Department of Electrical and Computer Engineering, Cornell University

ECE 4070: Physics of Semiconductors and Nanostructures

Spring 2014

Homework 9 **Due on Fri, May 09, 2012 at 5:00 PM**

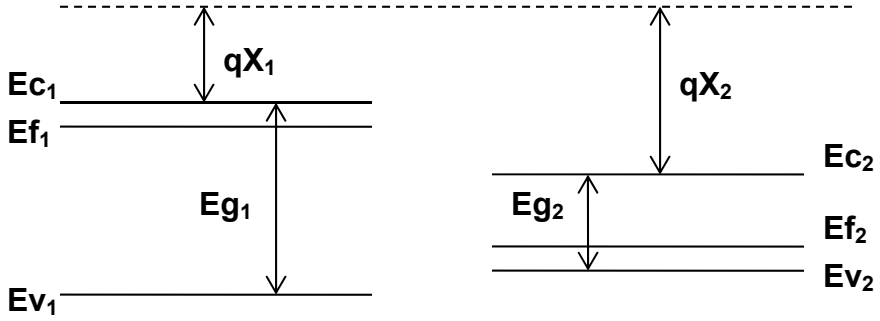
Suggested Readings:

a) Lecture notes
b) Note the change in the due date from Tuesday
c) This homework will count for 200 points total (double compared to other homework sets)

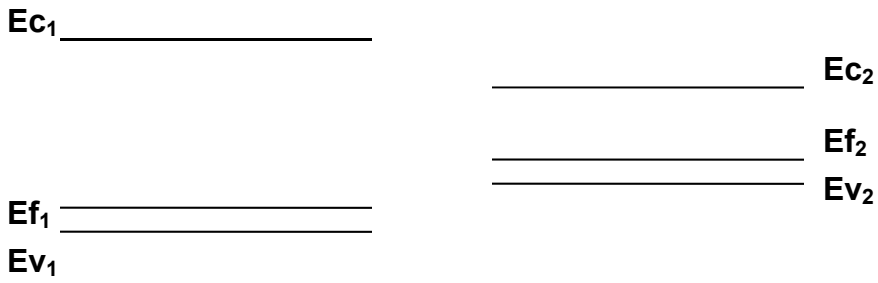
Problem 9.1: (Semiconductor heterostructures in equilibrium)

In each of the parts given below band diagrams of two different semiconductors are drawn (one on the left side and the other on the right side). In each case you are supposed to **sketch** the **equilibrium** band diagram when a heterojunction is formed between the two semiconductors. In each case indicate the **depletion** and/or **accumulation** and/or **inversion** regions that may exist in equilibrium on either side of the heterointerface. For your convenience, I have already taken the liberty of aligning the band diagrams such that the alignment shown corresponds to the electron affinity rule (as shown explicitly in part (a)). All the labels are also shown in more detail in part (a), and which you can use for the other parts as well.

a)



b)


E_{c_1}
E_{c_2}
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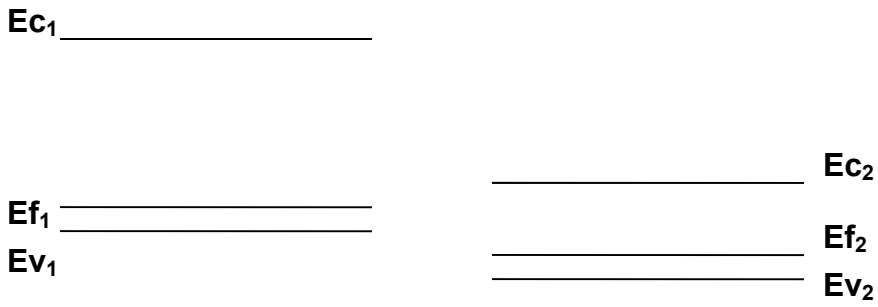
c)


E_{c_1}
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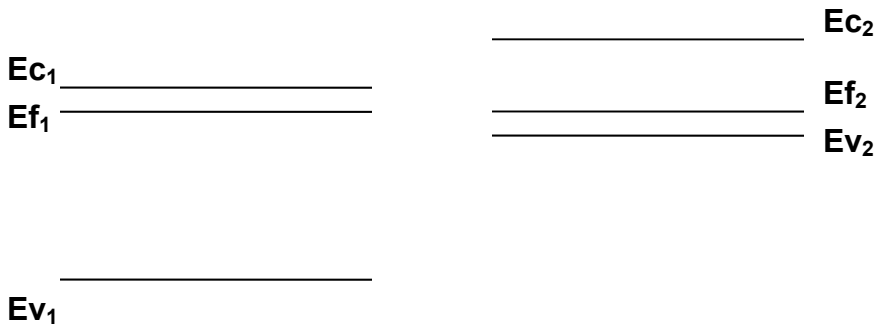
d)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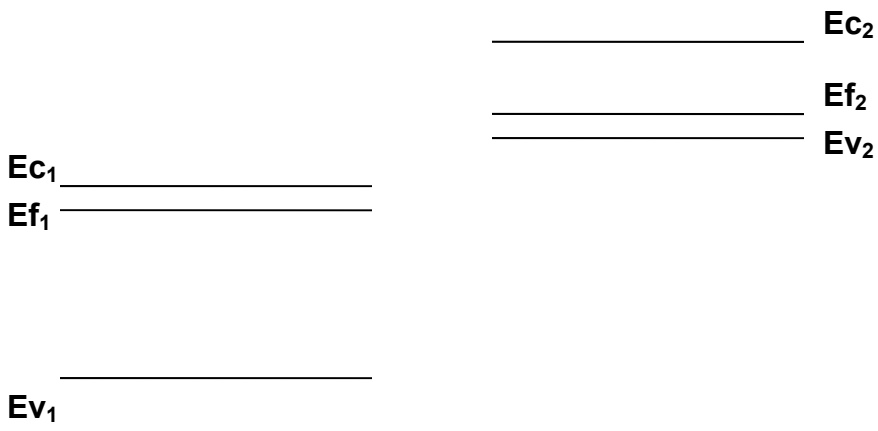
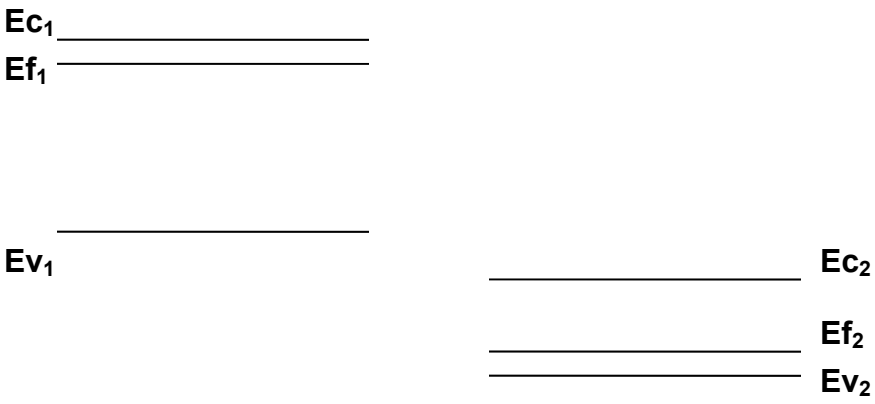
e)

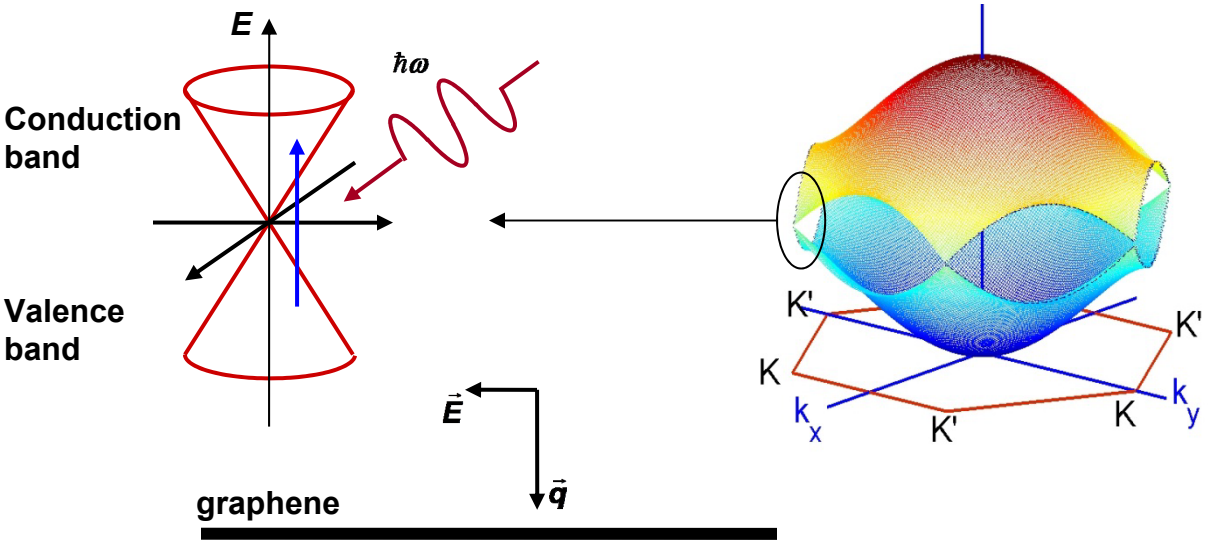

E_{c_2}
E_{f_2}
E_{v_2}
E_{c_1}
E_{f_1}
E_{v_1}

f)



Problem 9.2: (Interband optical absorption in graphene)

Graphene has two complete carrier pockets (or six 1/3rd carrier pockets) in the FBZ, as shown below.

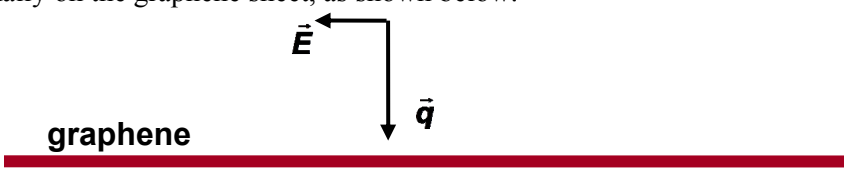


In each carrier pocket, the conduction and valence band dispersions are:

$$E_c(\vec{k}) = +\hbar v k = +\hbar v\sqrt{k_x^2 + k_y^2}$$

$$E_v(\vec{k}) = -\hbar v k = -\hbar v\sqrt{k_x^2 + k_y^2}$$

where the wavevector is measured, for simplicity, from the pocket center (as opposed to from the zone center) and the zero of energy is also chosen to coincide with E_p. Assume that the temperature is close to zero (i.e. $T \approx 0K$) and the valence band is full and the conduction band is empty. Light of frequency ω is incident normally on the graphene sheet, as shown below.



The average value of the momentum matrix element is:

$$\left\langle \left| \vec{P}_{vc} \cdot \hat{n} \right|^2 \right\rangle = \frac{m_o^2 v^2}{2}$$

Provided that the polarization unit vector of the incident field is in the plane of the graphene sheet. Assume that the intensity of the incident light is I_{inc}. Assume that for all practical purposes the photon momentum is small enough to be taken as zero (i.e. $\vec{q} = 0$).

a) Write an expression for the rate of stimulated absorption per unit area $R_{\uparrow}$ (units: 1/m^2-sec) in graphene in terms of the incident light Intensity I_{inc}. Make sure you include contributions from both spins and both carrier pockets. Write your answer as an integral over k-space.

b) Evaluate your integral in part (a) and show that $R_{\uparrow}$ can be written as:

$$R_{\uparrow} = \text{constant } \eta_o \left(\frac{I_{inc}}{\hbar\omega} \right) \qquad \left\{ \eta_o = \sqrt{\frac{\mu_o}{\varepsilon_o}} \right.$$

Find the value of the "constant" in the expression above and also specify the units of this "constant".

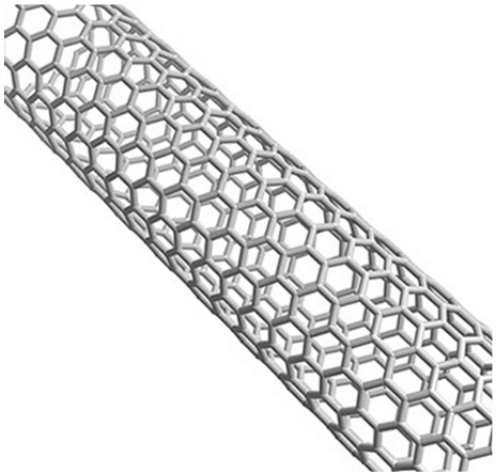
c) As the light crosses the graphene sheet, some photons are lost because of absorption in the graphene sheet. From your knowledge of $R_{\uparrow}$ and the incident light Inensity I_{inc} find out what fraction of the incident photon flux is absorbed in the graphene sheet. You will find, to your amazement perhaps, that this fraction is independent of the light frequency as well as of any material parameter value and depends only on few fundamental constants of physics. **Find a numerical value for this fraction.**

Problem 9.3: (Zigzag carbon nanotubes)

The energy band dispersion of graphene can be written as,

$$E\left(\vec{k}\right) = E_p \pm \hbar v \sqrt{k_x^2 + k_y^2}$$

Consider a semiconducting zizag nanotube with circumference given by $C = ma$ where the integer m is not a multiple of 3. $C = 2\pi R$ where R is the radius of the nanotube. Assume that $C >> a$.



a) Find the magnitude of the bandgap E_g and show that $E_g \approx \frac{2\hbar v}{3R}$. What is the magnitude (in eV) of the bandgap for a 1 nm radius zigzag nanotube?

b) The total electron density n (units: #/m) at the bottom of the two lowest (and degenerate) conduction bands can be written as:

$$n = \int_{E_p + \frac{E_g}{2}}^{\infty} g_{1D}(E)\, f(E - E_f)\, dE$$

Find the conduction band density of states function $g_{1D}(E)$ for the nanotube and sketch it. Don't forget to include band as well as spin degeneracies in $g_{1D}(E)$.

c) Suppose T=0K. Find an expression relating the electron density n to the Fermi energy E_F by evaluating the integral in part (b) exactly.

d) Sometimes it is helpful to assign effective masses to the carriers near the band edges in semiconducting nanotubes even though the energy band dispersion is not exactly parabolic. Show that the 1D energy subband dispersions for the conduction and valence bands of a semiconducting zigzag nanotube of radius R can be **approximately** written right near the band edges in the following parabolic forms:

$$E_c(k_x) \approx E_p + \frac{E_g}{2} + \frac{\hbar^2 k_x^2}{2m_e} \qquad E_v(k_x) \approx E_p - \frac{E_g}{2} - \frac{\hbar^2 k_x^2}{2m_h}$$

and find expressions for the electron and hole effective masses m_e and m_h. Would the electrons and holes be lighter or heavier in nanotubes of larger radii?