ECE4070: Final Exam Solutions (By Farhan Rana)

Problem 1 (Electron Dynamics and Transport)

a)
$$I = WD \times 2e \times \int_{-\infty}^{\infty} \frac{dk_x}{2\pi} \int_{-\infty}^{\infty} \frac{dk_y}{2\pi} \int_{0}^{\infty} \frac{dk_z}{2\pi} \left(\frac{\hbar k_z}{m_e}\right) \left[f\left(E_c(\vec{k}) - E_{fR}\right) - f\left(E_c(\vec{k}) - E_{fL}\right) \right]$$
$$= \frac{WD}{2} e^2 g_{3D}(E_F) v_F \; V = GV$$

b)
$$J = (+e) 2 \times \int_{-\pi/a}^{\pi/a} \frac{dk_x}{2\pi} \left(1 - f\left(k_x + \frac{e\tau}{\hbar} E_x\right)\right) v(k_x) = -2e \int_{-\pi/a}^{\pi/a} \frac{dk_x}{2\pi} \left(1 - f(k_x)\right) v_o \sin\left(k_x a - \frac{e\tau a}{\hbar} E_x\right)$$
$$= 2 \frac{e^2 \tau \; a}{\hbar} \int_{-\pi/a}^{\pi/a} \frac{dk_x}{2\pi} \left(1 - f(k_x)\right) v_o \cos(k_x a) E_x = 2 \frac{e^2 \tau \; a}{\hbar} v_o \int_{-\pi/2a}^{\pi/2a} \frac{dk_x}{2\pi} \cos(k_x a) E_x = \frac{2e^2 \tau}{\pi \hbar} v_o E_x$$
$$\Rightarrow \sigma = \frac{2e^2 \tau}{\pi \hbar} v_o$$

c)
$$\rho_{zz} = \frac{m_z}{ne^2 \tau}\left(1 + \omega_c^2 \tau^2\right) \qquad \left\{ \; \omega_c = \frac{eB_o}{\sqrt{m_y m_z}} \right.$$

Problem 2 (CNTs)

a)
$$E(k_x) = \pm \hbar v \sqrt{(k_x - K_x)^2 + \left(E_g / 2\hbar v\right)^2}$$

$$R_{\uparrow} = \frac{2\pi}{\hbar} \left(\frac{eA_o}{2m}\right)^2 \left\langle \left| \vec{P}_{cv} \,.\, \hat{n} \right|^2 \right\rangle \; 4 \times \int_{-\infty}^{\infty} \frac{dk_x}{2\pi} \quad \delta\left(E_c(k_x) - E_v(k_x) - \hbar\omega\right)$$
$$= \frac{2\pi}{\hbar} \left(\frac{e}{2m}\right)^2 \left(\frac{2\eta_o P_{inc}}{\omega^2}\right) (mv)^2 \quad \frac{2\omega / v\pi}{\sqrt{(\hbar\omega)^2 - E_g^2}} \; \theta\left(\hbar\omega - E_g\right)$$
$$= \frac{e^2}{4\hbar} \eta_o \left(\frac{P_{inc}}{\hbar\omega}\right) \frac{8v}{\sqrt{\omega^2 - \left(E_g/\hbar\right)^2}} \; \theta\left(\hbar\omega - E_g\right)$$

b)
$$E(k_x) = \pm \hbar v |k_x| \quad \Rightarrow \quad v(k_x) = v \; \text{sign}(k_x)$$
$$I = 4e \int_{-\infty}^{\infty} \frac{dk_x}{2\pi} f\left(k_x + \frac{e\tau}{\hbar} E_o\right) v(k_x) = \frac{4e^2 \tau \; v}{\pi \hbar} E_o = \sigma \; E_o$$

c)

There are two carrier pockets in the 1D FBZ for an armchair nanotube. For one of these pockets,
$$E(k_y) = \pm \hbar v \left| k_y - K_{y1} \right| \quad \Rightarrow \quad v(k_y) = v \; \text{sign}\left(k_y - K_{y1}\right)$$
The total current is therefore,
$$I = 4e \int_{0}^{\infty} \frac{dk_y}{2\pi} \left[f_L(E - E_{fL}) - f_R(E - E_{fR}) \right] v(k_y) = \frac{2e^2}{\pi \hbar} V_o = GV_o$$

Problem 3 (2D materials)

a) The answer follows from the lecture notes:

$$R_{\uparrow}(\omega) = \frac{2\pi}{\hbar}\left(\frac{eA_o}{2m_o}\right)^2 \left\langle \left|\vec{P}_{cv} \, . \, \hat{n}\right|^2 \right\rangle \; 2 \times \int_{-\infty}^{\infty} \frac{d^2\vec{k}}{(2\pi)^2} \; \delta\left(E_c\left(\vec{k}\right) - E_v\left(\vec{k}\right) - \hbar\omega\right)$$

b) First note that:

$$E_c\left(\vec{k}\right) - E_v\left(\vec{k}\right) = E_g + \frac{\hbar^2 k_x^2}{2m_{rx}} + \frac{\hbar^2 k_x^2}{2m_{ry}}$$

where the masses with the subscript "***r***" are the reduced masses in each dimension. Therefore the joint density of states equal $\sqrt{m_{rx} m_{ry}} \big/ \pi\hbar^2$ (independent of energy). So we can write,

$$R_{\uparrow} = \frac{2\pi}{\hbar}\left(\frac{eA_o}{2m}\right)^2 \left\langle \left|\vec{P}_{cv} \, . \, \hat{n}\right|^2 \right\rangle \; 4 \times \int_{-\infty}^{\infty} \frac{dk_x}{2\pi} \; \delta\left(E_c(k_x) - E_v(k_x) - \hbar\omega\right)$$

$$= \frac{2\pi}{\hbar}\left(\frac{e}{2m_o}\right)^2 \left(\frac{2\eta_o P_{inc}}{\omega^2}\right) \left(\frac{m_o E_p}{2}\right) \frac{\sqrt{m_{rx} m_{ry}}}{\pi\hbar^2} \; \theta\left(\hbar\omega - E_g\right)$$