Department of Electrical and Computer Engineering, Cornell University

ECE 4070: Physics of Semiconductors and Nanostructures

Spring 2014

Exam 2 ` **April 17, 2014**

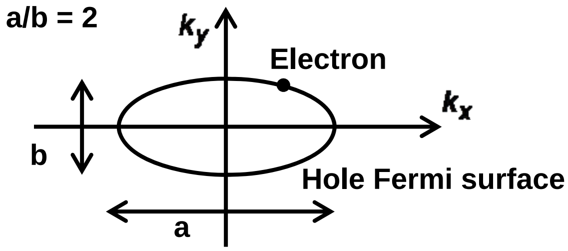
INSTRUCTIONS:

- **Every problem must be done in the exam booklet**
- **Only work done on the exam booklets will be graded – do not attach your own sheets to the exam booklets under any circumstances**
- **To get partial credit you must show all the relevant work**
- **Correct answers with wrong reasoning will not get points**
- **All questions do not carry equal points**
- **All questions do not have the same level of difficulty**

DO NOT WRITE IN THIS SPACE

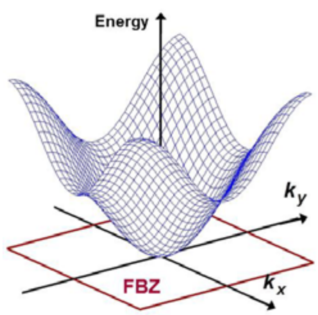
Problem 1 (Band electrons) – 40 points

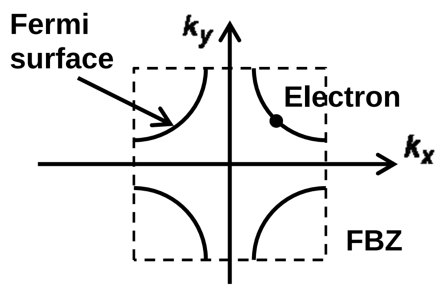
a) Consider a 2D crystal in the xy plane. The crystal has a single **valence band** with a **parabolic non-isotropic** dispersion relation. A constant and uniform magnetic field $\vec{B} = B_o \hat{z}$ is applied in the z-direction. Assume that the crystal has lots of holes in the valence band and the holes fill the valence band down to an energy E_F below the valence band maximum E_v. The rest of the valence band is full of electrons. Consider an **electron** in the valance band on the hole Fermi surface whose position in k-space at a certain point in time is as shown in the figure below.



Sketch the trajectory of the electron motion in real space, indicate the direction of electron motion with arrows, and indicate where the electron is on that trajectory when in k-space it is as shown above. (10 points)

b) Consider a 2D crystal with a square Bravais lattice in the xy plane. A constant and uniform magnetic field $\vec{B} = B_o \hat{z}$ is applied in the z-direction.





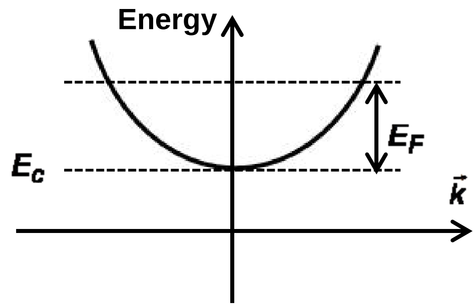
The conduction band dispersion is show above. At large electron number the Fermi surface in k-space is as shown above (solid lines) in the First Brillouin Zone. Now consider an electron on the Fermi surface whose position in k-space at a certain point in time is as shown in the figure above. Sketch the trajectory of the electron motion in real space, indicate the direction of electron motion with arrows, and indicate where the electron is on that trajectory when in k-space it is as shown above. (10 points)

c) Consider a 3D material with parabolic energy dispersion for the conduction band and the effective mass tensor for the conduction band electrons in the xyz coordinate system is known to be diagonal. A uniform and constant magnetic field of 0.1 Tesla is applied first in the x-direction and the cyclotron frequency is measured to be 4.66 GHz. The same magnetic field is then applied in the y-direction and then in the z-direction and in each case the cyclotron frequency is measured to be 9.32 GHz and 13.99 GHz, respectively. Find the effective mass tensor of the electrons. (10 points)

d) Consider a 3D crystal. The conduction band energy dispersion relation is not parabolic, and is given by the relation:

$$E(\vec{k}) = E_c + ak^4$$

Assume **near-zero temperature**. The Fermi energy is E_F (measured from the band bottom), and the Fermi wavevector is k_F. Assume that the electron scattering time is τ.



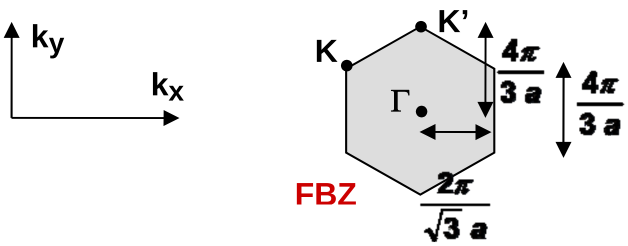
Find the conductivity tensor $\overline{\overline{\sigma}}$ of the material such that the current density can be written as $\vec{J} = \overline{\overline{\sigma}}\,\vec{E}$ for sufficiently small applied electric fields. (10 points)

Problem 2 (Miscellaneous) – 20 points

a) In the limit $\vec{q} \to 0$ (i.e. the long wavelength limit) the acoustic phonon frequencies go to zero whereas the optical phonon frequencies approach a non-zero value. Explain physically why in the $\vec{q} \to 0$ limit the acoustic phonon frequencies go to zero but not the optical phonon frequencies. The points awarded will depend on the quality and clarity of the explanation provided. (5 points)

b) How do you think the phonon dispersion relation for negative wavevectors is related to the phonon dispersion for positive wavevectors? More specifically, does the relation $\omega(-\vec{q}) = \omega(\vec{q})$ hold? If so, give a formal proof. If not, explain why not. The points awarded will depend on the quality and clarity of the explanation provided. (5 points)

c) Consider a 2D crystal (not graphene) with hexagonal symmetry and with the first Brillouin zone shown below.



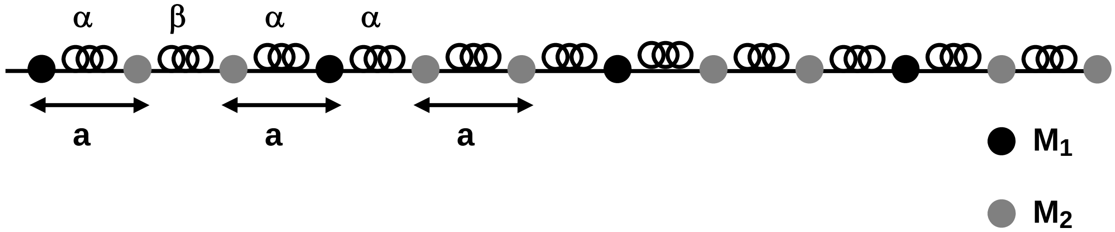
The material has conduction band minima at the three K and the three K' points. Near the minima, the conduction band energy dispersion is parabolic but not isotropic. At the K-point $\left(-\frac{2\pi}{\sqrt{3}a}, \frac{2\pi}{3a}\right)$ the inverse effective mass matrix is known to be:

$$M^{-1} = \begin{bmatrix} \frac{1}{m_1} & \left(\frac{1}{m_2} - \frac{1}{m_1}\right)\frac{\sqrt{3}}{2} \\ \left(\frac{1}{m_2} - \frac{1}{m_1}\right)\frac{\sqrt{3}}{2} & \frac{1}{m_2} \end{bmatrix}$$

Find the inverse effective mass matrix for the conduction band minimum at the K'-point $\left(0, \frac{4\pi}{3a}\right)$ and also for the minimum at the K-point $\left(\frac{2\pi}{\sqrt{3}a}, \frac{2\pi}{3a}\right)$. (10 points)

Problem 3 (Phonons in 1D) – 30 points

Consider the following 1D lattice of atoms consisting of two different kinds of atoms (black and gray).



a) Assume a solution form and then use it to write the equations for lattice wave in the standard form:

$$\overline{\overline{D}}(q)\begin{bmatrix} ? \end{bmatrix} = \omega^2 \begin{bmatrix} & ? & \end{bmatrix}\begin{bmatrix} ? \end{bmatrix}$$

Obtain all the elements of the dynamical matrix $\overline{\overline{D}}(q)$ and the other column vectors and matrices indicated by question marks in the expression above. Show your work.

(15 points)

b) If M_1=M_2, would you expect a zero bandgap between the acoustic band and the next higher band at the zone boundary? Give a physical explanation. Correct answer with wrong physics will not get you any points.

(5 points)

c) If M_1=M_2, and α=β, would you expect a zero bandgap between the acoustic band and the next higher band at the zone boundary? Give a physical explanation. Correct answer with wrong physics will not get you any points.

(5 points)

d) For If M_1=M_2, and α=β, find the frequencies of all the phonon bands at the zone center without solving the matrix equation obtained in part (a). Justify your approach/answer by a physical explanation.

(5points)