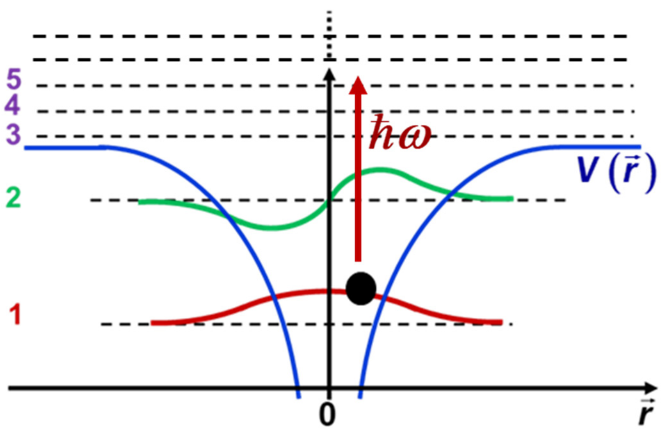
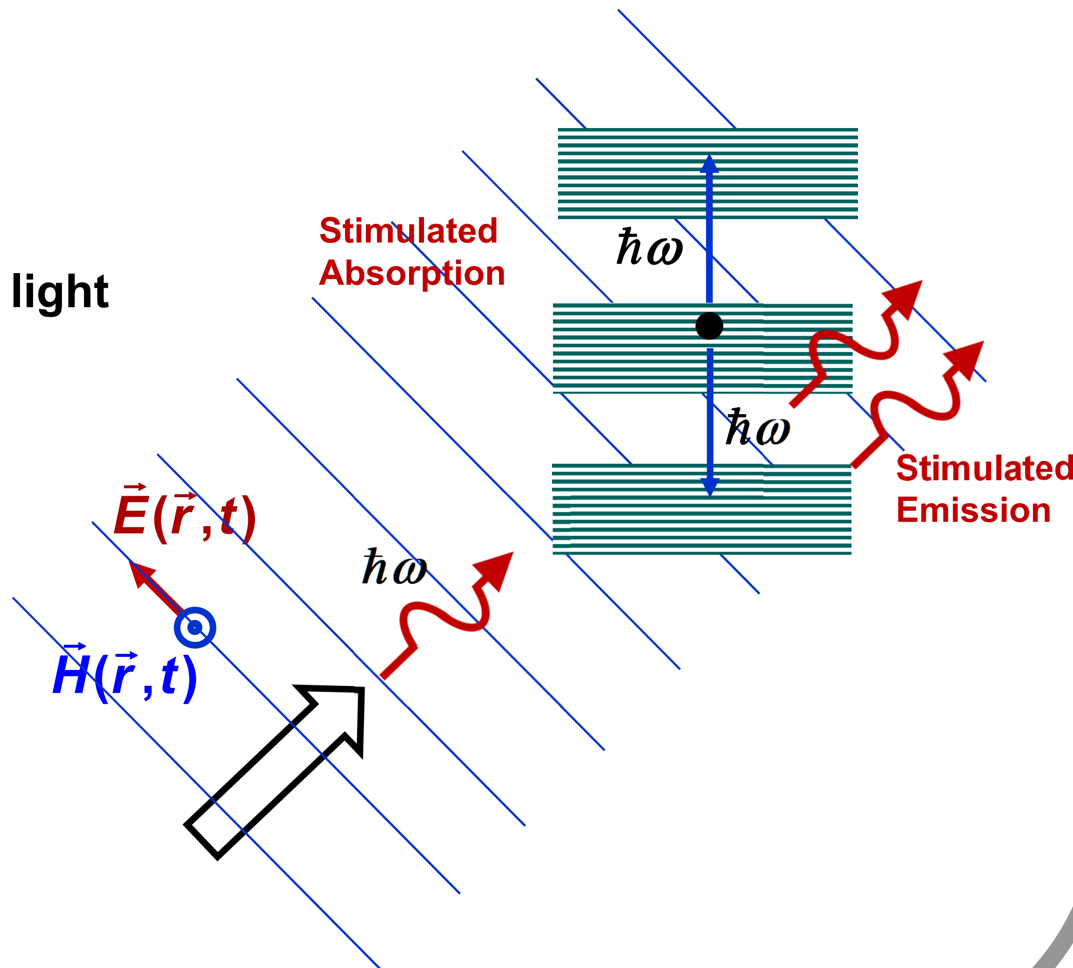
Lecture 19

Light-Matter Interaction and Optical Transitions – II
Time-Dependent Perturbation Theory and Fermi's Golden Rule

In this lecture you will learn:

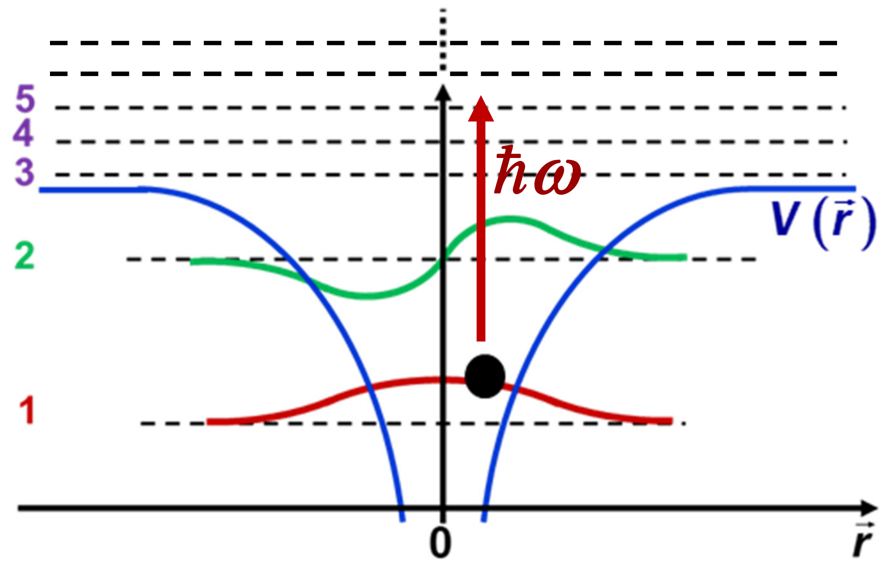
- **Time-dependent perturbation theory**
- **Transition rates**
- **Fermi's Golden Rule**
- **Stimulated emission and absorption of light**

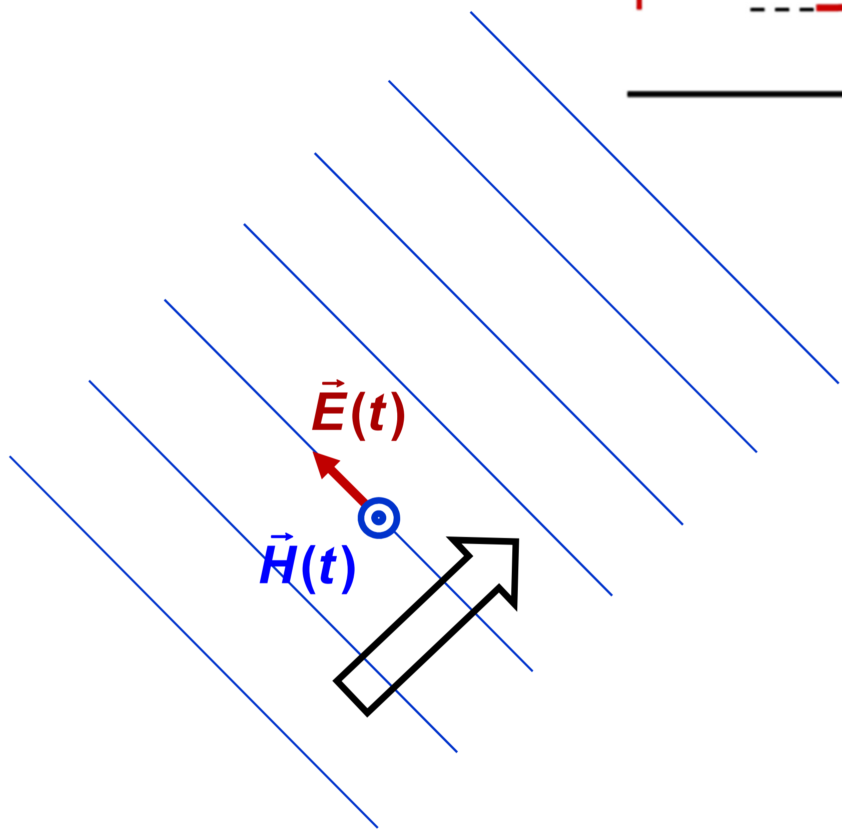




Optical Transition to a Continuum of Final States

What if the upper state is not an isolated discrete energy level but a continuum of energy levels ?



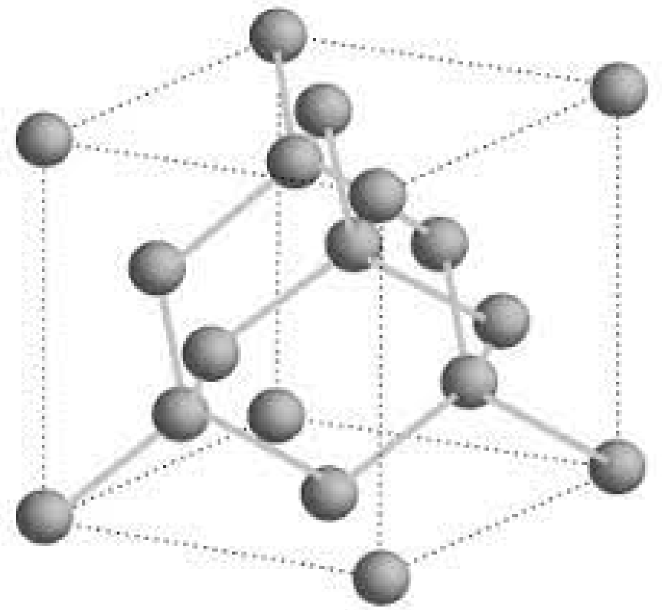


How to deal with the continuum of the upper energy states?

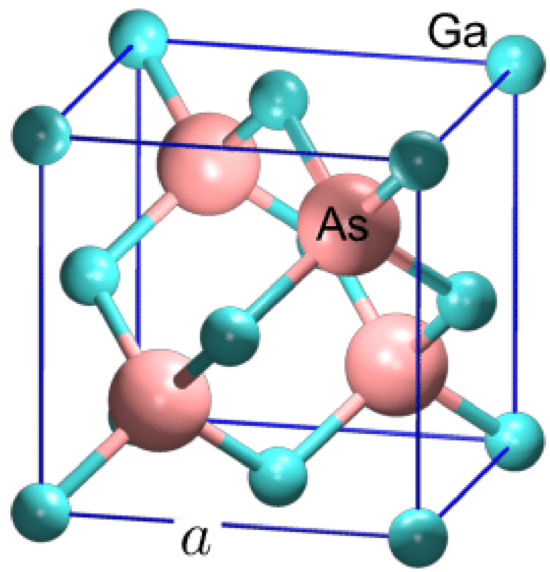
Will one still see Rabi oscillations?

Energy Bands in Crystalline Solids

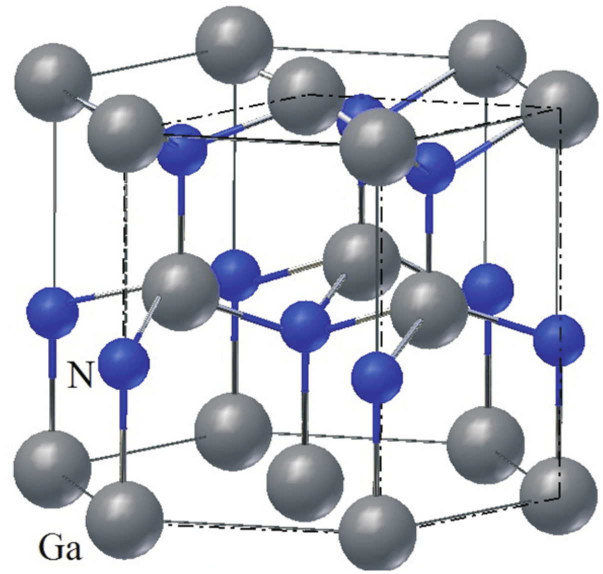
Silicon lattice



GaAs lattice



GaN lattice

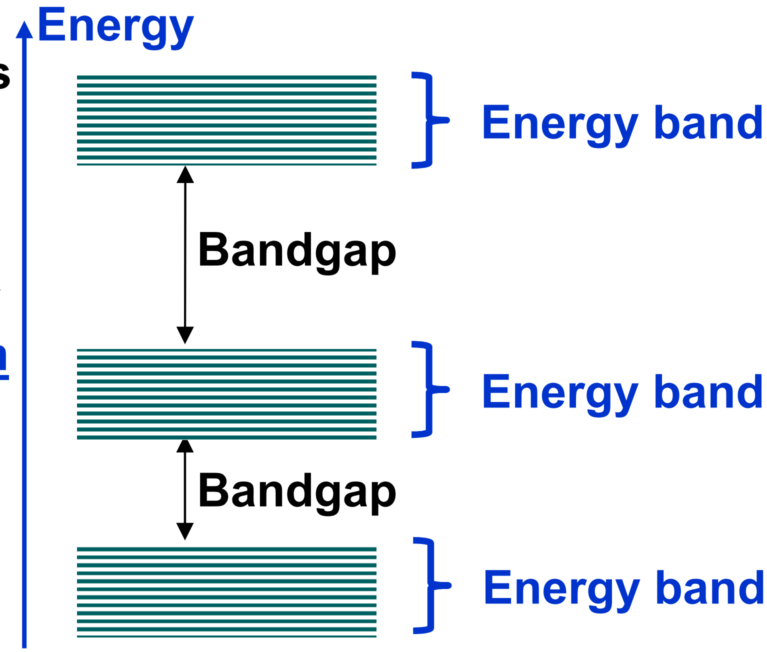


Energy Bands in Crystalline Solids

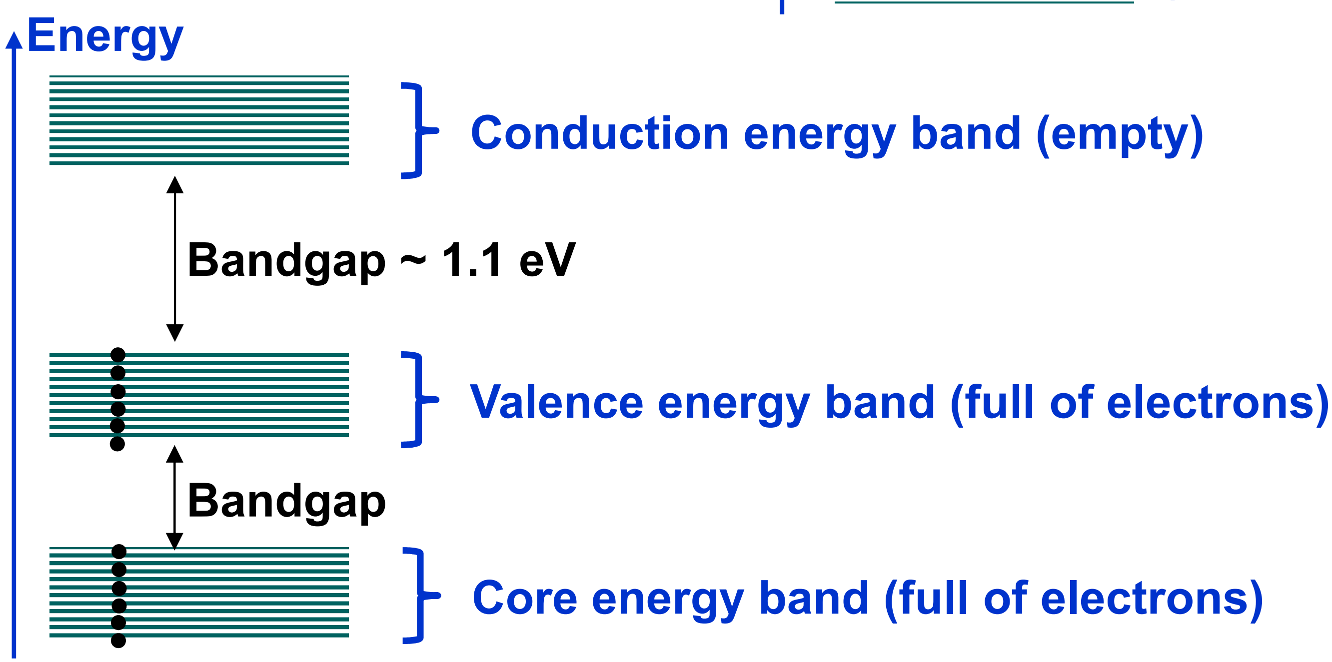
Crystalline materials have electron energy eigenstates distributed among different energy bands

An energy band is a collection of energy eigenstates whose energies are very close such that these energy eigenstates can be thought of as forming a continuum

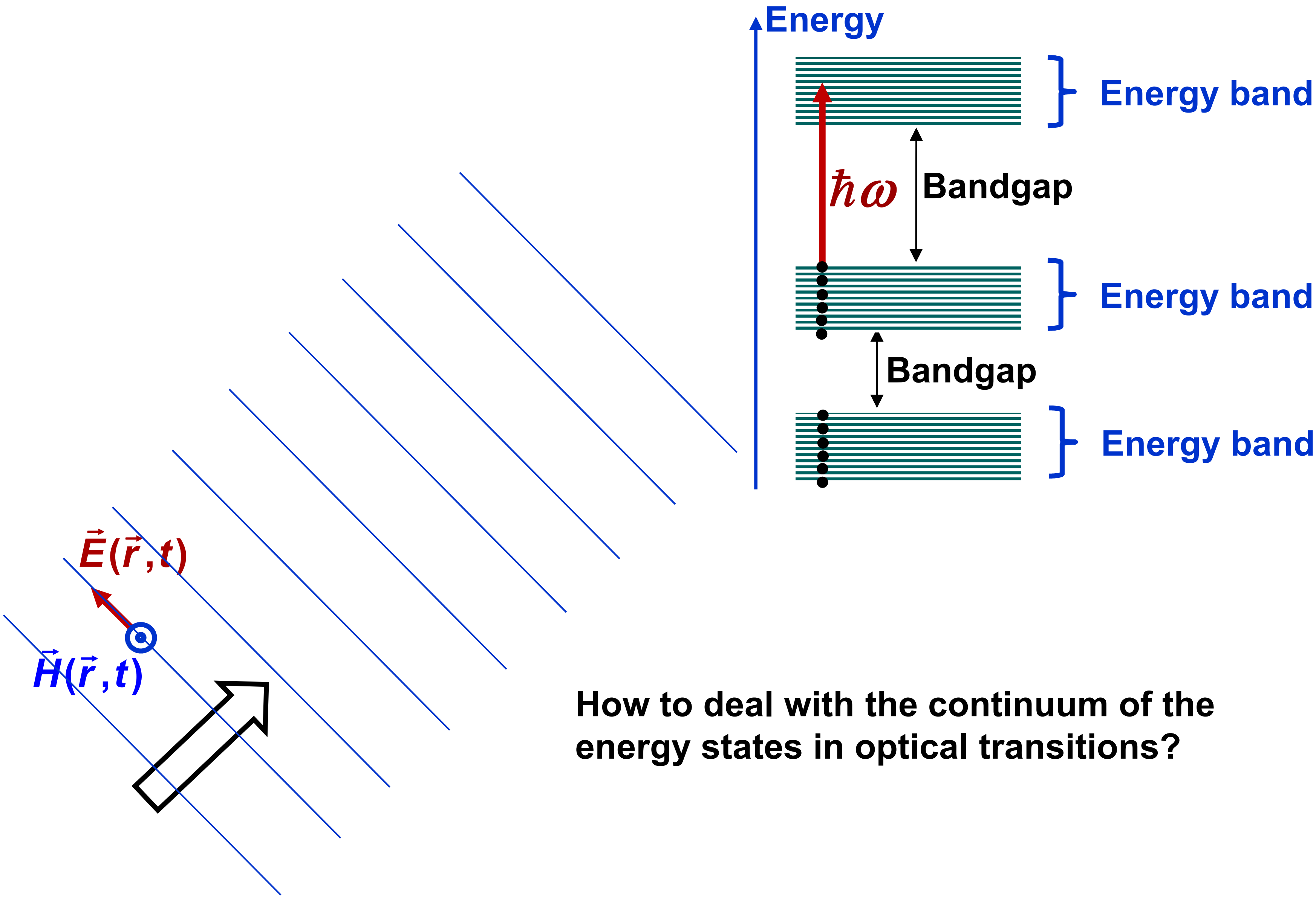
There are energy gaps between energy bands that are called bandgaps



Energy bands in Silicon



Optical Transitions in Crystalline Solids



How to deal with the continuum of the energy states in optical transitions?

Density of States (DOS) of Energy Bands

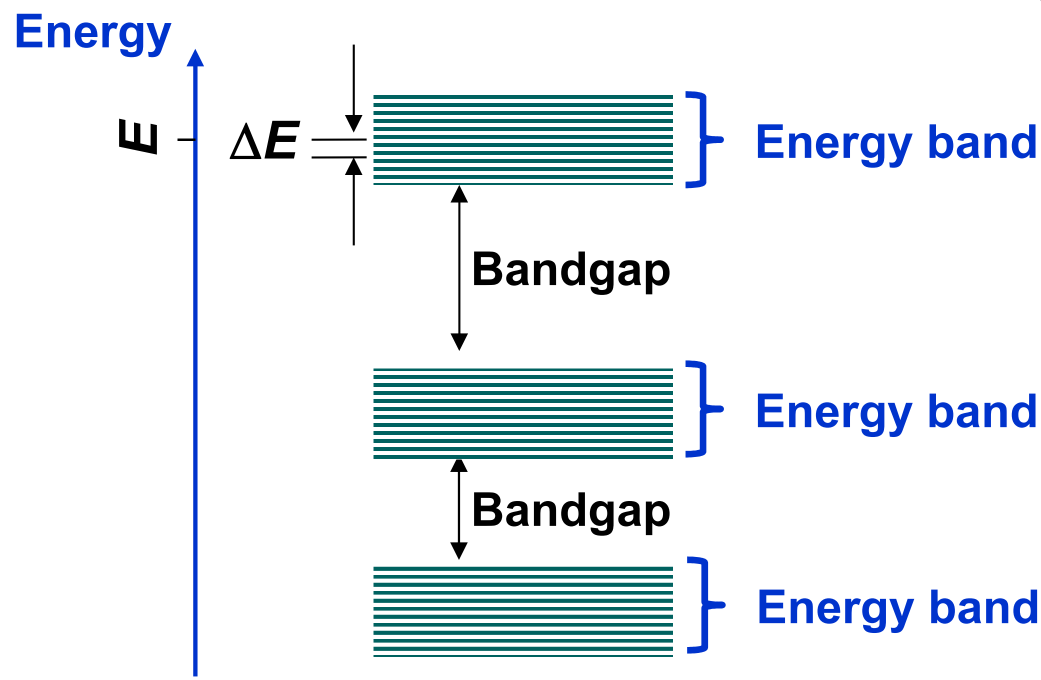
Consider a small interval ΔE of energy in an energy band at energy E

Question: How many energy eigenstates are in this energy interval?

Answer:
The number of energy eigenstates in a small energy interval ΔE equals:

$$D(E)\Delta E$$

The function $D(E)$ is the density of states (DOS) function of the energy band

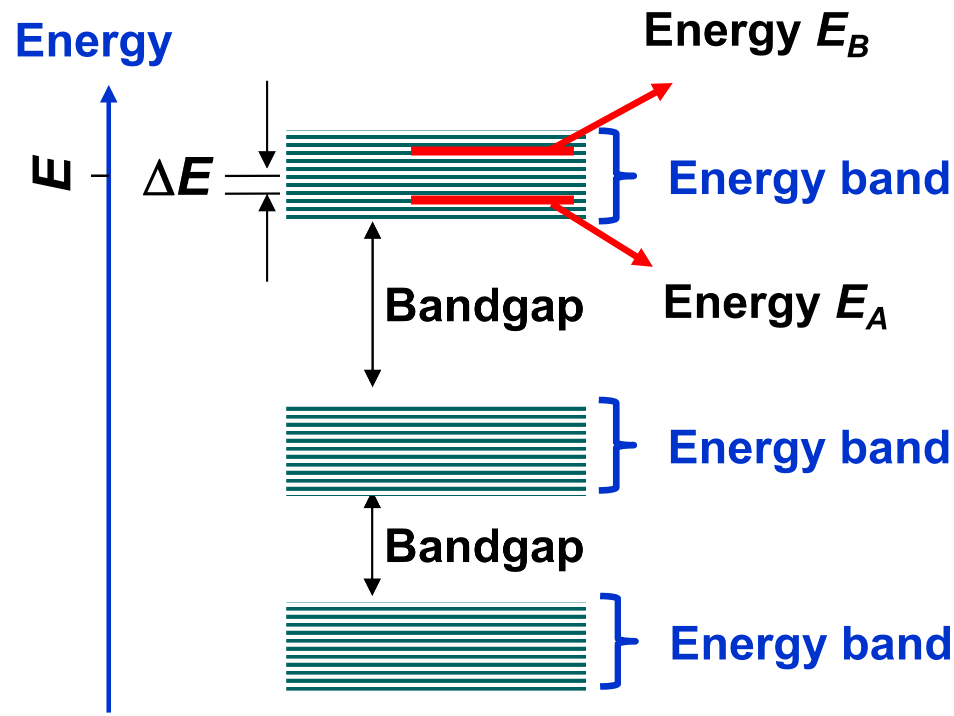


DOS function equals the number of energy eigenstates with energy E in an energy band per unit energy interval

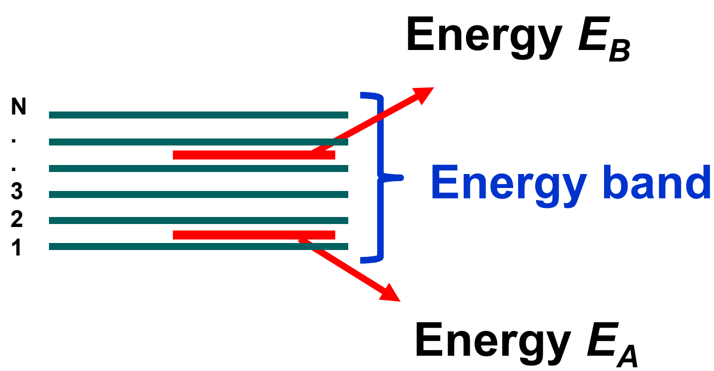
Density of States (DOS) of Energy Bands and Counting

The number of energy eigenstates in a small energy interval ΔE centered around energy E equals:

$$D(E)\Delta E$$



Suppose we need to count all the energy states between energy E_A and energy E_B, as shown

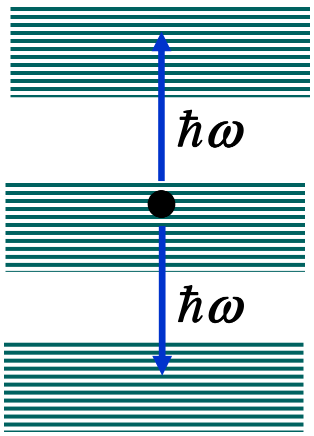


Answer:

$$\sum_{m}\{\text{All levels with energy between } E_A \text{ and } E_B\}$$

$$= \int_{E_A}^{E_B} dE\, D(E)$$

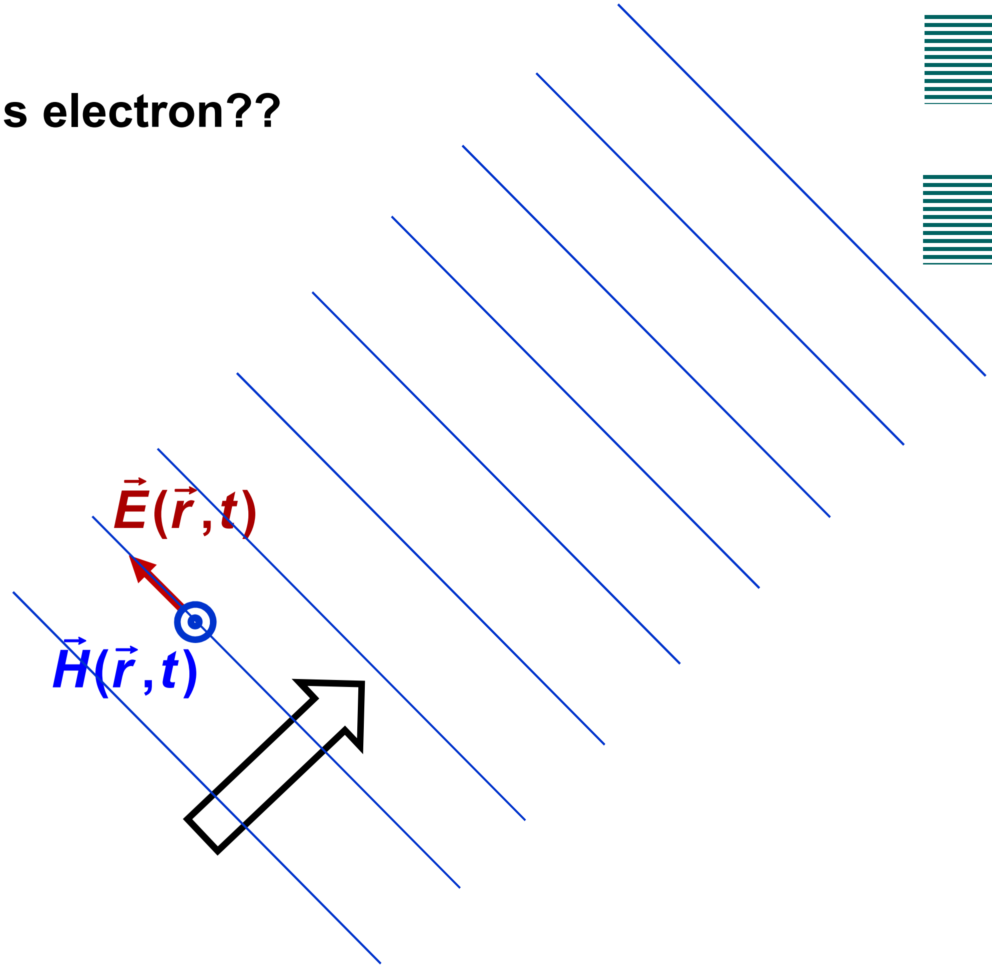
Light-Matter Interaction in Energy Bands



Consider an electron in a solid subjected to a time-dependent perturbation because light is passing through the solid

Question: What happens to this electron??

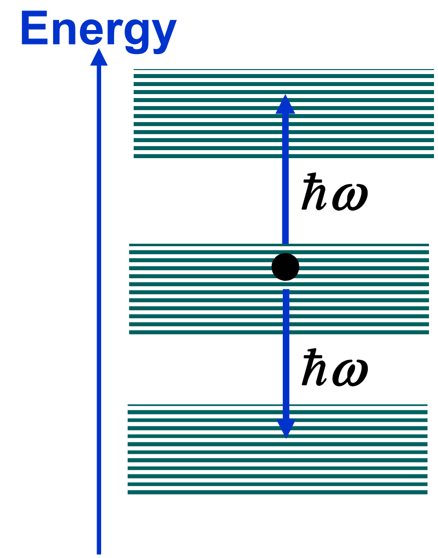
Assumption:
The perturbation is weak



The Hamiltonian is:

$$\hat{H}(t) = \frac{\hat{p}^2}{2m} + V\left(\hat{\vec{r}}\right) - q\vec{E}(t).\hat{\vec{r}}$$

Time-Dependent Perturbation Theory



$$\hat{H}(t) = \frac{\hat{p}^2}{2m} + V\left(\hat{\vec{r}}\right) - q\vec{E}(t).\hat{\vec{r}}$$

$$\vec{E}(\vec{r}_o, t) = \hat{n} E_o \cos(\omega t)$$

$$\hat{H}(t) = \underbrace{\frac{\hat{p}^2}{2m} + V\left(\hat{\vec{r}}\right)}_{\hat{H}_o} - qE_o \cos(\omega t)\,\hat{n}.\hat{\vec{r}}$$

$\hat{n}$: Light polarization direction unit vector

Lets generalize a bit:

$$\hat{H}(t) = \hat{H}_o + 2\hat{H}_p \cos(\omega t) = \hat{H}_o + \hat{H}_p\left[e^{i\omega t} + e^{-i\omega t}\right]$$

$\hat{H}_o$: Original Hamiltonian

$\hat{H}_p$: Perturbing Hamiltonian's operator part

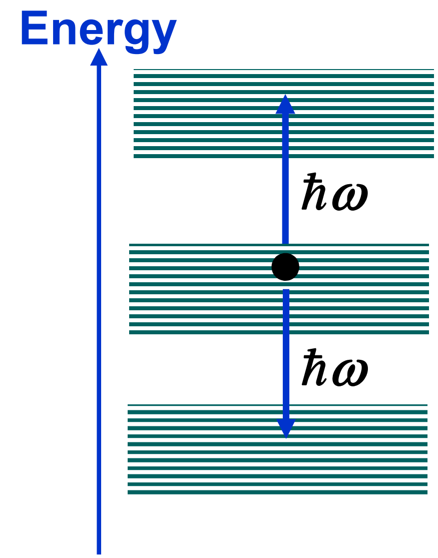
$e^{i\omega t}$: Makes the electron go down in energy

$e^{-i\omega t}$: Makes the electron go up in energy

For light-matter interaction:

$$\hat{H}_p = -q\frac{E_o}{2}\hat{n}.\hat{\vec{r}}$$

Time-Dependent Perturbation Theory



The full time-dependent Hamiltonian is:

$$\hat{H}(t) = \hat{H}_o + 2\hat{H}_p \cos(\omega t)$$

Energy eigenstates:

$$\hat{H}_o |e_j\rangle = E_j |e_j\rangle \qquad \sum_j |e_j\rangle\langle e_j| = \hat{1}$$

$$\langle e_j | e_k \rangle = \delta_{jk}$$

$$\hat{H}(t) = \hat{1}\,\hat{H}(t)\,\hat{1}$$

$$= \underbrace{\left(\sum_j E_j |e_j\rangle\langle e_j|\right)}_{\hat{H}_o} + 2\cos(\omega t) \sum_{j,k} \underbrace{\langle e_k | \hat{H}_p | e_j \rangle}_{\text{Matrix elements of the perturbing Hamiltonian}} |e_k\rangle\langle e_j|$$

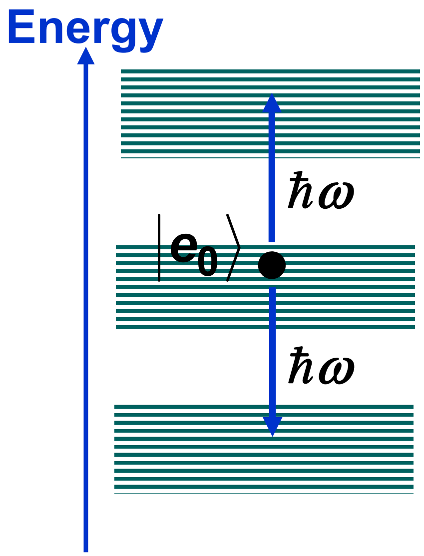
$$\langle e_k | \hat{H}_p | e_j \rangle = -\frac{qE_o d_{kj}}{2}$$

Time-Dependent Perturbation Theory

Need to solve:

$$i\hbar\frac{\partial|\psi(t)\rangle}{\partial t}=\hat{H}(t)|\psi(t)\rangle$$

Subject to the boundary condition: $|\psi(t=0)\rangle=|e_0\rangle$



Assume a solution of the form:

$$|\psi(t)\rangle=\sum_j c_j(t)e^{-i\frac{E_j}{\hbar}t}|e_j\rangle$$

Boundary condition →

$$|\psi(t=0)\rangle=|e_0\rangle$$
$$\Rightarrow c_j(t=0)=\delta_{j0}$$

And plug into the Schrödinger equation:

$$i\hbar\frac{\partial}{\partial t}|\psi(t)\rangle=\hat{H}(t)|\psi(t)\rangle$$

$$\Rightarrow\sum_j\left[e^{-i\frac{E_j}{\hbar}t}i\hbar\frac{\partial c_j(t)}{\partial t}+E_jc_j(t)e^{-i\frac{E_j}{\hbar}t}\right]|e_j\rangle=\left[\hat{H}_o+2\cos(\omega t)\hat{H}_p\right]\sum_j c_j(t)e^{-i\frac{E_j}{\hbar}t}|e_j\rangle$$

$$\Rightarrow\sum_j\left[e^{-i\frac{E_j}{\hbar}t}i\hbar\frac{\partial c_j(t)}{\partial t}+\cancel{E_jc_j(t)e^{-i\frac{E_j}{\hbar}t}}\right]|e_j\rangle=\cancel{\sum_j E_jc_j(t)e^{-i\frac{E_j}{\hbar}t}|e_j\rangle}$$

$$+2\cos(\omega t)\sum_j c_j(t)e^{-i\frac{E_j}{\hbar}t}\hat{H}_p|e_j\rangle$$

Time-Dependent Perturbation Theory

$$\sum_j \left[e^{-i\frac{E_j}{\hbar}t} i\hbar \frac{\partial c_j(t)}{\partial t} \right] |e_j\rangle = 2\cos(\omega t) \sum_j c_j(t) e^{-i\frac{E_j}{\hbar}t} \hat{H}_p |e_j\rangle$$

Multiply both sides from the left by $\langle e_m|$ (where $m \neq 0$) :

$$i\hbar \frac{\partial c_m(t)}{\partial t} = 2\cos(\omega t) \sum_j \langle e_m|\hat{H}_p|e_j\rangle c_j(t) e^{-i\frac{(E_j - E_m)}{\hbar}t}$$

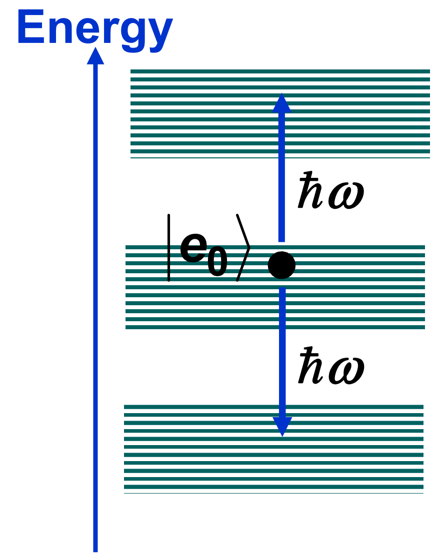
Keep only the j=0 term

Assume $c_{j=0}$ ~1

For times t not too large,

$$i\hbar \frac{\partial c_m(t)}{\partial t} = 2\cos(\omega t) \langle e_m|\hat{H}_p|e_0\rangle e^{-i\frac{(E_0 - E_m)}{\hbar}t}$$

$$\Rightarrow \frac{\partial c_m(t)}{\partial t} = -\frac{i}{\hbar} \langle e_m|\hat{H}_p|e_0\rangle \left[e^{-i\frac{(E_0 + \hbar\omega - E_m)}{\hbar}t} + e^{-i\frac{(E_0 - \hbar\omega - E_m)}{\hbar}t} \right]$$



Boundary condition

$$|\psi(t=0)\rangle = |e_0\rangle$$

$$\Rightarrow c_j(t=0) = \delta_{j0}$$

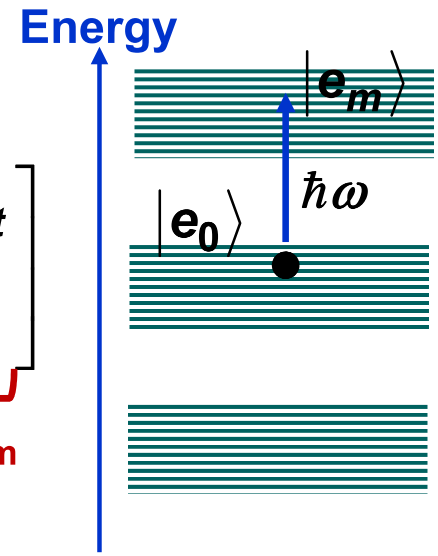
This equation shows that the coefficient $c_m(t)$ is increasing with time!

Time-Dependent Perturbation Theory: Upward Transitions

Focus on electron going up in energy (i.e. assume $E_m > E_0$):

$$\frac{\partial c_m(t)}{\partial t} = -\frac{i}{\hbar}\langle e_m|\hat{H}_p|e_0\rangle\left[\underbrace{e^{-i\frac{(E_0+\hbar\omega-E_m)}{\hbar}t}}_{\text{Resonant term}} + \underbrace{e^{-i\frac{(E_0-\hbar\omega-E_m)}{\hbar}t}}_{\text{Non-resonant term}}\right]$$

Ignore

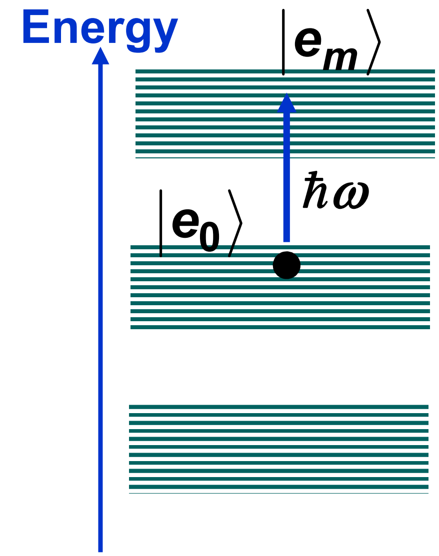


The non-resonant term is oscillating fast as a function of time and will not contribute much if the RHS is integrated wrt time to get the coefficient $c_m(t)$

Therefore, we can write:

$$\frac{\partial c_m(t)}{\partial t} = -\frac{i}{\hbar}\langle e_m|\hat{H}_p|e_0\rangle\underbrace{e^{-i\frac{(E_0+\hbar\omega-E_m)}{\hbar}t}}_{\text{Resonant term only}}$$

Time-Dependent Perturbation Theory: Upward Transitions



Now we integrate wrt time from *t*=0 to *t* :

$$\frac{\partial c_m(t)}{\partial t} = -\frac{i}{\hbar}\langle e_m|\hat{H}_p|e_0\rangle e^{-i\frac{(E_0+\hbar\omega-E_m)}{\hbar}t}$$

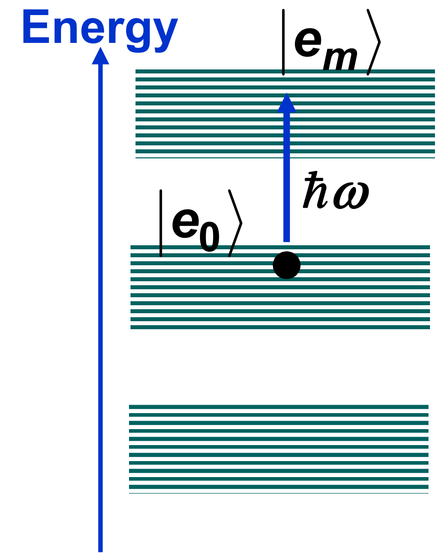
$$\Rightarrow c_m(t) = -\frac{i}{\hbar}\langle e_m|\hat{H}_p|e_0\rangle \int_0^t dt' e^{-i\frac{(E_0+\hbar\omega-E_m)}{\hbar}t'}$$

$$\Rightarrow c_m(t) = -\frac{i}{\hbar}\langle e_m|\hat{H}_p|e_0\rangle t\left[\frac{\sin\left[\frac{(E_0+\hbar\omega-E_m)}{2\hbar}t\right]}{\frac{(E_0+\hbar\omega-E_m)t}{2\hbar}}\right] e^{-i\frac{(E_0+\hbar\omega-E_m)}{2\hbar}t}$$

$$\Rightarrow |c_m(t)|^2 = \frac{1}{\hbar^2}\left|\langle e_m|\hat{H}_p|e_0\rangle\right|^2 t^2\left[\frac{\sin\left[\frac{(E_0+\hbar\omega-E_m)}{2\hbar}t\right]}{\frac{(E_0+\hbar\omega-E_m)t}{2\hbar}}\right]^2$$

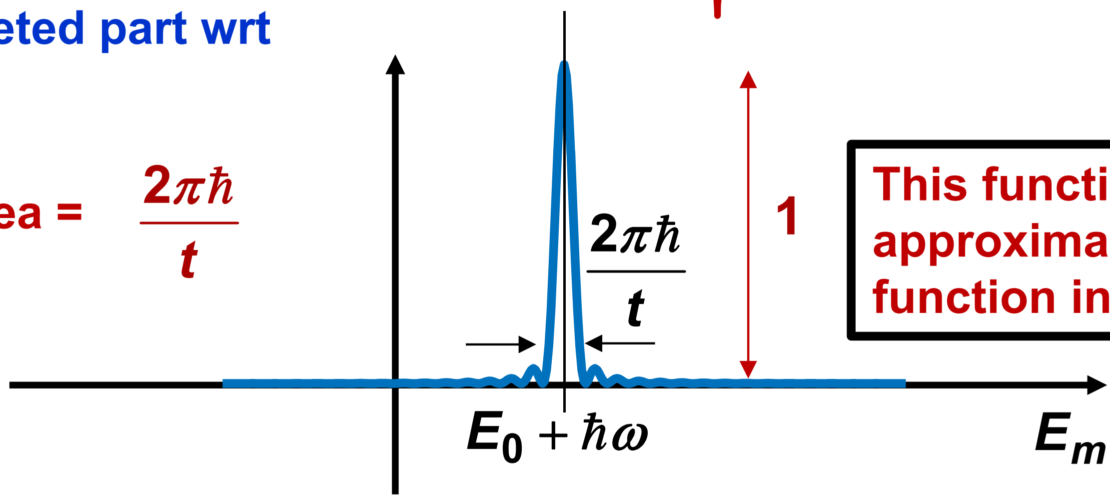
Time-Dependent Perturbation Theory: Upward Transitions

$$|c_m(t)|^2 = \frac{1}{\hbar^2}\left|\langle e_m|\hat{H}_p|e_0\rangle\right|^2 t^2 \left[\frac{\sin\left[\frac{(E_0+\hbar\omega-E_m)}{2\hbar}t\right]}{\frac{(E_0+\hbar\omega-E_m)t}{2\hbar}}\right]^2$$



Plot the bracketed part wrt to energy E_m :

Integrated area = $\frac{2\pi\hbar}{t}$

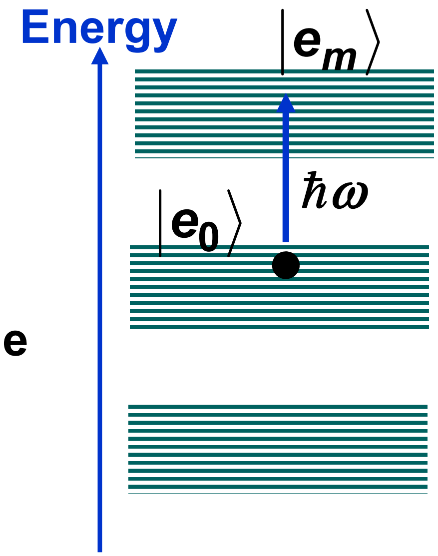


This function can be approximated as a delta function in energy !

$$|c_m(t)|^2 = \frac{1}{\hbar^2}\left|\langle e_m|\hat{H}_p|e_0\rangle\right|^2 t^2 \left[\frac{2\pi\hbar}{t}\delta(E_0+\hbar\omega-E_m)\right]$$

$$\Rightarrow \frac{d|c_m(t)|^2}{dt} = \frac{2\pi}{\hbar}\left|\langle e_m|\hat{H}_p|e_0\rangle\right|^2 \delta(E_0+\hbar\omega-E_m)$$

Time-Dependent Perturbation Theory: Upward Transition Rates



$$\Rightarrow \frac{d|c_m(t)|^2}{dt} = \frac{2\pi}{\hbar}\left|\langle e_m|\hat{H}_p|e_0\rangle\right|^2 \delta(E_0 + \hbar\omega - E_m)$$

$\frac{d|c_m(t)|^2}{dt}$ represents the rate of increase of the probability in the upper level m

The <u>total</u> transition rate for the electron to go to the higher energy states is given by:

$$R\uparrow = \sum_{m\neq 0} \frac{d|c_m(t)|^2}{dt} = \sum_{m\neq 0} \frac{2\pi}{\hbar}\left|\langle e_m|\hat{H}_p|e_0\rangle\right|^2 \delta(E_0 + \hbar\omega - E_m)$$

Replace summation over "m" by an integral over energy

$$= \int dE\, D(E)\, \frac{2\pi}{\hbar}\left|\langle e_E|\hat{H}_p|e_0\rangle\right|^2 \delta(E_0 + \hbar\omega - E)$$

$$= D(E_0 + \hbar\omega)\, \frac{2\pi}{\hbar}\left|\langle e_{E_0+\hbar\omega}|\hat{H}_p|e_0\rangle\right|^2$$

Fermi's Golden Rule

DOS evaluated at the electron final energy

Time-Dependent Perturbation Theory: Downward Transition Rates

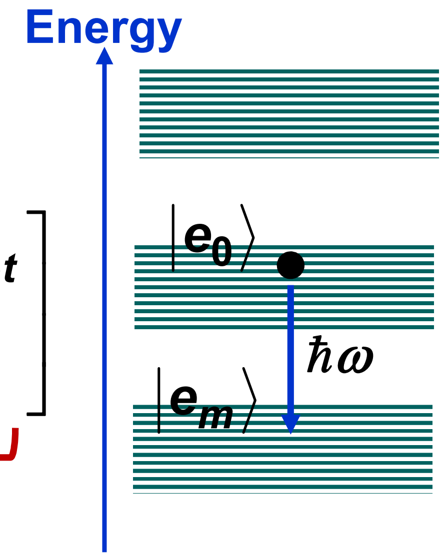
Focus on electron going down in energy now (i.e. assume $E_m < E_0$):

$$\frac{\partial c_m(t)}{\partial t} = -\frac{i}{\hbar}\langle e_m|\hat{H}_p|e_0\rangle\left[e^{-i\frac{(E_0+\hbar\omega-E_m)}{\hbar}t} + e^{-i\frac{(E_0-\hbar\omega-E_m)}{\hbar}t}\right]$$

Ignore

Non-resonant term

Resonant term



The total transition rate for the electron to go to the lower energy states is given by:

$$R\downarrow = \sum_{m\neq 0}\frac{d|c_m(t)|^2}{dt} = \sum_{m\neq 0}\frac{2\pi}{\hbar}\left|\langle e_m|\hat{H}_p|e_0\rangle\right|^2\delta(E_0-\hbar\omega-E_m)$$

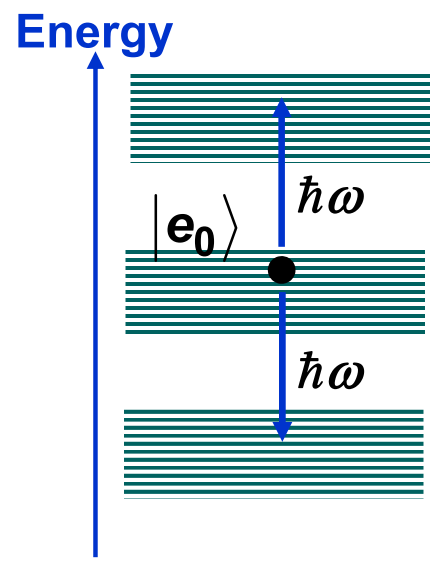
$$= \int dE\, D(E)\frac{2\pi}{\hbar}\left|\langle e_E|\hat{H}_p|e_0\rangle\right|^2\delta(E_0-\hbar\omega-E)$$

$$= D(E_0-\hbar\omega)\frac{2\pi}{\hbar}\left|\langle e_{E_0-\hbar\omega}|\hat{H}_p|e_0\rangle\right|^2$$

Fermi's Golden Rule

DOS evaluated at the electron final energy

Exponential Decay of the Initial State



With little extra work one can also find out how the probability of the electron being in the initial state is behaving with time:

$$\frac{d\left|c_0(t)\right|^2}{dt} = -\left(R\uparrow + R\downarrow\right)\left|c_0(t)\right|^2$$

Solution, subject to the boundary condition $c_0(t=0)=1$, is:

$$\left|c_0(t)\right|^2 = e^{-\left(R\uparrow + R\downarrow\right)t}$$

The probability of the electron being in the initial state decays exponentially with time and the decay constant is related to the transition rates to the higher and lower energy states

Notice also the probability conservation:

$$\frac{d\left|c_0(t)\right|^2}{dt} + \sum_{m\neq 0} \frac{d\left|c_m(t)\right|^2}{dt} = 0$$

$$\Rightarrow \left|c_0(t)\right|^2 + \sum_{m\neq 0} \left|c_m(t)\right|^2 = 1$$

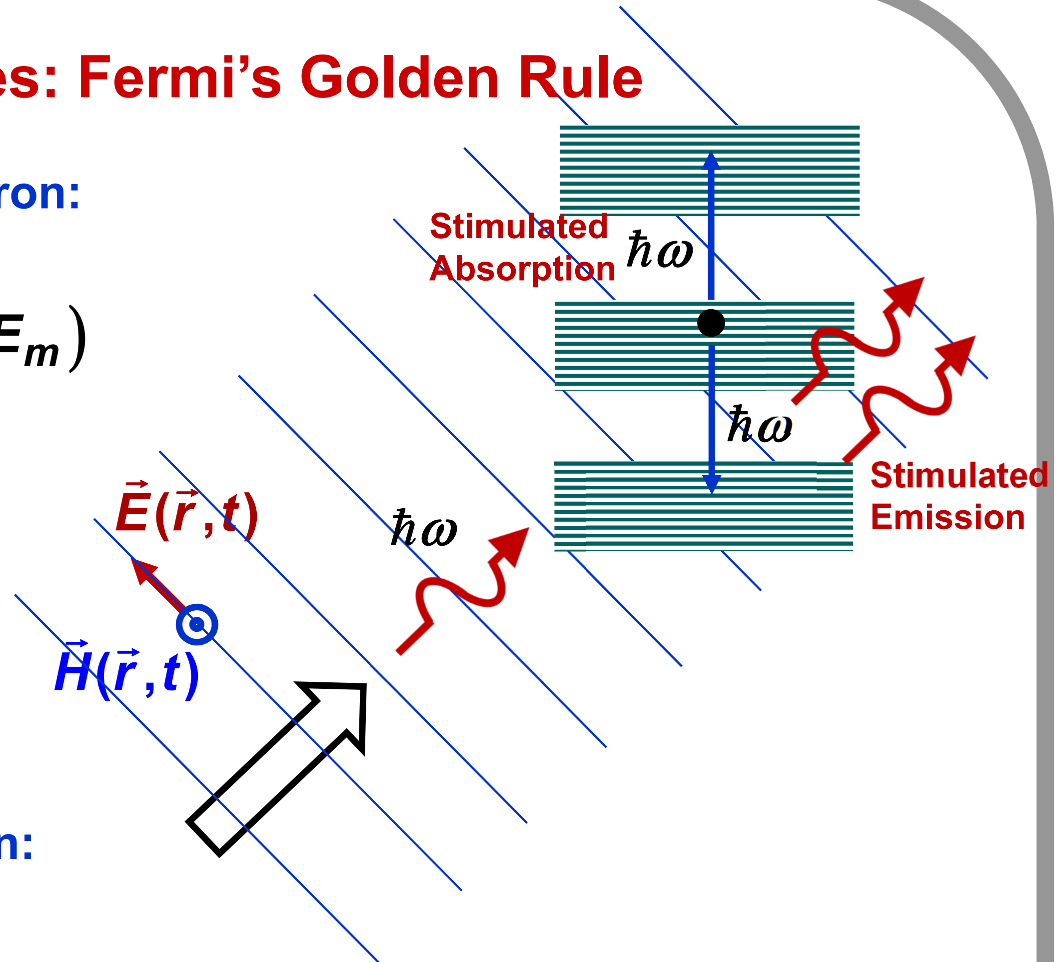
Optical Transition Rates: Fermi's Golden Rule

Stimulated Absorption of Light by an Electron:

$$R\uparrow = \sum_m \frac{2\pi}{\hbar}\left|\langle e_m|\hat{H}_p|e_0\rangle\right|^2 \delta\left(E_0+\hbar\omega-E_m\right)$$

$$= D\left(E_0+\hbar\omega\right)\frac{2\pi}{\hbar}\left|\langle e_{E_0+\hbar\omega}|\hat{H}_p|e_0\rangle\right|^2$$

$$= D\left(E_0+\hbar\omega\right)\frac{2\pi}{\hbar}\left|\frac{qE_o d}{2}\right|^2$$



Stimulated Emission of Light by an Electron:

$$R\downarrow = \sum_m \frac{2\pi}{\hbar}\left|\langle e_m|\hat{H}_p|e_0\rangle\right|^2 \delta\left(E_0-\hbar\omega-E_m\right)$$

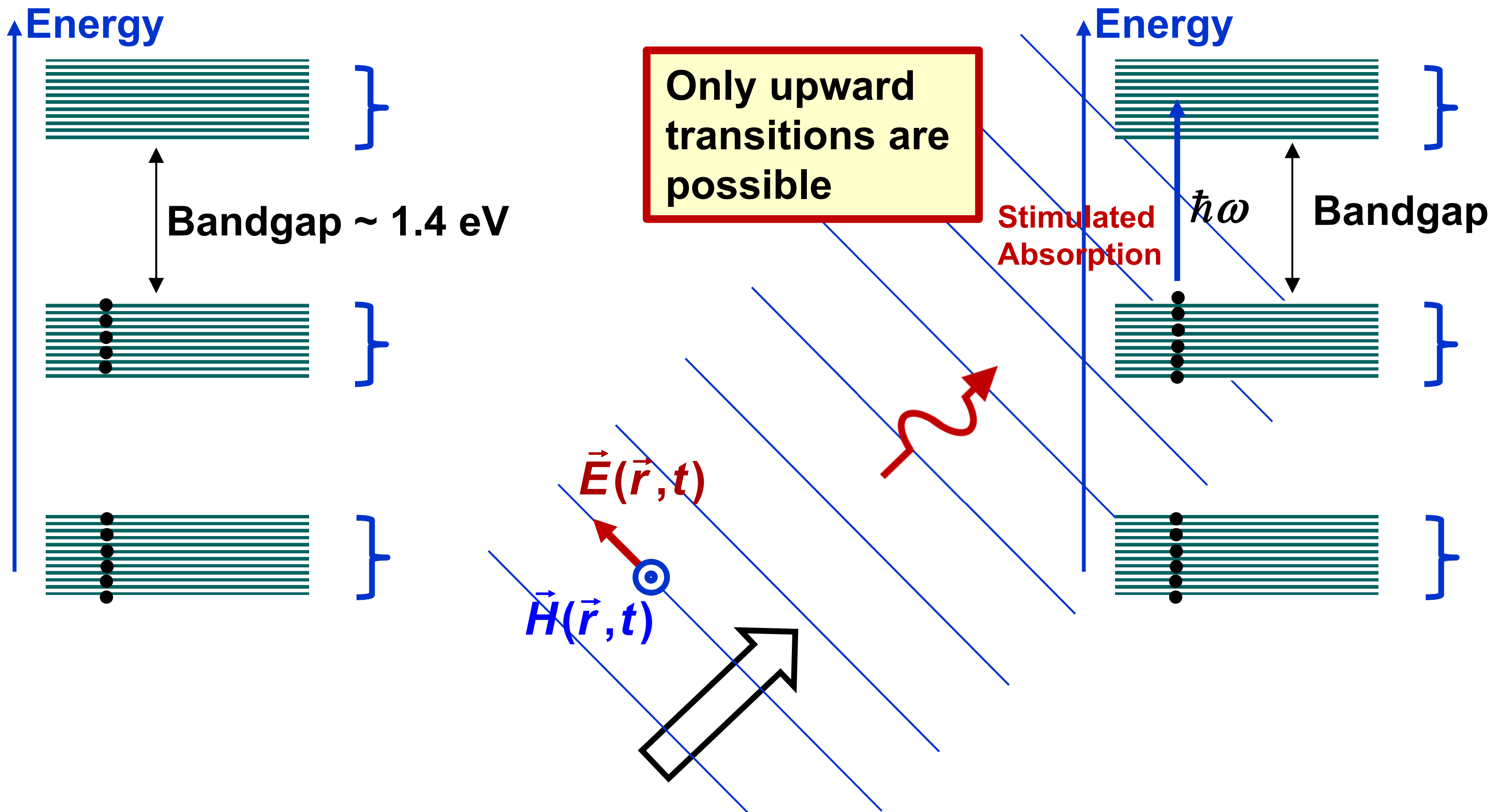
$$= D\left(E_0-\hbar\omega\right)\frac{2\pi}{\hbar}\left|\langle e_{E_0-\hbar\omega}|\hat{H}_p|e_0\rangle\right|^2$$

$$= D\left(E_0-\hbar\omega\right)\frac{2\pi}{\hbar}\left|\frac{qE_o d}{2}\right|^2$$

d **= dipole matrix element between the initial and final states of the electron**

Light Absorption in Crystalline Solids

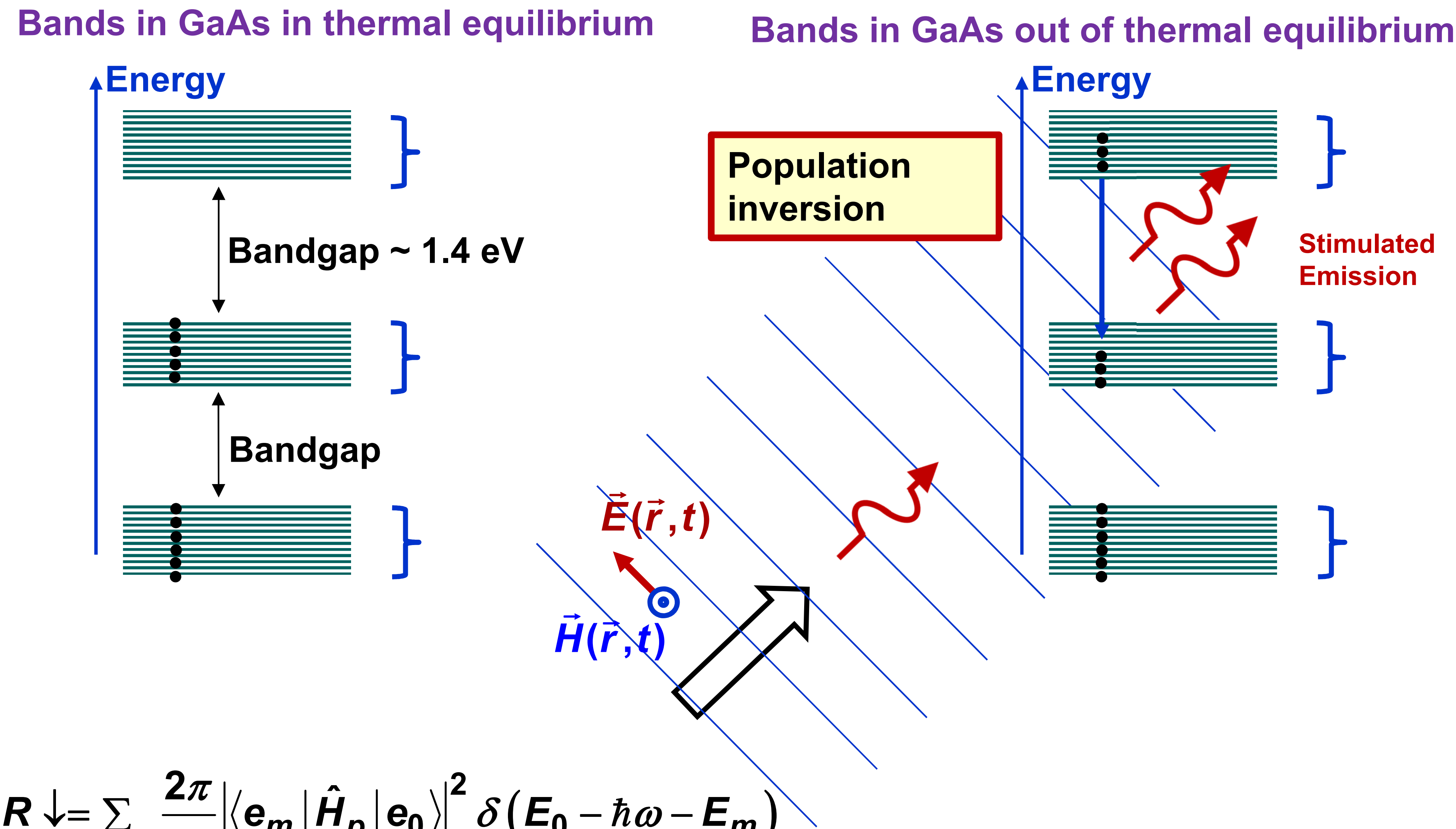
Bands in GaAs in thermal equilibrium



$$R\uparrow = \sum_{m\neq 0} \frac{2\pi}{\hbar}\left|\left\langle e_m \left| \hat{H}_p \right| e_0 \right\rangle\right|^2 \delta\left(E_0 + \hbar\omega - E_m\right)$$

$$= D\left(E_0 + \hbar\omega\right)\frac{2\pi}{\hbar}\left|\left\langle e_{E_0+\hbar\omega} \left| \hat{H}_p \right| e_0 \right\rangle\right|^2$$

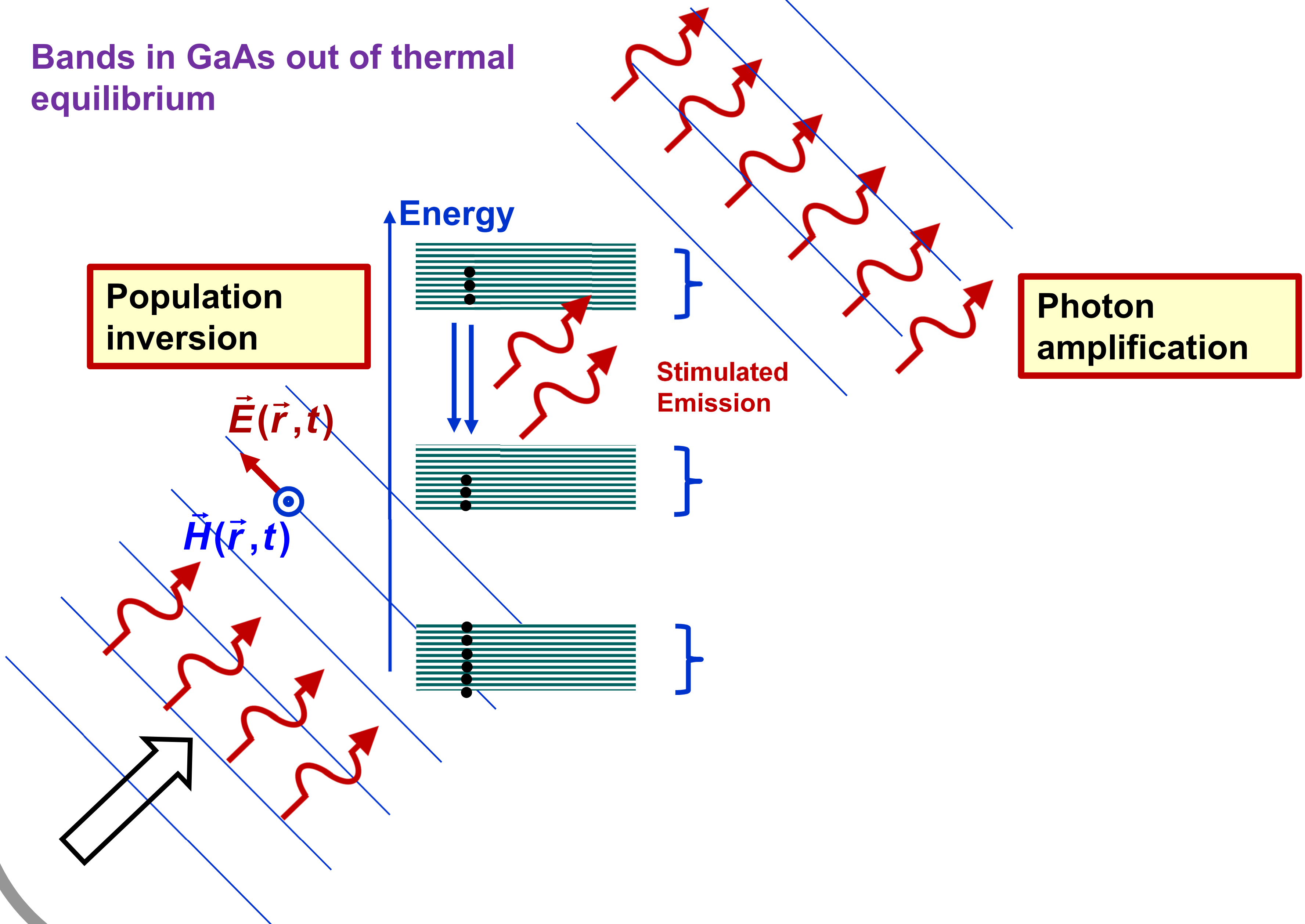
Light Amplification by Stimulated Emission of Radiation (LASER)



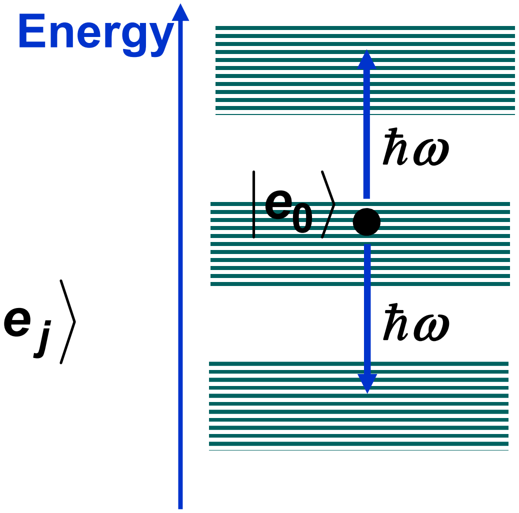
$$R\downarrow = \sum_m \frac{2\pi}{\hbar}\left|\langle e_m|\hat{H}_p|e_0\rangle\right|^2 \delta\left(E_0-\hbar\omega-E_m\right)$$

$$= D\left(E_0-\hbar\omega\right)\frac{2\pi}{\hbar}\left|\frac{qE_o d}{2}\right|^2$$

Light Amplification by Stimulated Emission of Radiation (LASER)



Appendix: Absence of Rabi Oscillations in Transitions to a Continuum



We start from the general equation derived earlier:

$$\sum_j \left[e^{-i\frac{E_j}{\hbar}t} i\hbar \frac{\partial c_j(t)}{\partial t} \right] |e_j\rangle = 2\cos(\omega t) \sum_j c_j(t) e^{-i\frac{E_j}{\hbar}t} \hat{H}_p |e_j\rangle$$

Multiply both sides from the left by $\langle e_m|$ **(where** $m \neq 0$ **) :**

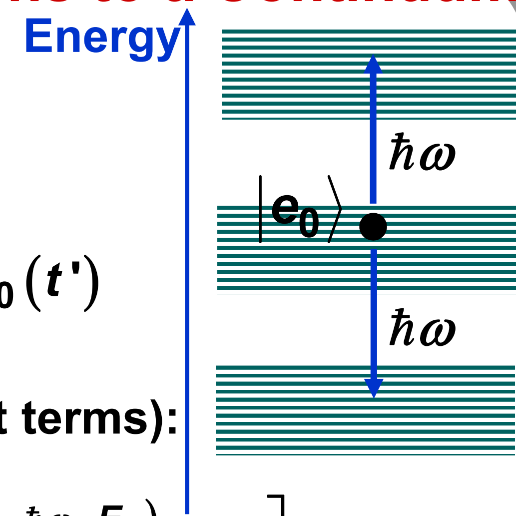
$$\frac{\partial c_m(t)}{\partial t} = -\frac{i}{\hbar}\langle e_m|\hat{H}_p|e_0\rangle \left[e^{-i\frac{(E_0+\hbar\omega-E_m)}{\hbar}t} + e^{-i\frac{(E_0-\hbar\omega-E_m)}{\hbar}t} \right] c_0(t) \longrightarrow (1)$$

Multiply both sides from the left by $\langle e_0|$ **in the general equation to get:**

$$\frac{\partial c_0(t)}{\partial t} = \sum_{m\neq 0} -\frac{i}{\hbar}\langle e_0|\hat{H}_p|e_m\rangle \left[e^{-i\frac{(E_m+\hbar\omega-E_0)}{\hbar}t} + e^{-i\frac{(E_m-\hbar\omega-E_0)}{\hbar}t} \right] c_m(t) \longrightarrow (2)$$

Now we will solve (1) and stick its solution in (2)

Appendix: Absence of Rabi Oscillations in Transitions to a Continuum



Solution of (1) by direct integration is:

$$c_m(t) = -\frac{i}{\hbar}\langle e_m|\hat{H}_p|e_0\rangle \int_0^t dt' \left[e^{-i\frac{(E_0+\hbar\omega-E_m)}{\hbar}t'} + e^{-i\frac{(E_0-\hbar\omega-E_m)}{\hbar}t'} \right] c_0(t')$$

Substitute the above in (2) to get (and ignoring some unimportant terms):

$$\frac{\partial c_0(t)}{\partial t} = -\sum_{m\neq 0} \frac{1}{\hbar^2}\left|\langle e_0|\hat{H}_p|e_m\rangle\right|^2 \int_0^t dt' \left[e^{-i\frac{(E_m+\hbar\omega-E_0)}{\hbar}(t-t')} + e^{-i\frac{(E_m-\hbar\omega-E_0)}{\hbar}(t-t')} \right] c_0(t')$$

The above is an equation for $c_0(t)$ alone!

Now change the order of time integration and summation over "*m*" and then convert the summation into an integral over energy using the density of states *D*(*E*) :

$$\frac{\partial c_0(t)}{\partial t} = -\int_0^t dt' \int dE\, D(E) \frac{1}{\hbar^2}\left|\langle e_0|\hat{H}_p|e_E\rangle\right|^2 \left[e^{-i\frac{(E+\hbar\omega-E_0)}{\hbar}(t-t')} + e^{-i\frac{(E-\hbar\omega-E_0)}{\hbar}(t-t')} \right] c_0(t')$$

Appendix: Absence of Rabi Oscillations in Transitions to a Continuum

$$\frac{\partial c_0(t)}{\partial t} = -\int_0^t dt' \left[\begin{array}{l} \int dE\, D(E) \frac{1}{\hbar^2} \left| \langle e_0 | \hat{H}_p | e_E \rangle \right|^2 e^{-i\frac{(E+\hbar\omega-E_0)}{\hbar}(t-t')} \\ + \quad \int dE\, D(E) \frac{1}{\hbar^2} \left| \langle e_0 | \hat{H}_p | e_E \rangle \right|^2 e^{-i\frac{(E-\hbar\omega-E_0)}{\hbar}(t-t')} \end{array} \right] c_0(t')$$

Note that the integration over time implies that in the first term (or second term) inside the brackets, energies that matter are those for which $E = E_0 - \hbar\omega$ (or $E = E_0 + \hbar\omega$)

So if $D(E)$ and $\left| \langle e_0 | \hat{H}_p | e_E \rangle \right|$ are not strong functions of energy around the energies $E = E_0 \pm \hbar\omega$ then one may write:

$$\frac{\partial c_0(t)}{\partial t} = -\frac{1}{\hbar^2}\int_0^t dt' \left[\begin{array}{l} D(E_0 - \hbar\omega) \left| \langle e_0 | \hat{H}_p | e_{E_0-\hbar\omega} \rangle \right|^2 \int dE\, e^{-i\frac{(E+\hbar\omega-E_0)}{\hbar}(t-t')} \\ + \quad D(E_0 + \hbar\omega) \left| \langle e_0 | \hat{H}_p | e_{E_0+\hbar\omega} \rangle \right|^2 \int dE\, e^{-i\frac{(E-\hbar\omega-E_0)}{\hbar}(t-t')} \end{array} \right] c_0(t')$$

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$$\frac{\partial c_0(t)}{\partial t} = -\frac{1}{\hbar^2}\int_0^t dt' \left[\begin{array}{l} D(E_0-\hbar\omega)\left|\langle e_0|\hat{H}_p|e_{E_0-\hbar\omega}\rangle\right|^2 \int dE\, e^{-i\frac{(E+\hbar\omega-E_0)}{\hbar}(t-t')} \\ + \quad D(E_0+\hbar\omega)\left|\langle e_0|\hat{H}_p|e_{E_0+\hbar\omega}\rangle\right|^2 \int dE\, e^{-i\frac{(E-\hbar\omega-E_0)}{\hbar}(t-t')} \end{array} \right] c_0(t')$$

Now note that:

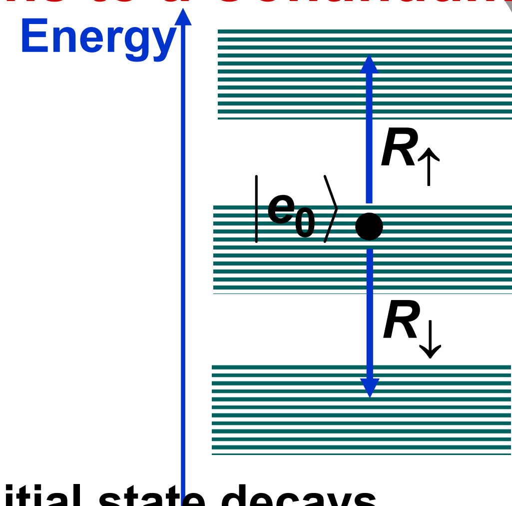
$$\int dE\, e^{-i\frac{(E\pm\hbar\omega-E_0)}{\hbar}(t-t')} = 2\pi\hbar\delta(t-t') \longrightarrow \text{(3)}$$

Carry out the time integrations and use (3) to get:

$$\frac{\partial c_0(t)}{\partial t} = -\left[\frac{\pi}{\hbar}D(E_0-\hbar\omega)\left|\langle e_0|\hat{H}_p|e_{E_0-\hbar\omega}\rangle\right|^2 + \frac{\pi}{\hbar}D(E_0+\hbar\omega)\left|\langle e_0|\hat{H}_p|e_{E_0+\hbar\omega}\rangle\right|^2\right]c_0(t)$$

$$= -\frac{1}{2}\left(R_\downarrow + R_\uparrow\right)c_0(t)$$

Appendix: Absence of Rabi Oscillations in Transitions to a Continuum



$$\frac{\partial c_0(t)}{\partial t} = -\frac{1}{2}\left(R_\downarrow + R_\uparrow\right)c_0(t)$$

Which implies:

$$\frac{\partial |c_0(t)|^2}{\partial t} = -\left(R_\downarrow + R_\uparrow\right)|c_0(t)|^2$$

This shows that the probability of the electron remaining in the initial state decays exponentially and does not exhibit any Rabi oscillations!

So why do we not see Rabi oscillations? Why doesn't $c_0(t)$ become large ever again? The answer lies in these two equations we had written down earlier:

$$\frac{\partial c_m(t)}{\partial t} = -\frac{i}{\hbar}\langle e_m|\hat{H}_p|e_0\rangle\left[e^{-i\frac{(E_0+\hbar\omega-E_m)}{\hbar}t} + e^{-i\frac{(E_0-\hbar\omega-E_m)}{\hbar}t}\right]c_0(t) \longrightarrow (1)$$

$$\frac{\partial c_0(t)}{\partial t} = \sum_{m\neq 0} -\frac{i}{\hbar}\langle e_0|\hat{H}_p|e_m\rangle\left[e^{-i\frac{(E_m+\hbar\omega-E_0)}{\hbar}t} + e^{-i\frac{(E_m-\hbar\omega-E_0)}{\hbar}t}\right]c_m(t) \longrightarrow (2)$$

Because different energy levels E_m have different detunings $\left(E_m \mp \hbar\omega + E_0\right)$, different coefficients $c_m(t)$ acquire different relative phases by (1). Consequently, they interfere destructively in (2) and, therefore, $c_0(t)$ never gets regenerated after it has decreased from its initial value