ECE4070 Homework 9 Solutions (By Farhan Rana)

Problem 9.2:

a) For graphene, $E(\vec{k}) = E_p \pm \hbar v \sqrt{(k_x - K_x)^2 + (k_y - K_y)^2}$

For zigzag nanotubes, $k_y = \frac{2\pi n}{ma}$. Suppose, $m = 3p$, then $k_y = \frac{2\pi n}{3pa}$. For $n = 2p$, $k_y = K_y$ and there is no bandgap. Now suppose $m = 3p \pm 1$. Then, $k_y - K_y = \frac{2\pi n}{(3p \pm 1)a} - \frac{4\pi}{3a}$. The smallest value of this difference will be when $n = 2p \pm 1$ and in this case,

$$k_y - K_y = \frac{2\pi(2p \pm 1)}{(3p \pm 1)a} - \frac{4\pi}{3a} = \mp \frac{4\pi}{3(3p \pm 1)a} = \pm \frac{2\pi}{3C} = \pm \frac{1}{3R}$$

The subband dispersions for this value of k_y are:

$$E(\vec{k}) = E_p \pm \hbar v \sqrt{(k_x - K_x)^2 + (1/3R)^2}$$

The bandgap is the difference between the energies of the conduction and valence subbands when $k_x = K_x$ and equals $2\hbar v/3R$. For a 1 nm radius nanotube, the bandgap is 0.44 eV.

b) Start from, $n = 4 \times \int_{-\infty}^{+\infty} \frac{dk_x}{2\pi} f(E(k_x) - E_f) = \frac{4}{\pi \hbar v} \int_{E_p + E_g/2}^{\infty} dE \frac{(E - E_p)}{\sqrt{(E - E_p)^2 - (E_g/2)^2}} f(E - E_f)$.

This implies, $g_{1D}(E) = \frac{4}{\pi \hbar v} \frac{(E - E_p)}{\sqrt{(E - E_p)^2 - (E_g/2)^2}}$.

c) At T=0K, $n = \frac{4}{\pi \hbar v} \int_{E_p + E_g/2}^{E_F} dE \frac{(E - E_p)}{\sqrt{(E - E_p)^2 - (E_g/2)^2}} = \frac{4}{\pi \hbar v} \sqrt{(E_F - E_p)^2 - (E_g/2)^2}$

d) Start from: $E(\vec{k}) = E_p \pm \hbar v \sqrt{(k_x - K_x)^2 + (1/3R)^2}$ and perform a Taylor expansion for small values of $(k_x - K_x)$ to get, $E(\vec{k}) \approx E_p \pm \frac{\hbar v}{3R}\left(1 + \frac{(k_x - K_x)^2}{2(1/3R)^2}\right) = E_p \pm \frac{\hbar v}{3R} \pm \hbar v \frac{(k_x - K_x)^2}{2(1/3R)}$. This implies, $m_e = m_h = \hbar/3Rv$. So the effective masses get smaller with increase in radius (or decrease in bandgap). This relation between bandgaps and effective masses is a common property of almost all semiconductor systems in 1D, 2D, and 3D.

Problem 9.3:

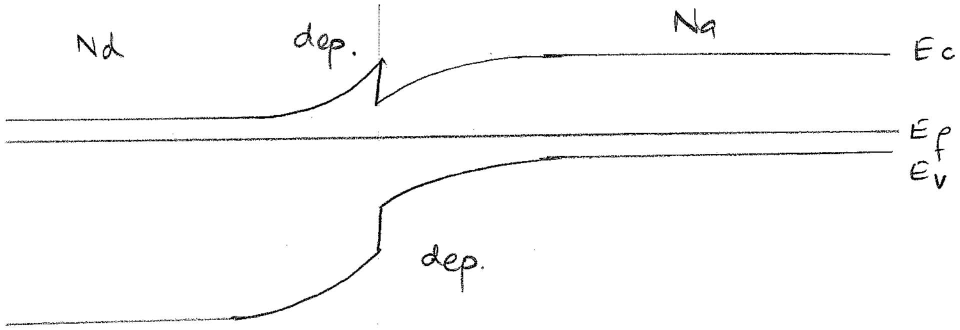
a+b) Use the expression from the handouts except that integration is now over 2D k-space and an extra factor of two comes in because of the two pockets in the FBZ:

$$
\begin{aligned}
R_{\uparrow} &= \frac{2\pi}{\hbar}\left(\frac{eA_o}{2m}\right)^2 \left\langle \left|\vec{P}_{cv}\,.\,\hat{n}\right|^2 \right\rangle \; 4\times\int \frac{d^2\vec{k}}{(2\pi)^2} \quad \delta\left(E_c\left(\vec{k}\right)-E_v\left(\vec{k}\right)-\hbar\omega\right) \\
&= \frac{2\pi}{\hbar}\left(\frac{e}{2m}\right)^2 \left(\frac{2\eta_o I_{inc}}{\omega^2}\right) \left\langle \left|\vec{P}_{cv}\,.\,\hat{n}\right|^2 \right\rangle \; 4\times\int \frac{d^2\vec{k}}{(2\pi)^2} \quad \delta\left(E_c\left(\vec{k}\right)-E_v\left(\vec{k}\right)-\hbar\omega\right) \\
&= \frac{2\pi}{\hbar}\left(\frac{e}{2m}\right)^2 \left(\frac{2\eta_o I_{inc}}{\omega^2}\right) \frac{m^2 v^2}{2} \; 4\times\int_0^\infty \frac{k\,dk}{(2\pi)}\,\delta\left(2\hbar v k-\hbar\omega\right) \\
&= \frac{e^2}{4\hbar}\eta_o\left(\frac{I_{inc}}{\hbar\omega}\right)
\end{aligned}
$$

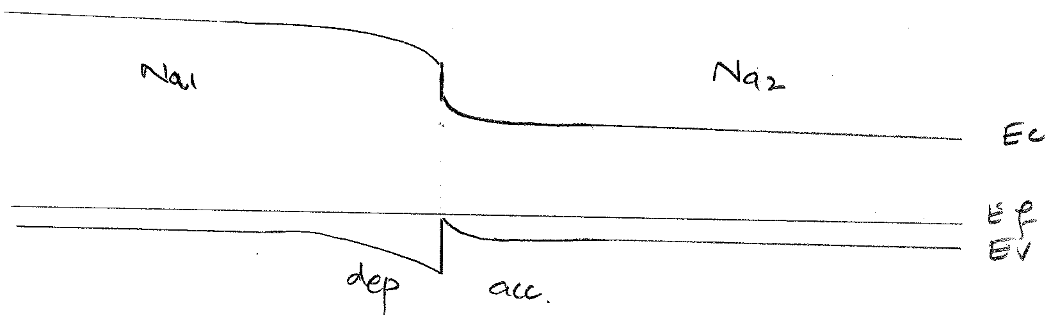
c) Incident photon flux per unit area is, $I_{inc}/\hbar\omega$. The photon absorption rate per unit area is $R_{\uparrow}$. Therefore, the fraction of incident photons absorbed in the graphene sheet is,
$\hbar\omega\, R_{\uparrow}/I_{inc} = \left(e^2/4\hbar\right)\eta_o \approx .023$.
It follows that ~2.3% of the incident photons are absorbed by graphene through interband transitions (irrespective of the wavelength!).

Problem 9.1

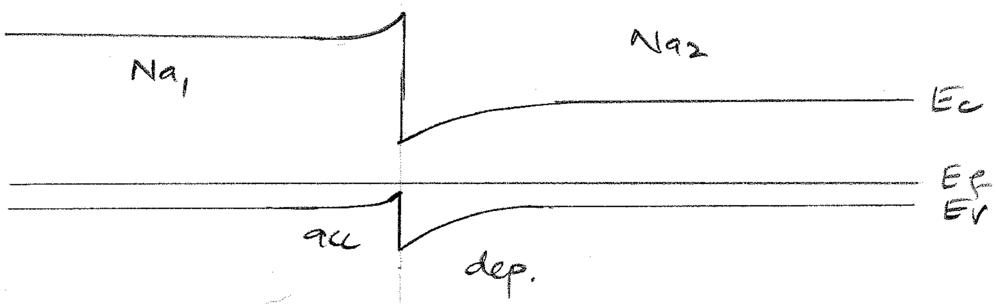
a)



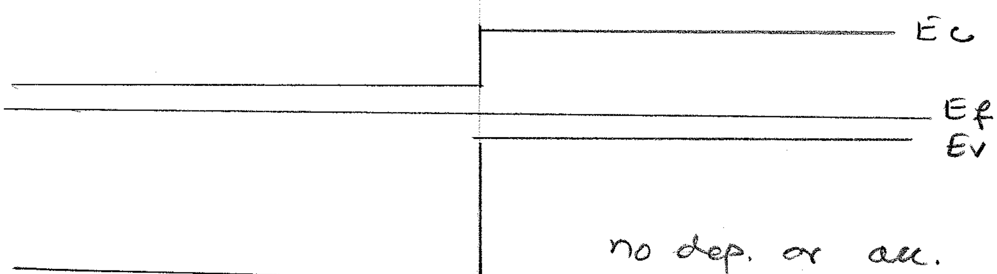
b)



c)

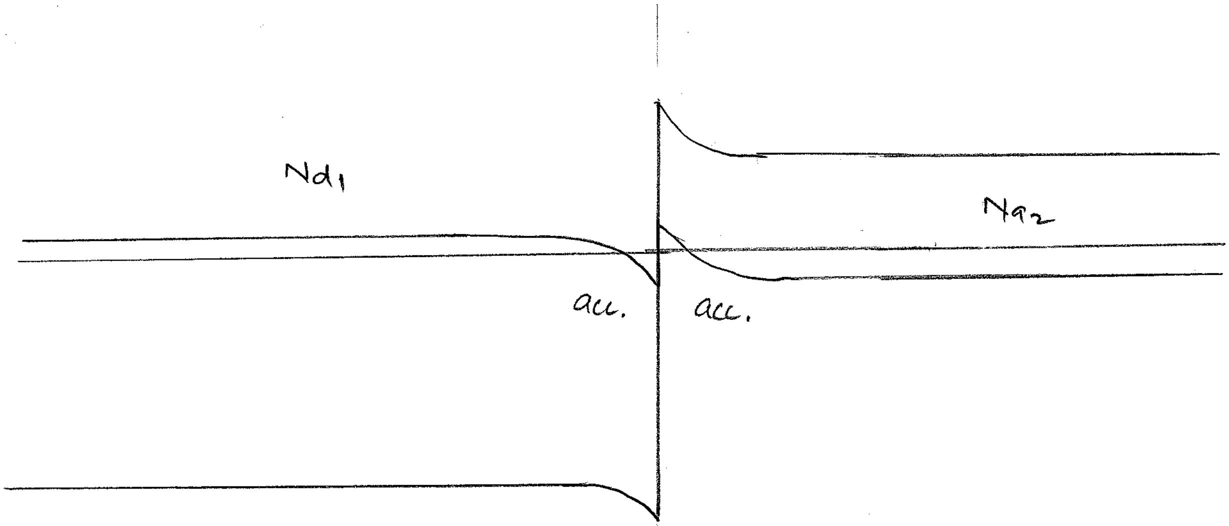


d) Fermi levels already aligned.



no dep. or acc. regions. $V_{bi} = 0$.

e)



f) Here you can have inversion Layers forming on one or both sides.