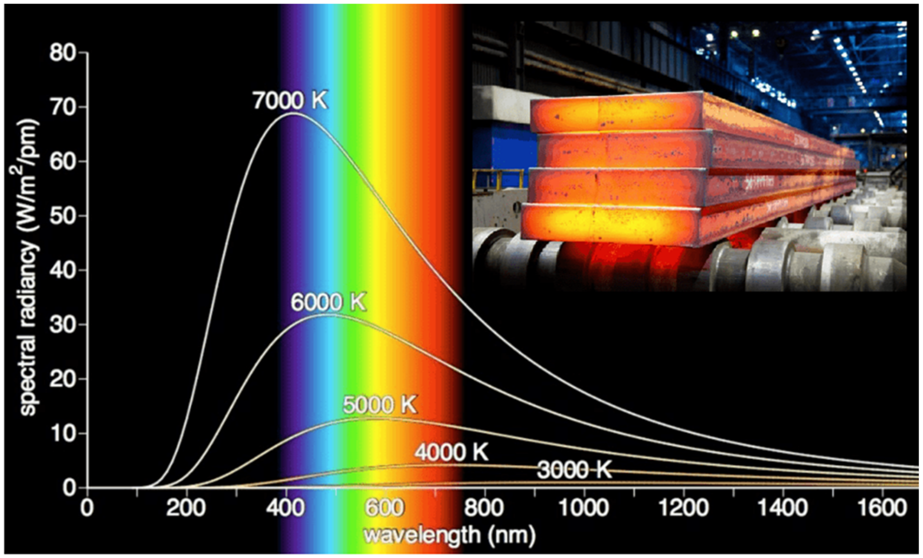
Lecture 3

The Quantum Story: How It All Started



In this lecture you will learn:

- **The historical developments leading to the quantum theory**

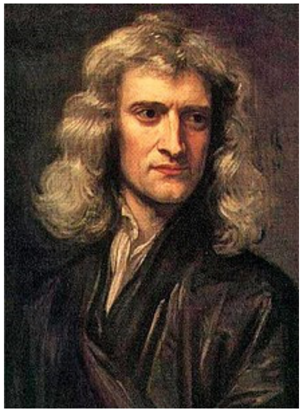
The Nature of Light

The nature of light has been a puzzle for centuries

The corpuscular theory of light, arguably set forward by Descartes in 1637, states that light is made up of small discrete particles called "corpuscles" (little particles) which travel in a straight line with a finite velocity

Rene Descartes (1596-1650)

Sir Issac Newton was a big champion of the corpuscular theory of light



Issac Newton (1642-1726)

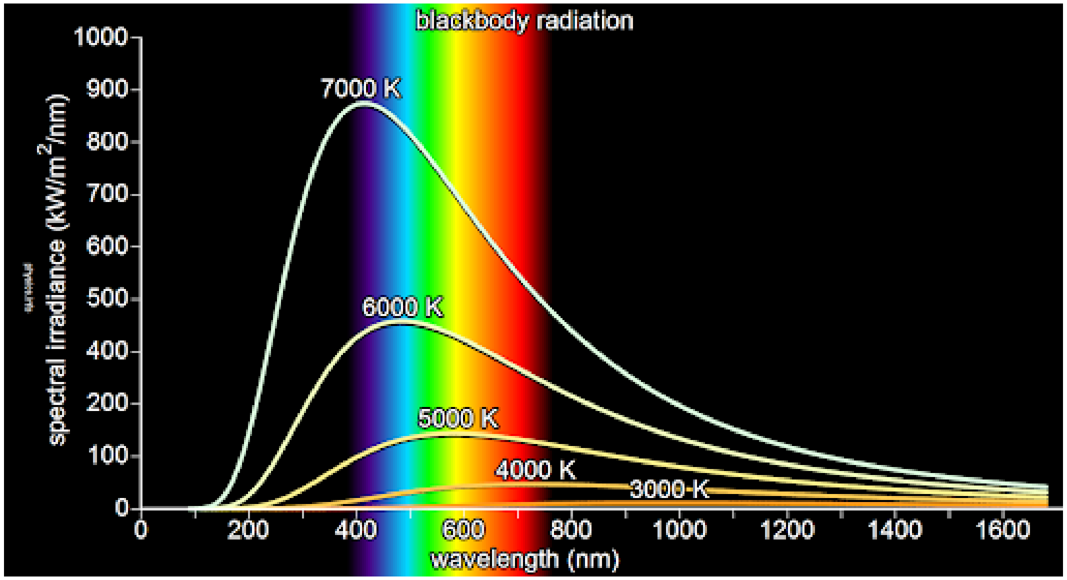
But the corpuscular theory of light could not explain phenomena like interference and diffraction which had been demonstrated for light by early 1800s by Thomas Young

Christiaan Huygens worked out a mathematical wave theory of light in 1678, and published it in his Treatise on light in 1690. He proposed that light was a wave in a medium called the Luminiferous ether. Huygens theory could explain interference and diffraction of light very well.

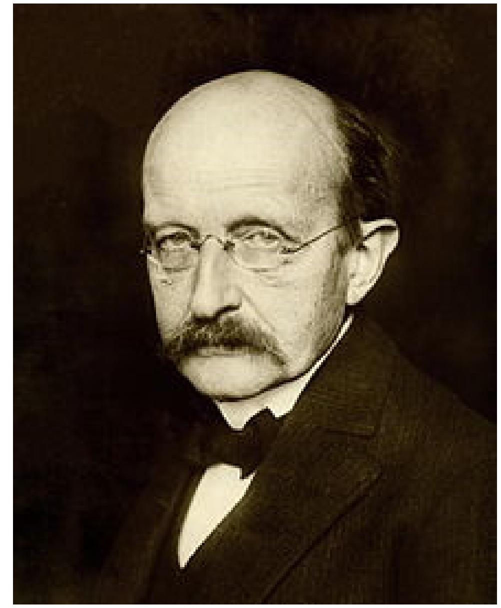
Christiaan Huygens (1629–1695)

Max Planck and the Blackbody Radiation Problem

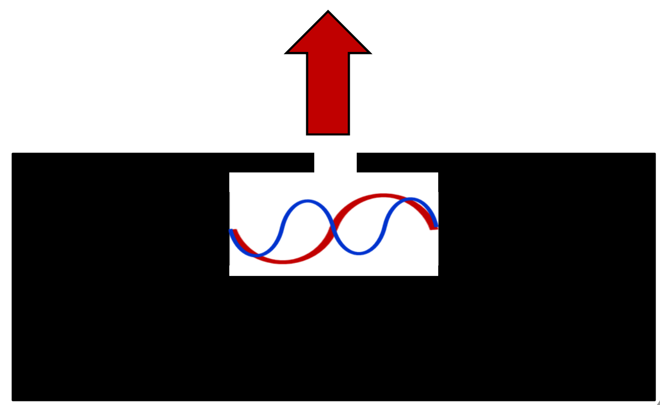
Around 1900, A German physicist was trying to explain the spectrum of light emitted by a blackbody



This problem was proposed by Kirchoff in 1859, but nobody had found a solution

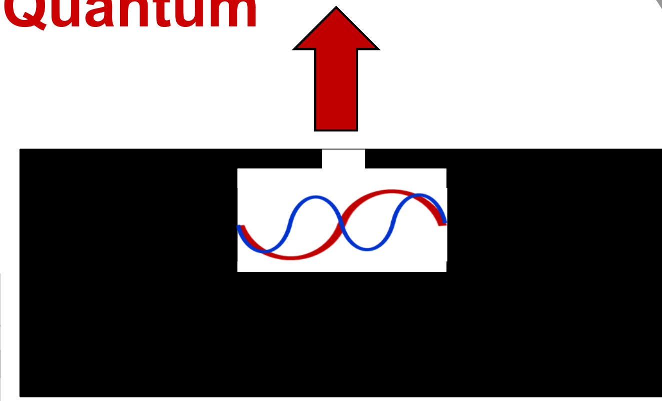


Max Planck (1858 – 1947)
Nobel Prize: 1918



"How does the intensity of the electromagnetic radiation emitted by a black body (a perfect absorber, also known as a cavity radiator) depend on the frequency (or wavelength) of the radiation (i.e., the color of the light) and the temperature of the body?"

Max Planck and the Birth of the Quantum



At that time it was believed that the energy of electromagnetic waves can be continuously increased to any value by increasing the amplitude of the electric and magnetic fields of the wave

$$\text{Electromagnetic energy} = \int dV \left[\frac{1}{2} \varepsilon_o \vec{E} . \vec{E} + \frac{1}{2} \mu_o \vec{H} . \vec{H} \right]$$

But the above notion could not explain the blackbody radiation spectra!

Planck's Breakthrough:

Planck assumed that the energy of an electromagnetic wave inside a blackbody cavity cannot be continuously increased, but can be increased in jumps of $\hbar\omega$ where ω is the frequency of the wave

In other words, the smallest unit by which energy can be added or taken away from an electromagnetic wave is $\hbar\omega$ which Planck called a "quantum" of energy:

$$\hbar\omega = \hbar 2\pi f = \hbar \frac{2\pi c}{\lambda}$$

$$\hbar = 1.05458 \times 10^{-34} \text{ Joules-sec}$$ **Planck's constant**

Planck's assumption lead to a remarkable agreement with the experimental data!

Planck's Constant

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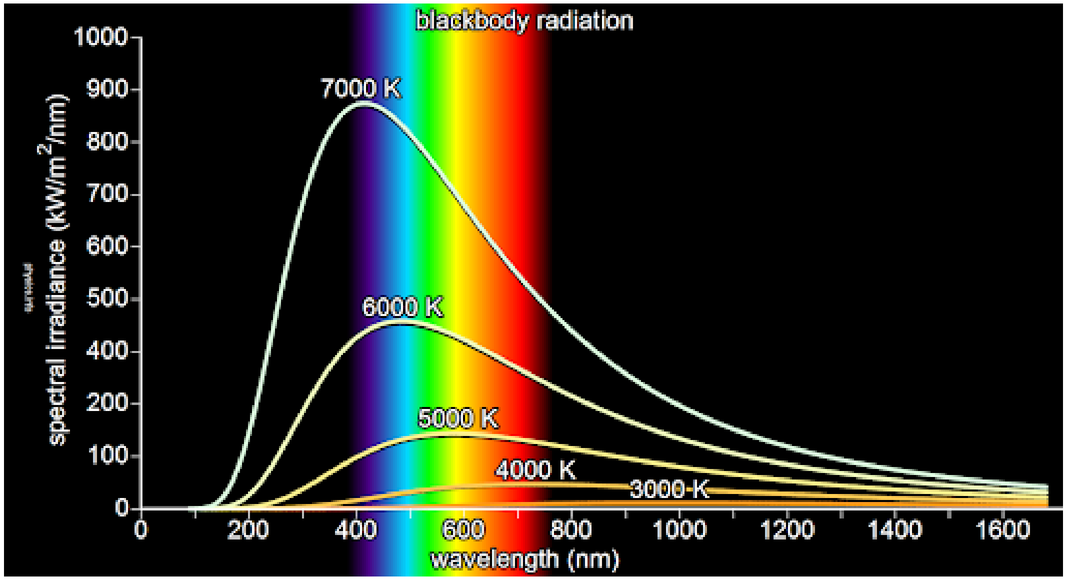


Lets look at green light:

Wavelength = λ ~550 nm

Energy of a single quantum of green light:

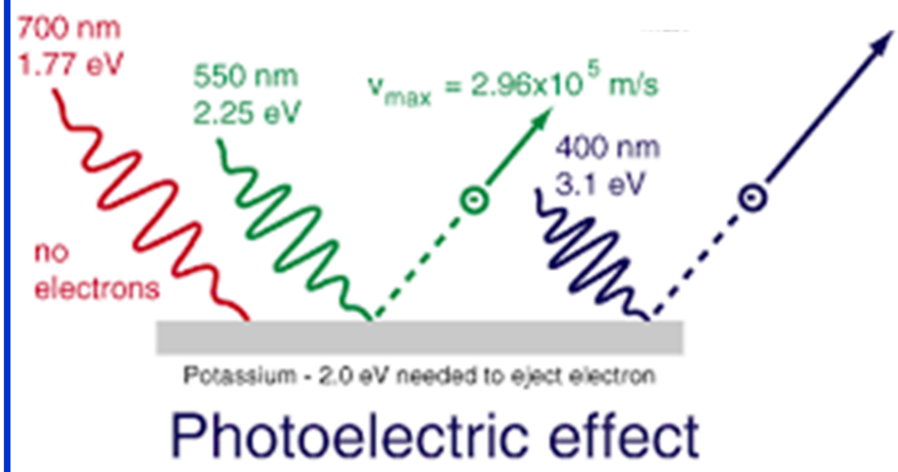
$$\hbar\omega = \hbar 2\pi f = \hbar \frac{2\pi c}{\lambda} = 3.6 \times 10^{-19} \text{ Joules} = 2.26 \text{ eV}$$

Very small !
Because $\hbar$ is so small !

Einstein and the Photoelectric Effect

The Photoelectric Effect:

When light of frequency ω larger than a threshold frequency ω_o is shone upon a metal, electrons are ejected from the metal. And if ω is smaller than the threshold frequency ω_o, no electrons are ejected however intense the light may be.



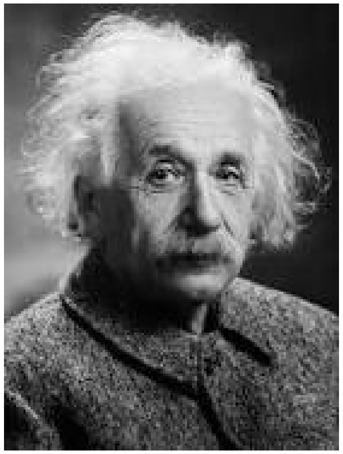
In 1905, Einstein used Max Planck's discovery and postulated that light of frequency ω is made up of particles, each with energy $\hbar\omega$.

An electron inside the metal needs a certain minimum energy E_o to come out of the metal. And an electron may only absorb one particle of light at a time.

So, an electron will come out by absorbing a particle of light if:

$$\hbar\omega > E_o = \hbar\omega_o$$

So Einstein's explanation favored the particle theory of light !!

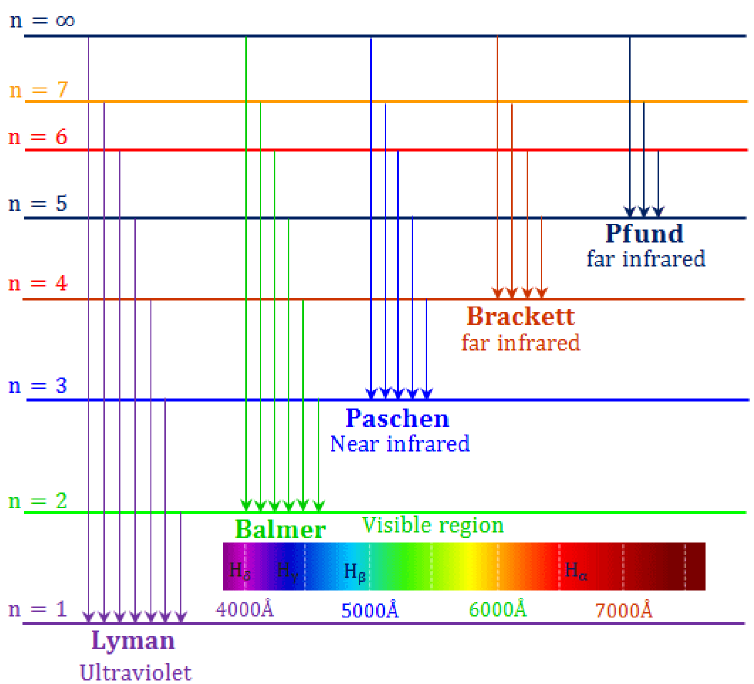


Albert Einstein (1879-1955)

Nobel Prize: 1921

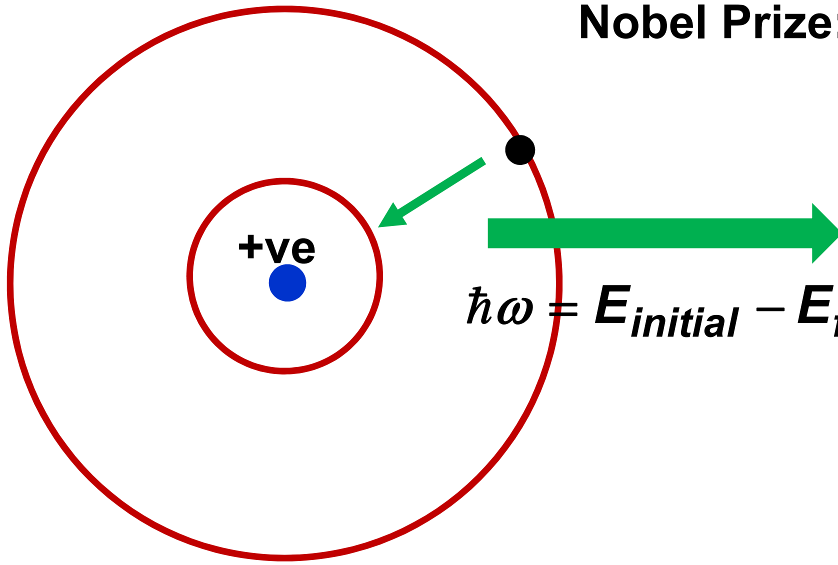
Neils Bohr and the Hydrogen Atom

During the 1910s and 1920s, Niels Bohr, a Danish Physicist, was studying the behavior of electrons in atoms and the light emitted by atoms



The spectrum of light emitted by the lightest element, Hydrogen, was found to consist of very sharp and well defined frequencies (or wavelengths)

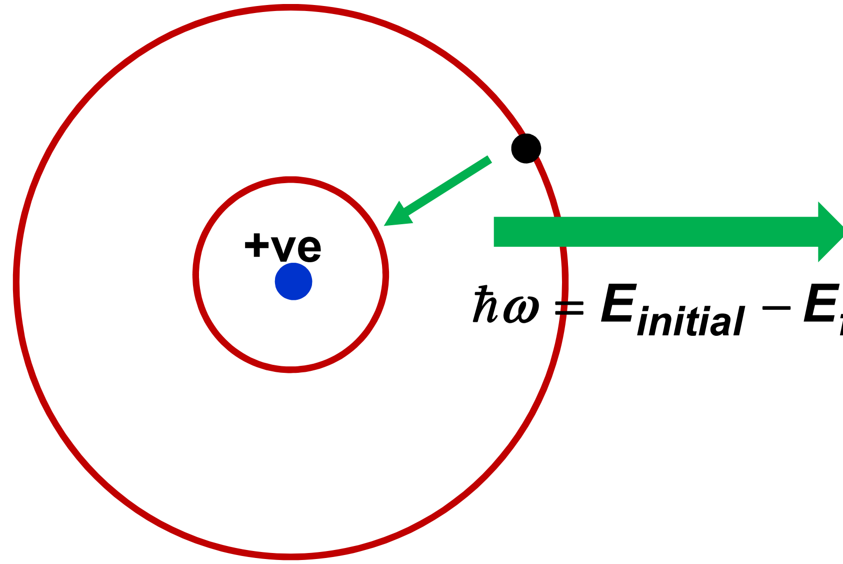
Niels Bohr (1885-1962) Nobel Prize: 1922



$$\hbar\omega = E_{initial} - E_{final}$$

So what are the energies that an electron can have in a Hydrogen atom?

Hydrogen Atom: What Does the Classical Physics Say?



$$\hbar\omega = E_{initial} - E_{final}$$

Classical physics says:

Centripetal force = Coulomb force

$$\Rightarrow \frac{e^2}{4\pi\varepsilon_o r^2} = \frac{mv^2}{r}$$

$$\Rightarrow r = \frac{e^2}{4\pi\varepsilon_o mv^2}$$

Energy of the electron:

$$E = \frac{1}{2}mv^2 - \frac{e^2}{4\pi\varepsilon_o r} = \frac{1}{2}mv^2 - mv^2$$

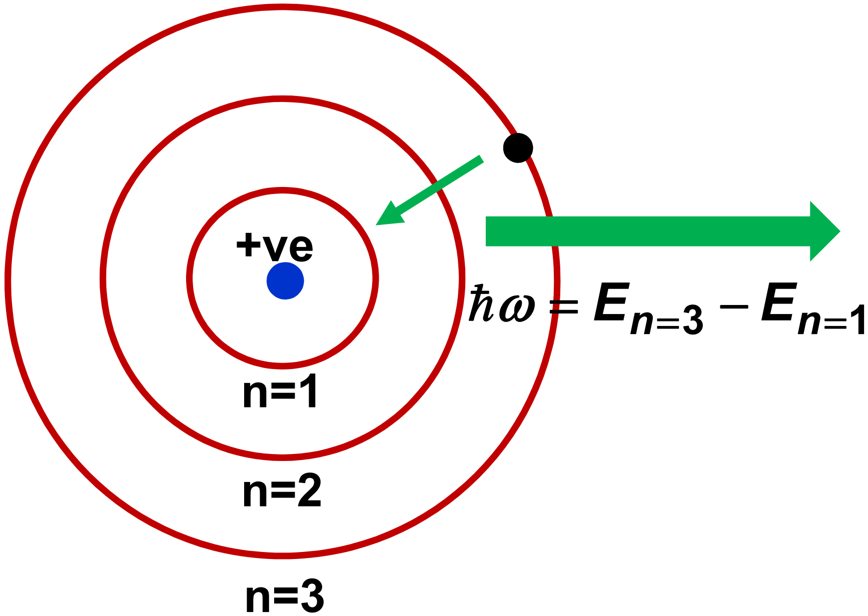
$$\Rightarrow E = -\frac{1}{2}mv^2 = -\frac{e^2}{8\pi\varepsilon_o r}$$

Angular momentum of the electron:

$$L = mvr = \frac{e^2}{4\pi\varepsilon_o v}$$

- The electron in the Hydrogen atom can have any energy value and any angular momentum value provided centripetal force and Coulomb force are related as above
- Higher the total energy, larger the radius of the orbit, and smaller the velocity of the electron

Bohr Quantization Rule: Planck's Constant Appears in Matter Physics



Niels Bohr
(1885-1962)

Bohr could explain the radii of the electron orbits, and the energy of the electron in each orbit, and the measured frequency of the light emitted when an electron transitioned from a higher energy orbit to a lower energy orbit PROVIDED he assumed that the angular momentum $L = mvr$ of an electron in the n-th orbit is ASSUMED to be given as:

$$L = mvr = n\hbar$$

Angular momentum is "quantized"!! But why???

For the first time, Planck's constant appeared in phenomenon related to matter. Previously it had been associated with light only.

Electromagnetic Momentum and Wavelength

By 1900s, it was known that any electromagnetic wave packet of energy *E* and moving with velocity *c* carried a momentum given by:

$$p = \frac{E}{c}$$

(Follows from classical Maxwell's equations)

Since the above expression holds for light packets of all big and small energies, it must also hold for the packet with the smallest energy for which:

$$E = \hbar\omega = \frac{\hbar 2\pi c}{\lambda}$$

Therefore, the momentum of the smallest energy light packet or light "quantum" is:

$$p = \frac{E}{c} = \frac{\hbar\omega}{c} = \frac{\hbar 2\pi}{\lambda}$$

This means that associated with a light particle of momentum *p* there is a wave of wavelength λ that is given by:

$$\lambda = \frac{2\pi\hbar}{p}$$

What does this mean??
Is light a wave or made up of particles??

The De Broglie Hypothesis

Louis De Broglie
(1892-1987)
Nobel Prize: 1929

In 1924, a French physicist Louis De Broglie hypothesized that since light seems to display certain particle-like features, it might also be possible that particles display certain wave-like qualities

To quantify this hypothesis, De Broglie assumed that "associated" with a particle of momentum $p=mv$, there is a wave of some sort with a wavelength λ equal to:

$$\lambda = \frac{\hbar 2\pi}{p}$$

Basically, he just extended the relation known previously for light particles to matter particles

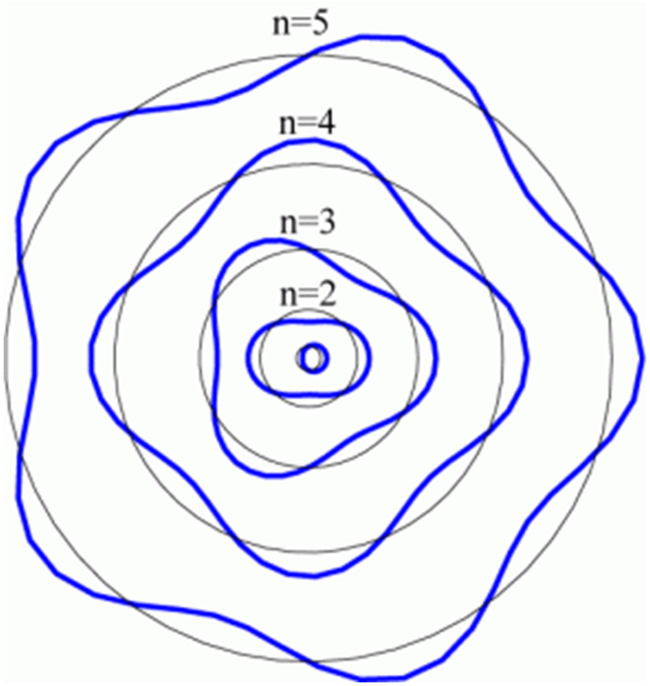
De Broglie further assumed that "associated" with a particle of energy E, there is a wave of some sort with a frequency ω equal to:

$$\omega = \frac{E}{\hbar}$$

Again, he just extended the relation known previously for light particles to matter particles

These were some really wild assumptions !!

The De Broglie Hypothesis and the Atom



According to the Be Broglie hypothesis, since the electron has an associated wave, the circumference of the electron orbit in an atom must be an integral multiple of the electron wave's wavelength for the associated wave to smoothly fit in the orbit:

$$2\pi r = n\lambda \qquad \left\{ n = 1,2,3,\ldots\ldots \right.$$

But:

$$\lambda = \frac{\hbar 2\pi}{p} = \frac{\hbar 2\pi}{mv}$$

Therefore:

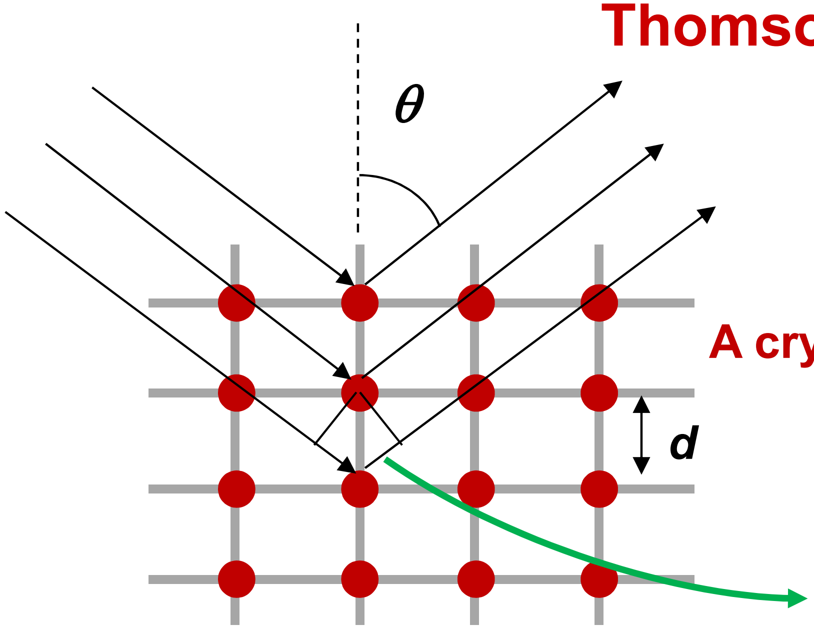
$$2\pi r = n\frac{\hbar 2\pi}{mv}$$

$$\Rightarrow L = mvr = n\hbar$$

Bohr quantization rule!!!

The fact that the De Broglie hypothesis led to the Bohr quantization rule was a phenomenal success for the hypothesis!

The De Broglie Hypothesis and the Davisson-Germer-Thomson Experiments



A crystal lattice of atoms

$d \cos\theta$

Clinton Davisson (Nobel Prize 1937) and Lester Germer

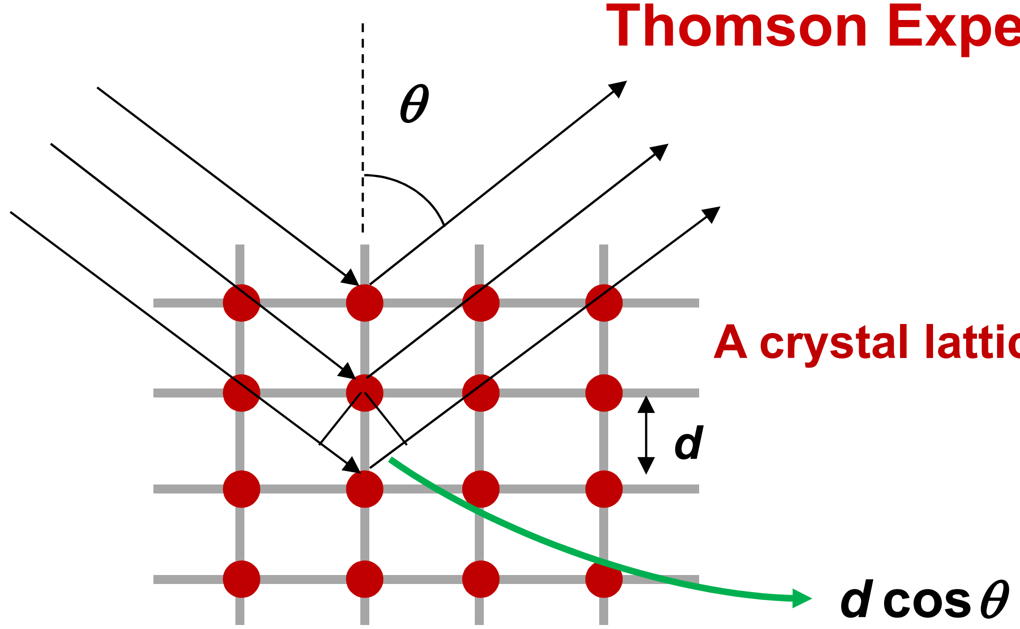
Sir George P. Thomson Nobel Prize 1937

People knew that x-rays can reflect from crystals in certain directions (for certain angles θ) very strongly – which happens when x-rays reflected from adjacent atomic planes add in phase for that direction (i.e. constructive interference)

Condition for enhanced reflection: $2d\cos\theta = n\lambda$

Why x-rays? Because *d*~3-6 Angstroms for most crystals and only x-rays have that short wavelengths

The De Broglie Hypothesis and the Davisson-Germer-Thomson Experiments



Can this principle of wave interference work for "electron waves" too?

According to the De Broglie hypothesis: $\lambda = \dfrac{\hbar 2\pi}{p} = \dfrac{\hbar 2\pi}{mv}$ **But:** $E = \dfrac{1}{2} m v^2$

So: $\lambda = \dfrac{2\pi\hbar}{\sqrt{2mE}}$

If we want λ~4 Angstroms, the electron energy *E* has to be around ~10 eV

The constructive interference in electron reflection from crystal atomic planes was experimentally seen by Davisson-Germer-Thomson during 1923-1927 experiments!!

Where Did Things Stand by 1927?

Is light a wave or is it made up of particles?

Is electron a wave or a particle?

Is the rest of matter waves or particles?

What does this De Broglie wave associated with an electron do? Or is the electron itself a wave? Or is the wave just accompanying the electron and guiding it along?

Every wave satisfies a wave equation. So what is the equation satisfied by the wave associated with an electron?