Department of Electrical and Computer Engineering, Cornell University

ECE 4070: Physics of Semiconductors and Nanostructures

Spring 2014

Final Exam ` **May 13, 2014**

INSTRUCTIONS:

• **Every problem must be done in the blue booklet**

• **Only work done on the blue exam booklets will be graded – do not attach your own sheets to the exam booklets under any circumstances**

• **To get partial credit you must show all the relevant work**

• **Correct answers with wrong reasoning will not get points**

• **All questions do not carry equal points**

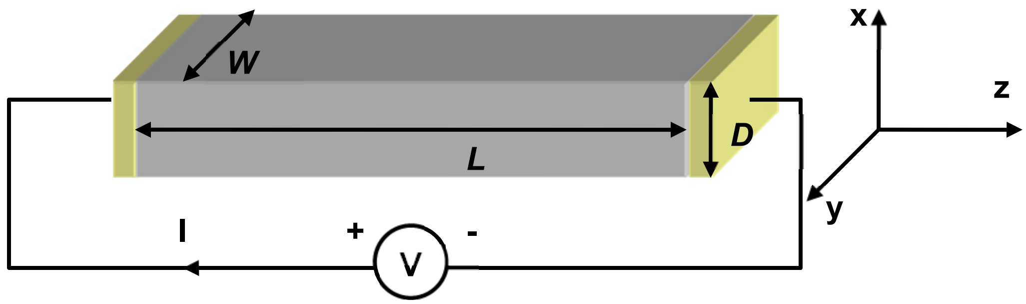
• **All questions do not have the same level of difficulty**

DO NOT WRITE IN THIS SPACE

Problem 1 (Electron Dynamics and Transport) – 35 points

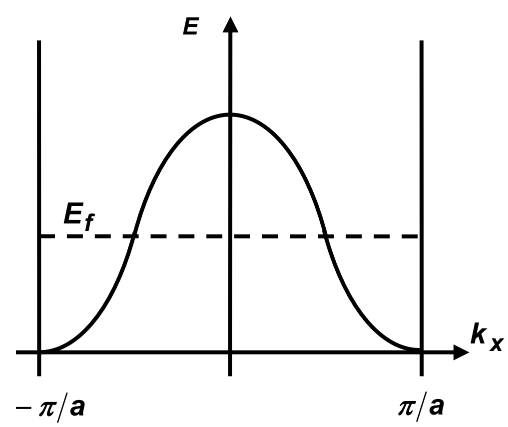
Note that parts (a) and (b) and (c) are unrelated.

a) Consider a **3D** p-doped semiconductor of length L, width W, and thickness D, as shown:

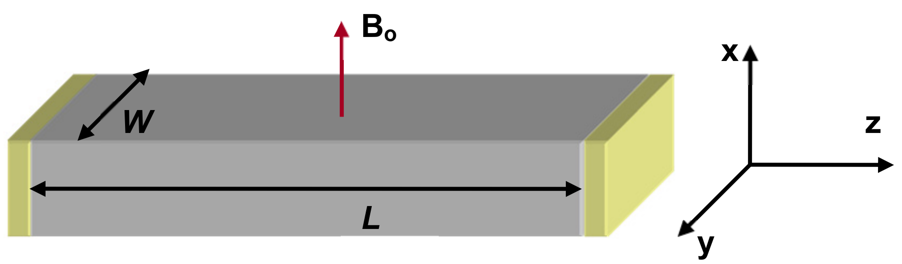


The length of the semiconductor is much smaller than the mean free path so the transport through the semiconductor is Ballistic. Assume that the temperature is close to zero (i.e. $T \approx 0K$). Assume that the energy band dispersion for the valence band is: $E(\vec{k}) = E_v - \hbar^2 k^2 / 2m_h$. The current in the external circuit can be written as $I = GV$. Find an expression for the conductance G.

b) Consider a **1D** conductor with a band having the following energy dispersion: $E(k_x) = \frac{\hbar v_o}{a}\left[1 + \cos(k_x a)\right]$.
The temperature is near zero and the Fermi level is $E_f = \hbar v_o / a$ so the band is exactly half full. Find an expression for the conductivity assuming that the scattering time is τ. There should be no unevaluated integral(s) in your final answer.



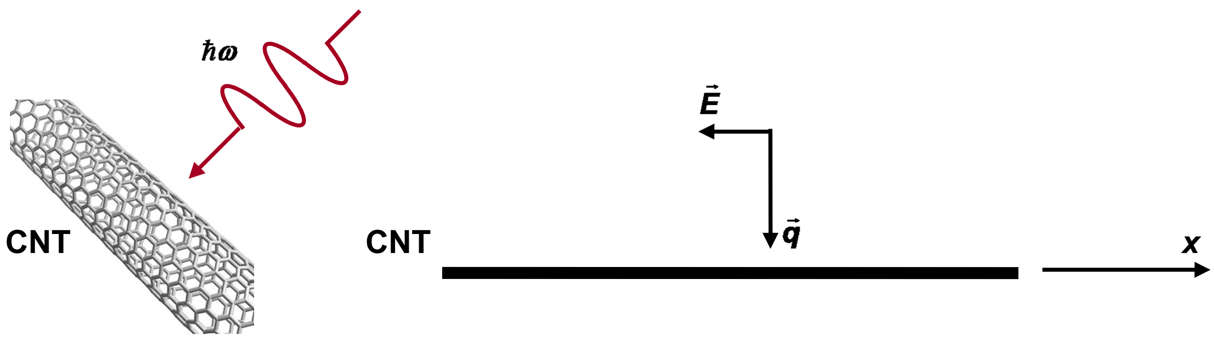
c) Consider a **3D** n-doped semiconductor, as shown. Assume that the energy band dispersion for the conduction band is: $E(\vec{k}) = E_c + \hbar^2 k_x^2 / 2m_x + \hbar^2 k_y^2 / 2m_y + \hbar^2 k_z^2 / 2m_z$. An electric field $\vec{E} = E_z \hat{z}$ is externally applied in the z-direction. A magnetic field $\vec{B} = B_o \hat{x}$ is applied in the x-direction. Assume that the scattering time is τ and the electron density is n .



Find the resistivity ρ_{zz} defined as E_z / J_z .

Problem 2 (CNTs) – 35 points

a) Consider a zizag nanotube with circumference given by $C = na$ where the integer n is NOT a multiple of 3. $C = 2\pi R$ where R is the radius of the nanotube. Assume that $C >> a$. The bandgap is E_g and $E_g \approx \frac{2\hbar v}{3R}$. And the electron and hole effective masses are $m_e = m_h \approx \frac{\hbar}{3Rv}$.
Assume **T≈0K**.



Assume light incident on the nanotube with field polarized along the length of the nanotube, as shown. Near the band extrema, the average value of the momentum matrix element between the valence and conduction band states is give by the relation,

$$\left\langle \left| \vec{P}_{vc} \cdot \hat{n} \right|^2 \right\rangle = m_o^2 v^2$$

Assume that the incident light power is P_{inc} (units: Watt/m^2). Find the frequency dependent rate $R_{\uparrow}(\omega)$ of photon absorption per unit length of the nanotube in terms of P_{inc} and the given nanotube parameters. There should be no unevaluated integral(s) in your answer.

b) Consider a semiconducting zizag nanotube with circumference given by $C = na$ where the integer n is a multiple of 3 and therefore the bandgap is E_g is zero. Assume that the scattering time is τ.

Supppose **T≈0K** and suppose the nanotube is heavily n-doped with a Fermi level that is well inside the conduction band. A DC electric field of strength E_o is applied in the x-direction. The current I in the nanotube can be written as $I = \sigma E_o$ where σ is the conductivity (units: Siemens-meter in 1D). Find an expression for the conductivity. There should be no unevaluated integral(s) in your answer.

c) Consider a semiconducting armchair nanotube with circumference given by $C = n\sqrt{3}a$. <u>Assume that there is no scattering within the length of the nanotube.</u>

Supppose **T≈0K** and suppose the nanotube is heavily n-doped with a Fermi level that is well inside the conduction band. A DC potential difference V_o is applied to the two ends of the nanotube. The current I in the nanotube can be written as $I = G V_o$ where G is the conductance (units: Siemens). Find an expression for the conductance. There should be no unevaluated integral(s) in your answer.

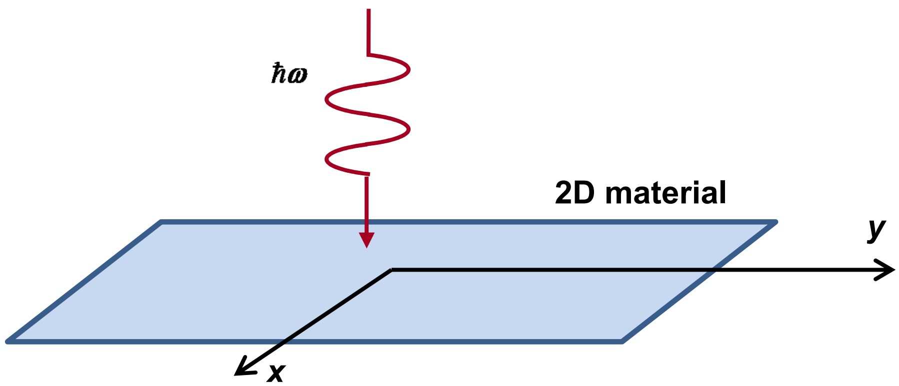
Problem 3 (Optical Absorption in 2D Materials) – 30 points

Consider a 2D material (like BN) with parabolic conduction and valence bands, and the band energy dispersions near the band extrema are given by,

$$E_c(\vec{k}) = E_c + \frac{\hbar^2 k_x^2}{2m_{ex}} + \frac{\hbar^2 k_y^2}{2m_{ey}}$$

$$E_v(\vec{k}) = E_v - \frac{\hbar^2 k_x^2}{2m_{hx}} - \frac{\hbar^2 k_y^2}{2m_{hy}}$$

Assume only a single valence band maximum and a single conduction band minimum in the FBZ. The 2D material occupies the x-y plane. Assume that the temperature is close to zero (i.e. **T ≈ 0K**) and the valence band is full and the conduction band is empty. Light of frequency ω is incident normally on the 2D material, as shown below.



The average value of the momentum matrix element is:

$$\left\langle \left|\vec{P}_{vc} \cdot \hat{n}\right|^2 \right\rangle = \frac{m_o\, E_p}{2}$$

provided that the polarization unit vector of the incident field is in the plane of the 2D material. Assume that the intensity (power per unit area) of the incident light is I_{inc}.

a) Write an expression for the rate of stimulated absorption per unit area $R_{\uparrow}$ (units: 1/m^2-sec) in terms of the incident light Intensity I_{inc} and the given parameters. Make sure you include contributions from both spins. Write your answer as an integral over k-space.

b) Evaluate the k-space integral obtained in part (a) above and sketch $R_{\uparrow}(\omega)$ as a function of the incident photon energy $\hbar\omega$. If you have the right concepts, you might be able to sketch correctly even if your answer to part (a) is incorrect.