon(\infty) + i\frac{ne^2\tau/m_e}{\omega(1 - i\omega\tau)} \approx \varepsilon(\infty) - \frac{ne^2/m_e}{\omega^2} \qquad \{\ \omega\tau >> 1$$

The condition, $\varepsilon_{eff}(\omega) = 0$ gives:

$$\omega = \omega_p = \sqrt{\frac{ne^2}{\varepsilon(\infty)m_e}}$$

The longitudinal plasmon-polaritons are just the plasma waves!

Transverse Polaritons

Suppose the E-field has a wave-like form:

$$\vec{E} = \hat{n}E_o e^{i\vec{q}.\vec{r} - i\omega t} \qquad \{\ \nabla.\vec{E} = 0 \quad \Rightarrow \quad \vec{q}.\hat{n} = 0$$

The D-field is given as:

$$\vec{D} = \varepsilon_o\vec{E} + \vec{P} = \varepsilon_{eff}(\omega)\vec{E}$$

For transverse polaritons we must have:

$$\nabla.\vec{E} = \nabla.\vec{D} = 0$$

The electromagnetic wave equation when $\nabla.\vec{E} = 0$ is:

$$\nabla \times \nabla \times \vec{E} = \omega^2 \mu_o \varepsilon_{eff}(\omega)\vec{E}$$
$$\Rightarrow \nabla\left(\cancelto{0}{\nabla.\vec{E}}\right) - \nabla^2\vec{E} = \omega^2 \mu_o \varepsilon_{eff}(\omega)\vec{E}$$
$$\Rightarrow -\nabla^2\vec{E} = \omega^2 \mu_o \varepsilon_{eff}(\omega)\vec{E}$$

The plane wave is a solution of the wave equation if:

$$\omega^2 \frac{\varepsilon_{eff}(\omega)}{\varepsilon_o} = q^2c^2$$

The above equation gives the dispersion of the transverse polaritons

Transverse Phonon-Polaritons

Consider a non-conducting polar medium (polar semiconductor or a polar insulator) whose dielectric constant at frequencies much smaller than the material bandgap energies is approximately,

$$\varepsilon_{eff}(\omega) = \varepsilon(\infty) - \frac{nf^2/M_r}{\omega^2 - \omega_{TO}^2} = \varepsilon(\infty) - \omega_{TO}^2 \frac{(\varepsilon(0) - \varepsilon(\infty))}{\omega^2 - \omega_{TO}^2} = \varepsilon(\infty)\left[\frac{\omega^2 - \omega_{LO}^2}{\omega^2 - \omega_{TO}^2}\right]$$

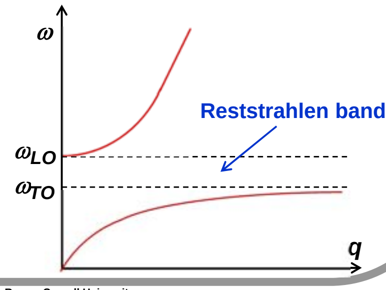
The dispersion relation: $\omega^2 \frac{\varepsilon_{eff}(\omega)}{\varepsilon_o} = q^2 c^2$

gives the following equation:

$$\omega^4 - \omega^2\left(\omega_{LO}^2 + q^2 c^2\, \varepsilon_o/\varepsilon(\infty)\right) + q^2 c^2 \omega_{TO}^2\, \varepsilon_o/\varepsilon(\infty) = 0$$

The resulting dispersion relation is plotted in the Figure

Note that there is a band of frequencies in which no electromagnetic wave can propagate in the medium
(no propagating transverse wave mode exists)
This band is called the Restsrahlen band



Transverse Plasmon-Polaritons

Consider a conducting medium (like gold, silver) whose dielectric constant at frequencies much larger than the phonon frequencies but much smaller than the material bandgap energies is approximately,

$$\varepsilon_{eff}(\omega) = \varepsilon(\infty) + i\frac{\sigma(\omega)}{\omega} = \varepsilon(\infty) + i\frac{ne^2\tau/m_e}{\omega(1 - i\omega\tau)} \approx \varepsilon(\infty) - \frac{ne^2/m_e}{\omega^2} \qquad \{\ \omega\tau >> 1$$

The dispersion relation: $\omega^2 \frac{\varepsilon_{eff}(\omega)}{\varepsilon_o} = q^2 c^2$

gives the following equation:

$$\omega^2 = \omega_p^2 + q^2 c^2 \frac{\varepsilon_o}{\varepsilon(\infty)} \qquad \left\{\ \omega_p = \sqrt{\frac{ne^2}{\varepsilon(\infty) m_e}}\right.$$

The resulting dispersion relation is plotted in the Figure

Note that no transverse electromagnetic wave can propagate in the medium with a frequency smaller than the plasma frequency