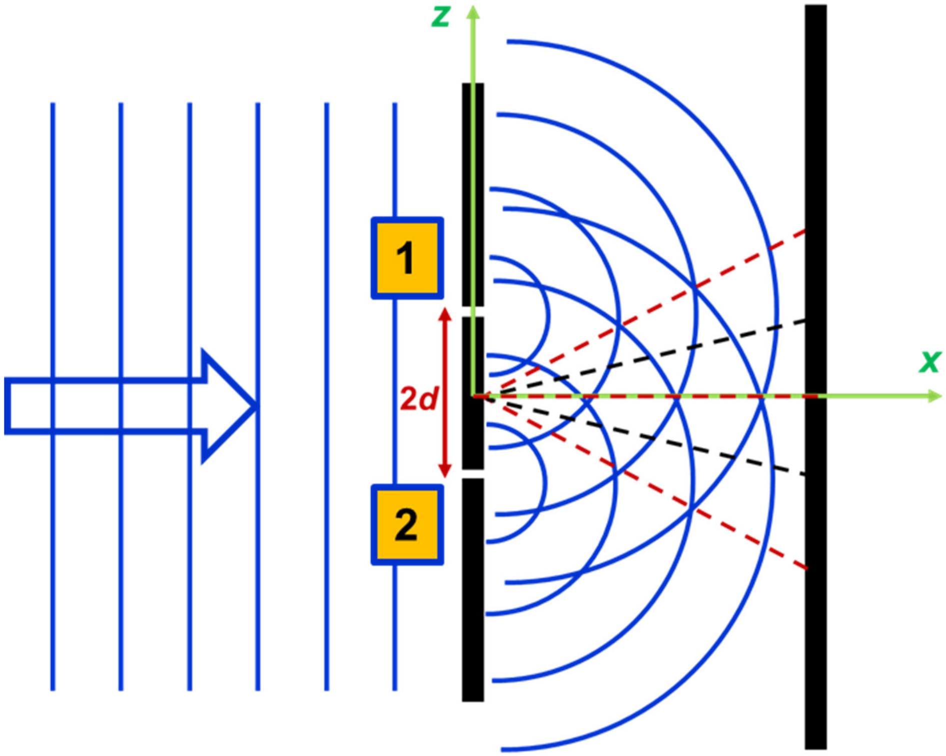
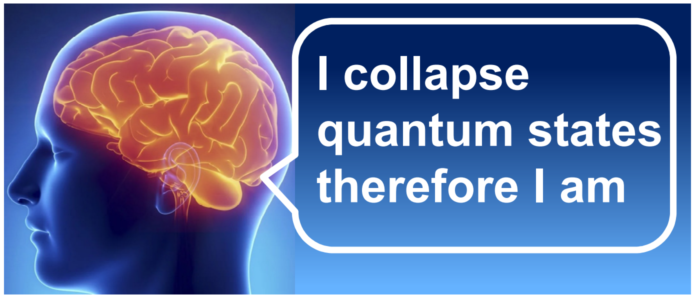
Lecture 17

Quantum Decoherence, Entanglement, and the Conscious Observer

In this lecture you will learn:

- Quantum superpositions
- Quantum decoherence
- Entanglement and decoherence
- Role of conscious observers (if any!)
- The Copenhagen Interpretation
- Schrödinger's cat paradox





Quantum Superpositions and Decoherence

An important property of quantum physics is superposition

Quantum state of physical system can be in a superposition (different realities can co-exist)

$$\begin{tikzcd} & |\psi\rangle = |z\uparrow\rangle \\ |\psi\rangle = \frac{1}{\sqrt{2}}\left[|z\uparrow\rangle + |z\downarrow\rangle\right] \arrow[ur] \arrow[dr, "\text{Decoherence}"] & \\ & |\psi\rangle = |z\downarrow\rangle \end{tikzcd}$$

But certain quantum superpositions are notoriously short lived in practice (BUT WHY?)

Certain quantum superpositions collapse pretty fast (BUT WHY?)

Quantum decoherence is the process whereby quantum superpositions collapse

Quantum Superpositions and Decoherence

Question:

But hold on!!
What about this?

$$\begin{tikzcd}
 & {|\psi\rangle = |z\uparrow\rangle} \\
{|\psi\rangle = \frac{1}{\sqrt{2}}\left[|z\uparrow\rangle + |z\downarrow\rangle\right]} \arrow[ur] \arrow[dr, "\text{Decoherence}"] & \\
 & {|\psi\rangle = |z\downarrow\rangle}
\end{tikzcd}$$

Written differently:

$$\begin{tikzcd}
 & {|\psi\rangle = \frac{1}{\sqrt{2}}\left[|x\uparrow\rangle + |x\downarrow\rangle\right]} \\
{|\psi\rangle = |x\uparrow\rangle} \arrow[ur] \arrow[dr, "\text{Decoherence}"] & \\
 & {|\psi\rangle = \frac{1}{\sqrt{2}}\left[|x\uparrow\rangle - |x\downarrow\rangle\right]}
\end{tikzcd}$$

It is not true that ALL quantum superspositions get destroyed because of decoherence

Recall that any quantum state can written as a superposition of other states (change of basis)

Some superpositions collapse vey fast and some have longer lifetimes (BUT WHY?)

Quantum Superposition and Decoherence

Clue:

We know that measurements, whereby a conscious observer gains information, collapses quantum superspositions:

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left[|z\uparrow\rangle + |z\downarrow\rangle\right]$$

S_z measurement

$$|\psi\rangle = |z\uparrow\rangle$$

$$|\psi\rangle = |z\downarrow\rangle$$

Question:

What if there are no conscious observers making measurements?
Would quantum superpositions still collapse?

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left[|\phi_1\rangle + |\phi_2\rangle\right]$$

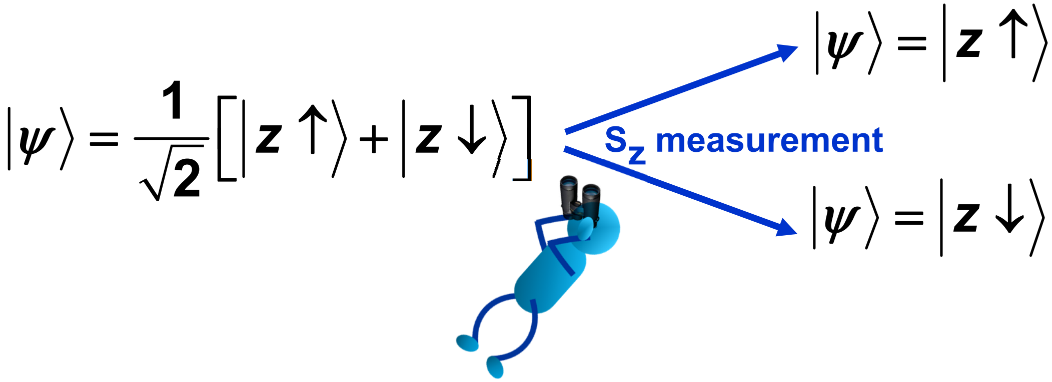
I am not looking.

Is the superposition state still going to collapse?

Quantum State Collapse

But wait a minute …….

Why does a quantum state collapse to begin with, when a conscious observer makes a measurement and gains information …….??



$$|\psi\rangle = \frac{1}{\sqrt{2}}\left[|z\uparrow\rangle + |z\downarrow\rangle\right]$$

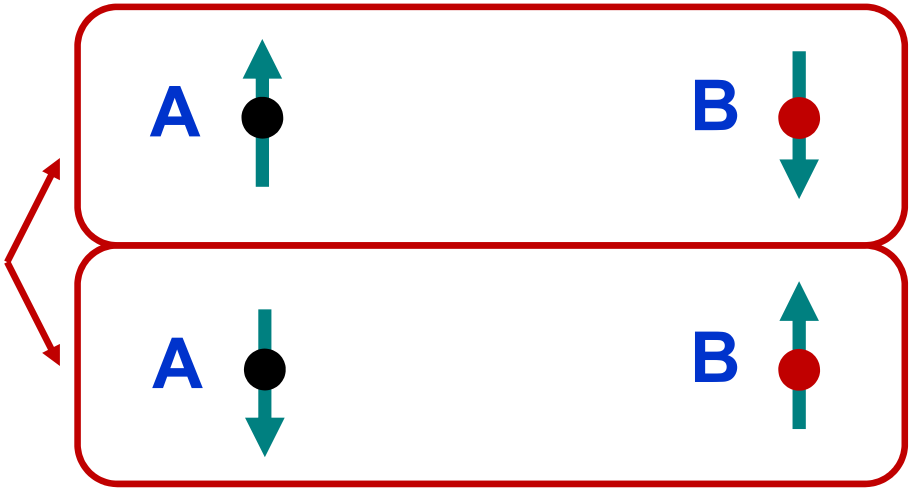
S_z measurement

$$|\psi\rangle = |z\uparrow\rangle$$

$$|\psi\rangle = |z\downarrow\rangle$$

Quantum Entanglement Recap

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left[|z\uparrow\rangle_A \otimes |z\downarrow\rangle_B + |z\downarrow\rangle_A \otimes |z\uparrow\rangle_B \right]$$



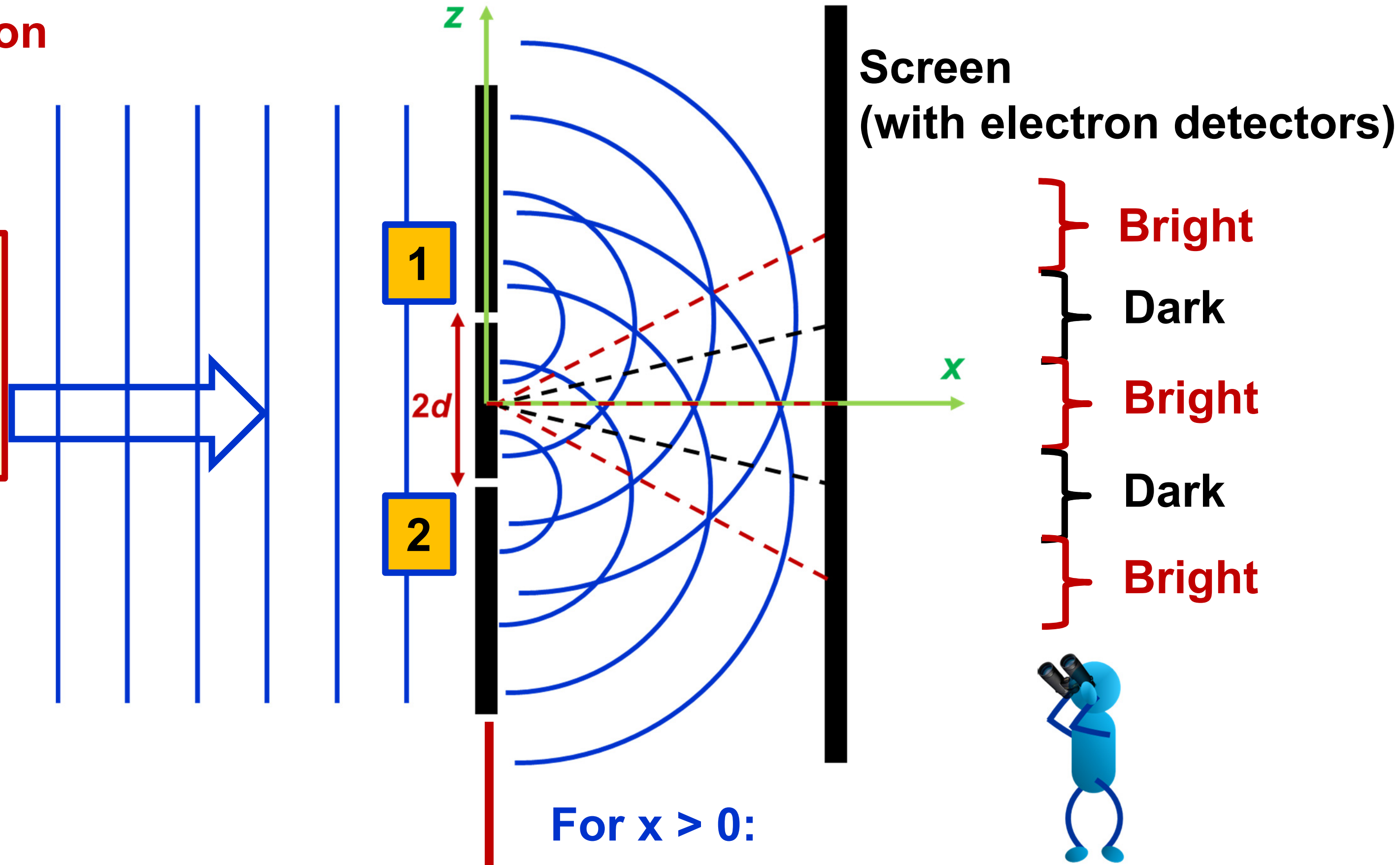
Entangled states of two systems A and B represent an entangled quantum supersposition of different realities!

The Quantum Double Split Experiment

This double-split experiment is a scheme to detect the effect of superpositions

Consider an electron going towards a double slit

We will solve the time-independent Schrodinger equation

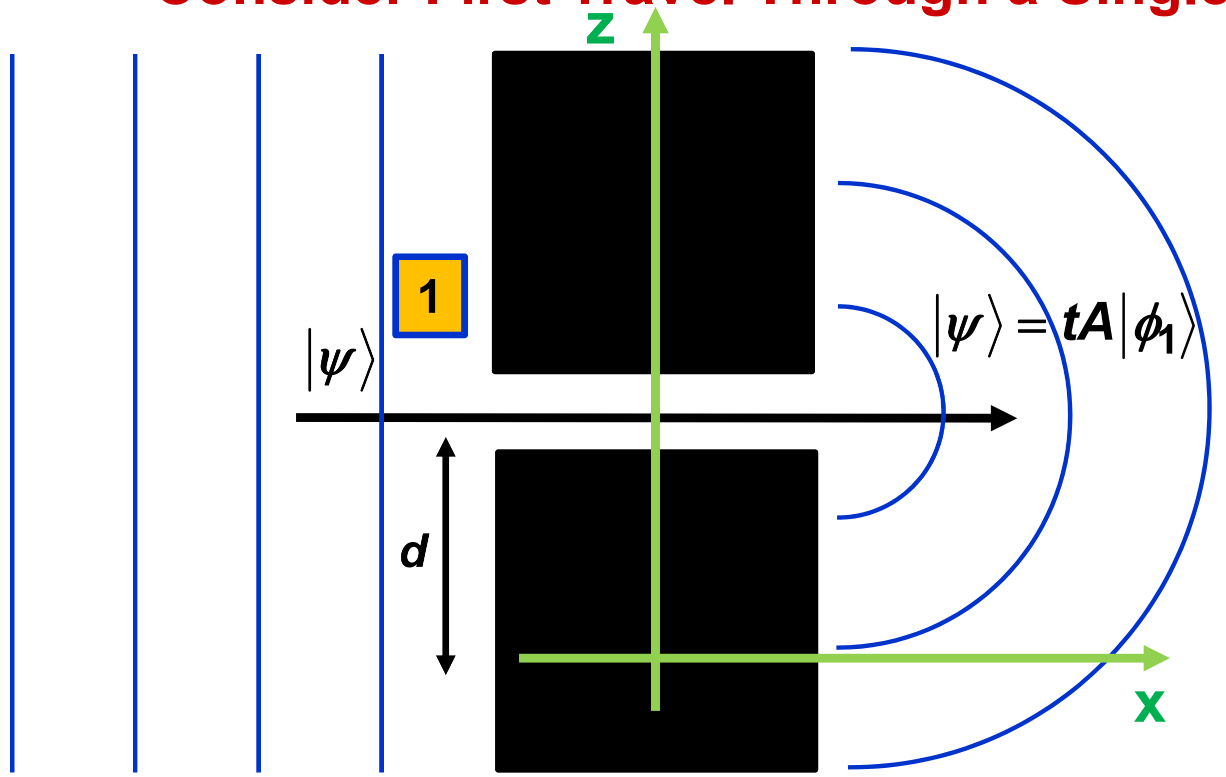


For x < 0:

$$\psi\left(\vec{r}\right) = \left\langle \vec{r} \middle| \psi \right\rangle = Ae^{ikx} + \text{ reflected wave}$$

For x > 0:

Consider First Travel Through a Single Slit



For x < 0:

$$\psi(\vec{r}) = \langle \vec{r}|\psi\rangle = Ae^{ikx} + \text{ reflected wave}$$

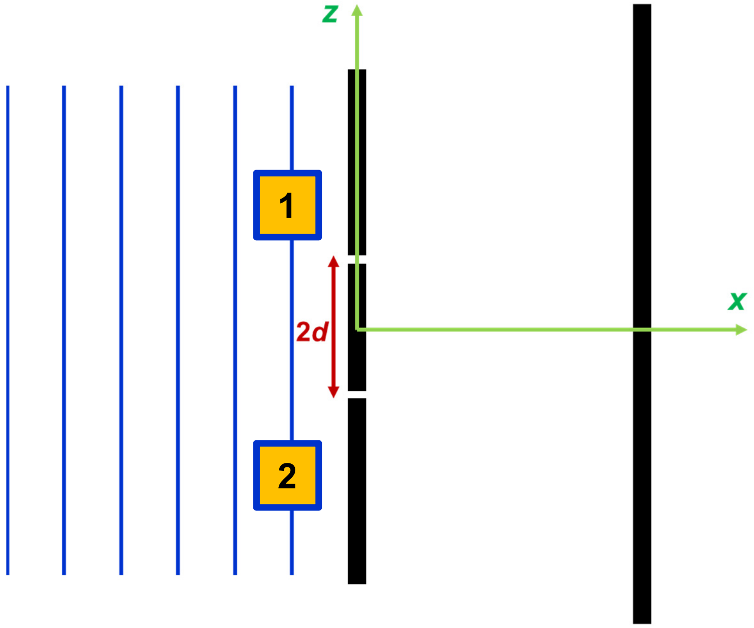
For x > 0:

$$\psi(\vec{r}) = \langle \vec{r}|\psi\rangle = tA\frac{e^{ik|\vec{r}-d\hat{z}|}}{|\vec{r}-d\hat{z}|}$$

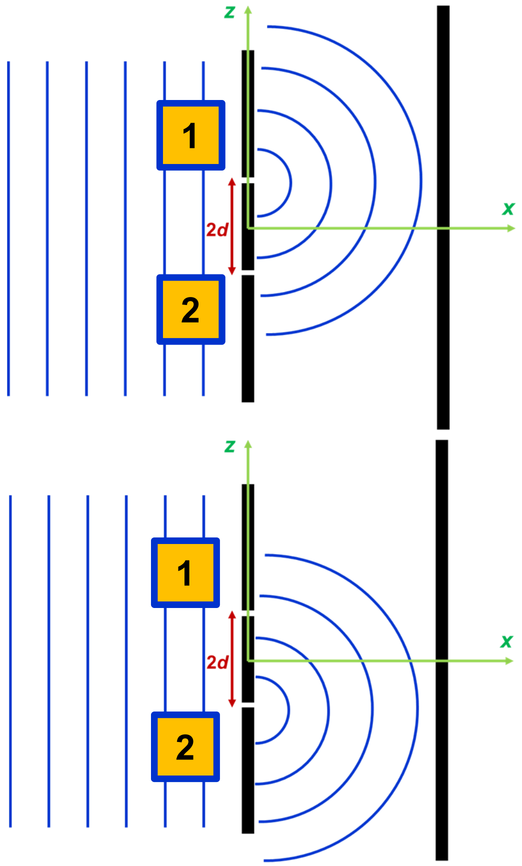
$$= tA\langle \vec{r}|\phi_1\rangle$$

$$\Rightarrow |\psi\rangle \approx tA|\phi_1\rangle$$

The Quantum Double Slit Experiment



Output is a superposition of these two outcomes



For x < 0:

$$\psi(\vec{r}) = \langle \vec{r} | \psi \rangle = Ae^{ikx} + \text{ reflected wave}$$

For x > 0:

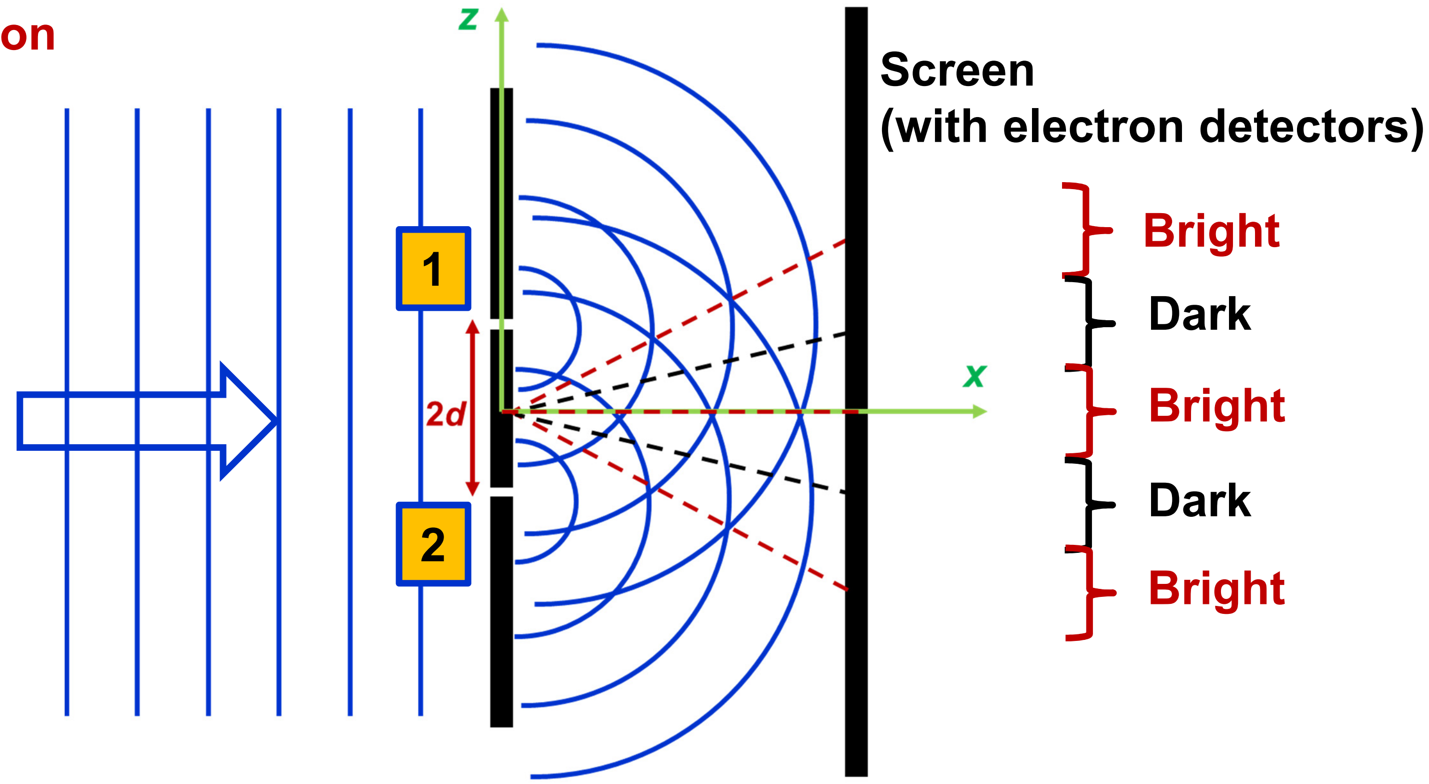
$$\psi(\vec{r}) = \langle \vec{r} | \psi \rangle \approx \frac{tA}{\sqrt{2}} \frac{e^{ik|\vec{r}-d\hat{z}|}}{|\vec{r}-d\hat{z}|} + \frac{tA}{\sqrt{2}} \frac{e^{ik|\vec{r}+d\hat{z}|}}{|\vec{r}+d\hat{z}|}$$

$$= \frac{tA}{\sqrt{2}} \langle \vec{r} | \phi_1 \rangle + \frac{tA}{\sqrt{2}} \langle \vec{r} | \phi_2 \rangle$$

$$\Rightarrow |\psi\rangle \approx \frac{tA}{\sqrt{2}} \left[|\phi_1\rangle + |\phi_2\rangle \right]$$

The Quantum Double Split Experiment

Consider an electron going towards a double slit



For x < 0:

$$\psi\left(\vec{r}\right)=\left\langle \vec{r}\,\middle|\,\psi\right\rangle = Ae^{ikx} + \text{ reflected wave}$$

For x > 0:

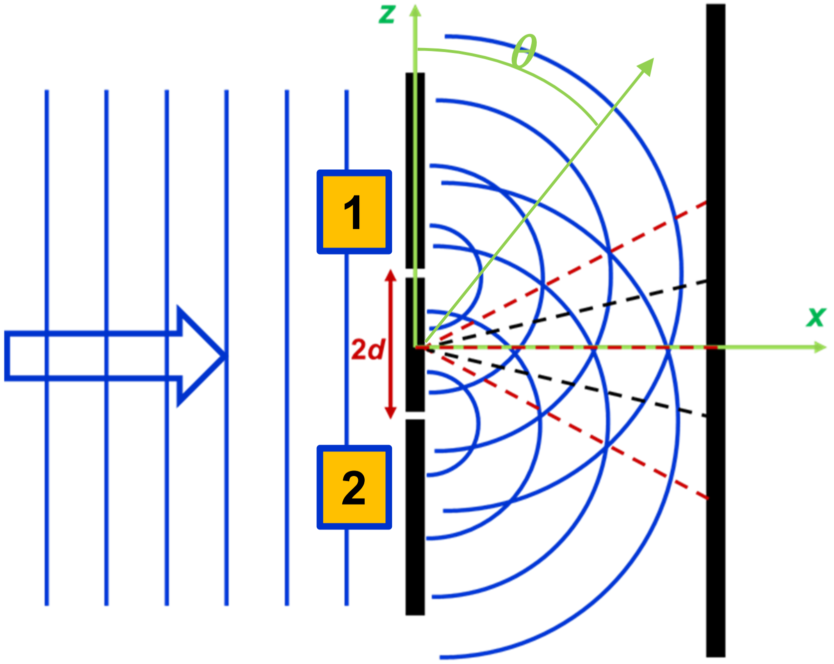
$$\psi\left(\vec{r}\right)=\left\langle \vec{r}\,\middle|\,\psi\right\rangle \approx \frac{tA}{\sqrt{2}}\frac{e^{ik\left|\vec{r}-d\hat{z}\right|}}{\left|\vec{r}-d\hat{z}\right|}+\frac{tA}{\sqrt{2}}\frac{e^{ik\left|\vec{r}+d\hat{z}\right|}}{\left|\vec{r}+d\hat{z}\right|}$$

$$=\frac{tA}{\sqrt{2}}\left\langle \vec{r}\,\middle|\,\phi_1\right\rangle+\frac{tA}{\sqrt{2}}\left\langle \vec{r}\,\middle|\,\phi_2\right\rangle$$

$$\Rightarrow \left|\psi\right\rangle \approx \frac{tA}{\sqrt{2}}\left[\left|\phi_1\right\rangle+\left|\phi_2\right\rangle\right]$$

The Quantum Double Split Experiment

$$\vec{r} = r\left[\sin\theta\cos\phi\hat{x} + \sin\theta\sin\phi\hat{x} + \cos\theta\hat{z}\right]$$



Assume $r \gg d$

$$\left|\vec{r} \pm d\hat{z}\right|$$

$$= \sqrt{\left(\vec{r} \pm d\hat{z}\right).\left(\vec{r} \pm d\hat{z}\right)}$$

$$\approx \sqrt{r^2 \pm 2d\vec{r}.\hat{z}}$$

$$= r\left(1 \pm \frac{d}{r}\cos\theta\right)$$

$$= r \pm d\cos\theta$$

For x < 0:

$$\psi\left(\vec{r}\right) = \left\langle\vec{r}\middle|\psi\right\rangle = Ae^{ikx} + \text{ reflected wave}$$

For x > 0:

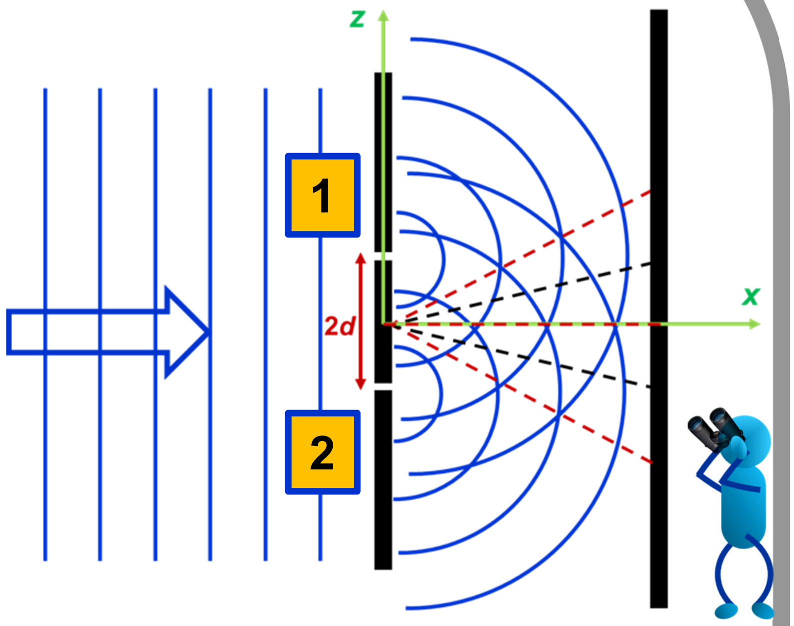
$$\psi\left(\vec{r}\right) = \left\langle\vec{r}\middle|\psi\right\rangle = \frac{tA}{\sqrt{2}}\left\langle\vec{r}\middle|\phi_1\right\rangle + \frac{tA}{\sqrt{2}}\left\langle\vec{r}\middle|\phi_2\right\rangle$$

$$\approx \frac{tA}{\sqrt{2}}\frac{e^{ik\left|\vec{r}-d\hat{z}\right|}}{\left|\vec{r}-d\hat{z}\right|} + \frac{tA}{\sqrt{2}}\frac{e^{ik\left|\vec{r}+d\hat{z}\right|}}{\left|\vec{r}+d\hat{z}\right|}$$

$$\approx \frac{tA}{\sqrt{2}}\frac{e^{ik\left(r-d\cos\theta\right)}}{r} + \frac{tA}{\sqrt{2}}\frac{e^{ik\left(r+d\cos\theta\right)}}{r}$$

Origin of Interference is in the Superposition

$$|\psi\rangle \approx \frac{tA}{\sqrt{2}}\left[|\phi_1\rangle + |\phi_2\rangle\right]$$



Probability of finding the electron at location $\vec{r}$ beyond the screen:

$$|\psi(\vec{r})|^2 = |\langle\vec{r}|\psi\rangle|^2 = \frac{|tA|^2}{2}|\langle\vec{r}|\phi_1\rangle + \langle\vec{r}|\phi_2\rangle|^2$$

$$\approx \frac{|tA|^2}{2r^2}\left|e^{ik(r-d\cos\theta)} + e^{ik(r-d\cos\theta)}\right|^2$$

$$= \frac{|tA|^2}{2r^2}\left[1+1+\underbrace{e^{i2kd\cos\theta} + e^{-i2kd\cos\theta}}\right]$$

Interference terms!

$$= \frac{|tA|^2}{r^2}\left[1+\cos(2kd\cos\theta)\right] \longrightarrow$$

Maxima:

$4d\cos\theta = n\lambda \quad \{n = 0, \pm2, \pm4.....$

Minima:

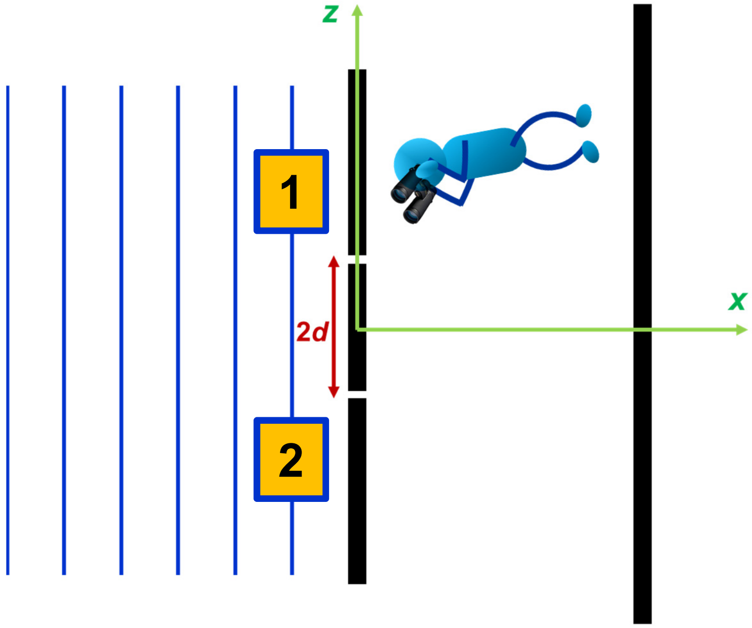
$4d\cos\theta = n\lambda \quad \{n = \pm1, \pm3, ...$

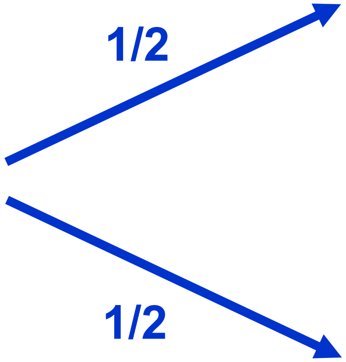
Interference between the two spherical waves, each emerging from one of the holes, gives rise to the fringes on the screen

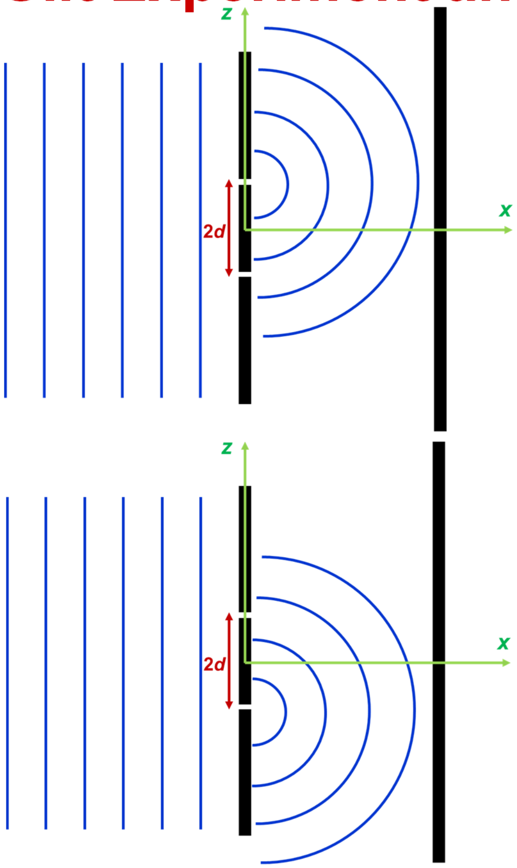
Interference happens because the electron state is a superposition of two spherical waves

Interference is a detection of superposition!

“Which Path” Measurement in the Double Slit Experiment and State Collapse







For x < 0:

$$\psi(\vec{r}) = \langle \vec{r} | \psi \rangle = Ae^{ikx} + \text{ reflected wave}$$

For x > 0:

Measurement

$$|\psi\rangle \approx \frac{tA}{\sqrt{2}}\left[|\phi_1\rangle + |\phi_2\rangle\right]$$

$$\xrightarrow{1/2} \quad |\psi\rangle \approx tA|\phi_1\rangle$$

$$\xrightarrow{1/2} \quad |\psi\rangle \approx tA|\phi_2\rangle$$

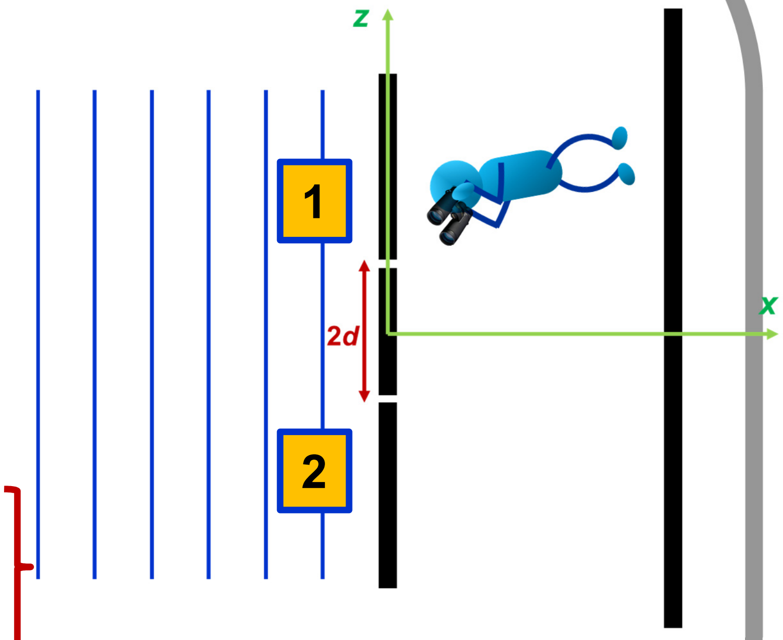
“Which Path” Measurement in the Double Slit Experiment and State Collapse

Measurement

$$\left|\psi\right\rangle \approx \frac{tA}{\sqrt{2}}\Big[\left|\phi_1\right\rangle + \left|\phi_2\right\rangle\Big]$$

$\xrightarrow{1/2}$ $\left|\psi\right\rangle \approx tA\left|\phi_1\right\rangle$

$\xrightarrow{1/2}$ $\left|\psi\right\rangle \approx tA\left|\phi_2\right\rangle$



Total a-priori probability of finding the electron at location $\vec{r}$ beyond the screen (assuming, as always, that the experiment is conducted many times) =

{Probability of finding the electron at location $\vec{r}$ beyond the screen given that it came through slit 1} + {Probability of finding the electron at location $\vec{r}$ beyond the screen given that it came through slit 2}

$$= \frac{1}{2}\left|\left\langle \vec{r}\,\middle|\, tA \middle| \phi_1\right\rangle\right|^2 + \frac{1}{2}\left|\left\langle \vec{r}\,\middle|\, tA \middle| \phi_2\right\rangle\right|^2$$

$$= \frac{|tA|^2}{2}\left[\left|\left\langle \vec{r}\,\middle|\,\phi_1\right\rangle\right|^2 + \left|\left\langle \vec{r}\,\middle|\,\phi_2\right\rangle\right|^2\right]$$

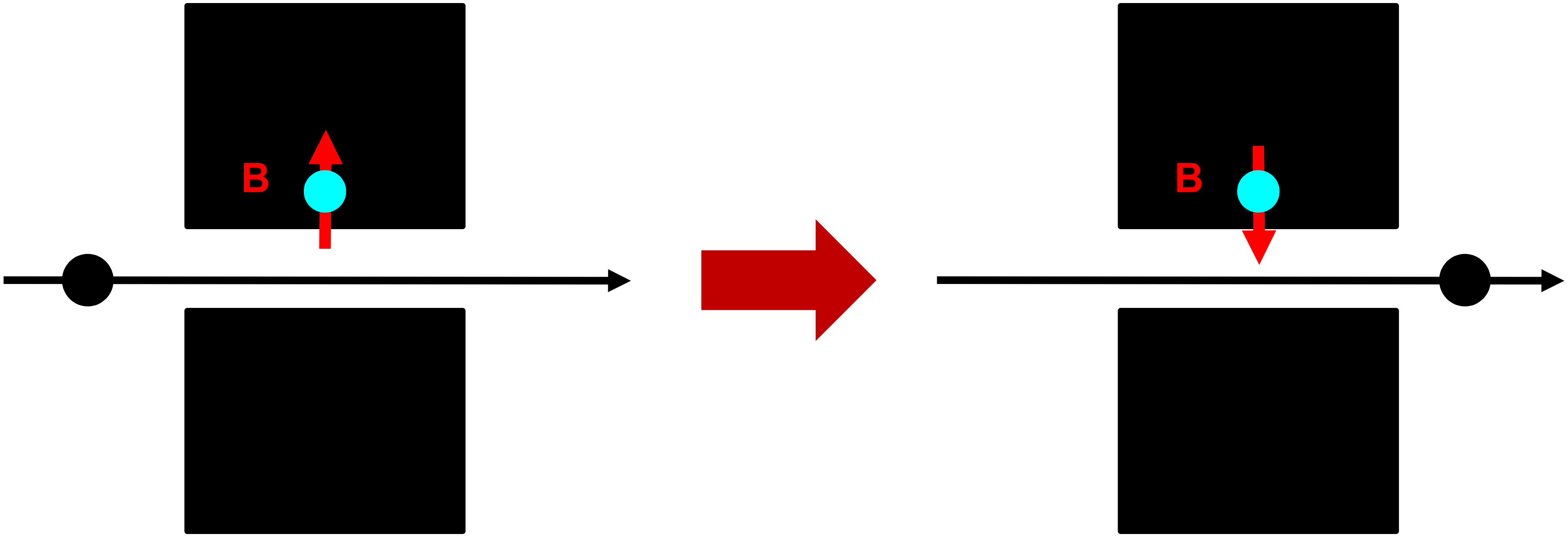
$$= \frac{|tA|^2}{2r^2}\left[\left|e^{ik(r-d\cos\theta)}\right|^2 + \left|e^{ik(r-d\cos\theta)}\right|^2\right]$$

$$= \frac{|tA|^2}{2r^2}[1+1] = \frac{|tA|^2}{r^2}$$

Now we get no interference pattern!

An Unconscious Observer Inside the Slit

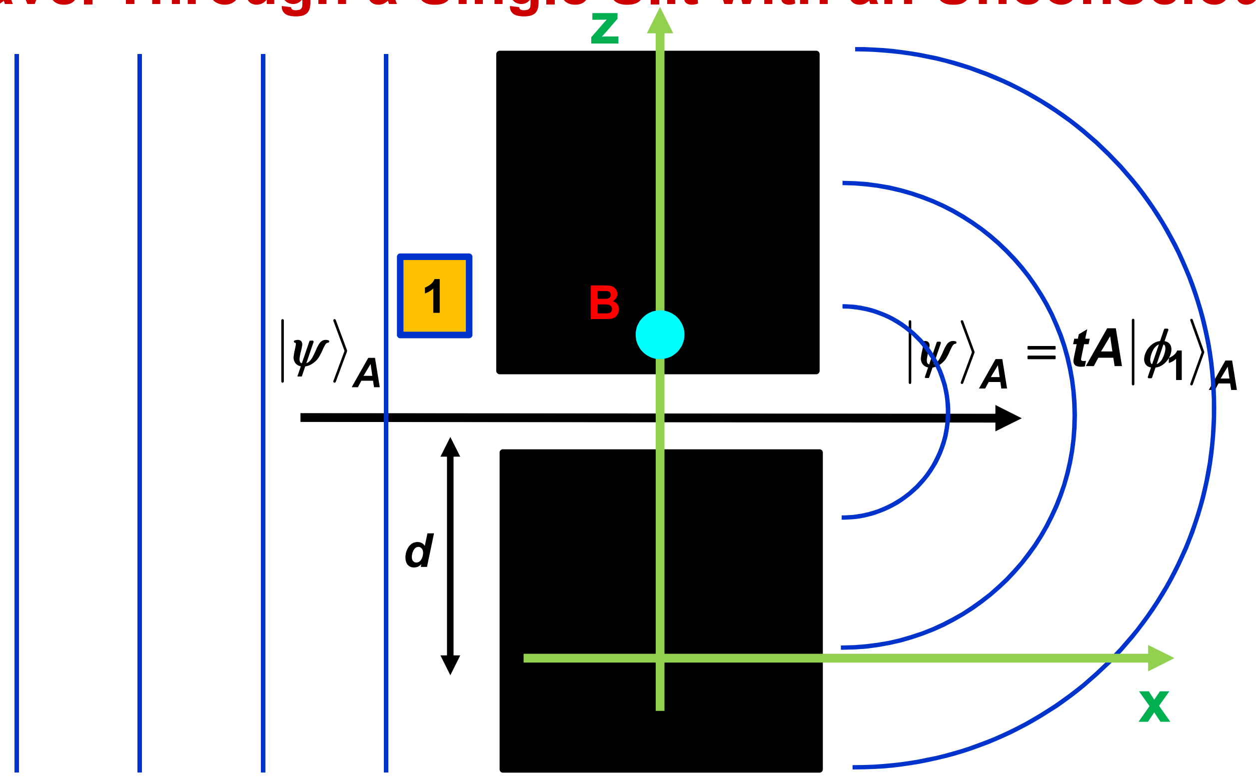
Lets try to make up an observer that is not conscious …………



An electron passing through the slit changes the spin state of another electron (the observer B) which is fixed and embedded inside the slit

PS: You can assume that the magnetic field produced by the moving electron flips the magnetic moment (and the spin) of the fixed electron

Travel Through a Single Slit with an Unconscious Observer



$$|\chi\rangle = |\psi\rangle_A \otimes |z\uparrow\rangle_B$$

For x < 0:

$$\psi_A(\vec{r}) = \langle\vec{r}|\psi\rangle_A = Ae^{ikx}$$

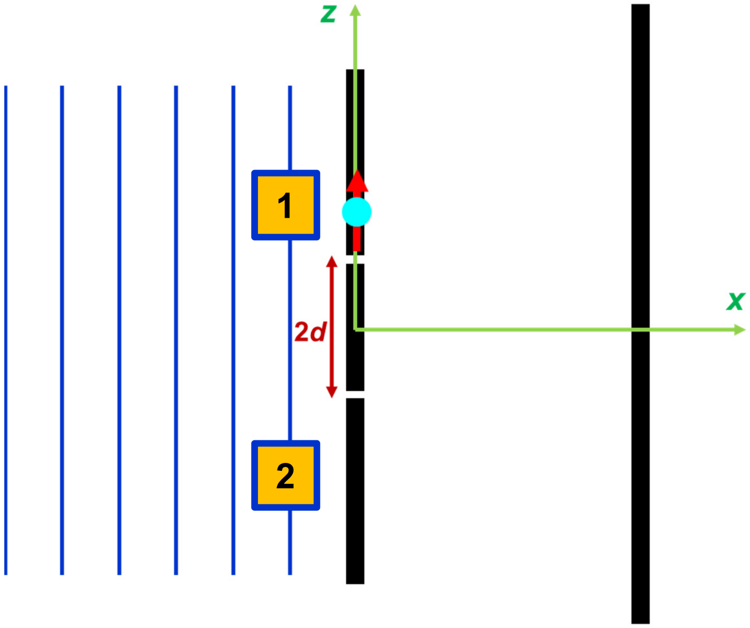
+ reflected wave

$$|\chi\rangle = |\psi\rangle_A \otimes |z\downarrow\rangle_B = tA|\phi_1\rangle_A \otimes |z\downarrow\rangle_B$$

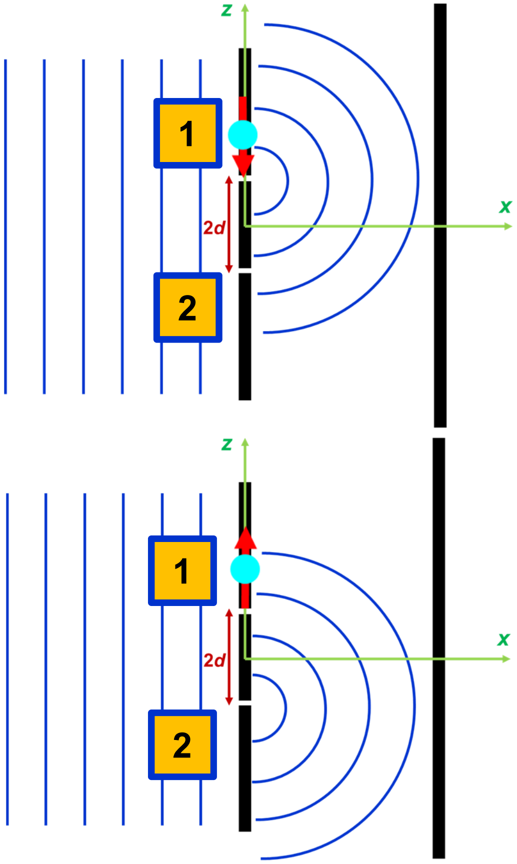
For x > 0:

$$\psi_A(\vec{r}) = \langle\vec{r}|\psi\rangle_A = tA\phi_1(\vec{r}) = tA\frac{e^{ik|\vec{r}-d\hat{z}|}}{|\vec{r}-d\hat{z}|}$$

The Quantum Double Slit Experiment with an Unconscious Observer



Output will be a superposition of these two outcomes



For x < 0:

$$|\chi\rangle = |\psi\rangle_A \otimes |z\uparrow\rangle_B$$

$$\psi_A(\vec{r}) = \langle \vec{r}|\psi\rangle_A = Ae^{ikx}$$

$+$ reflected wave

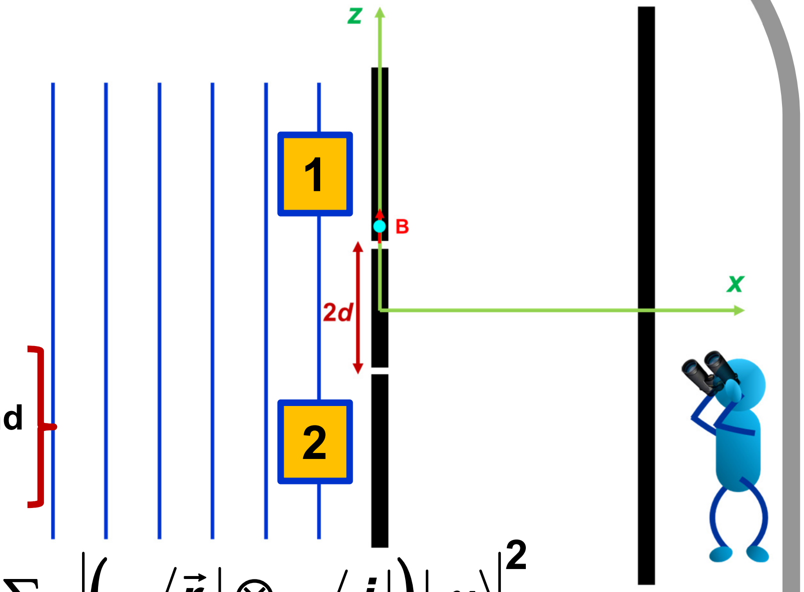
For x > 0:

$$|\chi\rangle \approx \frac{tA}{\sqrt{2}}\left[|\phi_1\rangle_A \otimes |z\downarrow\rangle_B + |\phi_2\rangle_A \otimes |z\uparrow\rangle_B\right]$$

Entangled state!!!

The state of the electron and of the observer have become entangled

The Quantum Double Slit Experiment with an Unconscious Observer



$$|\chi\rangle \approx \frac{tA}{\sqrt{2}}\left[|\phi_1\rangle_A \otimes |z\downarrow\rangle_B + |\phi_2\rangle_A \otimes |z\uparrow\rangle_B\right]$$

Total probability of finding the electron at location $\vec{r}$ beyond the screen =

{probability of finding the electron at location $\vec{r}$ beyond the screen and the spin in down state} + {probability of finding the electron at location $\vec{r}$ beyond the screen and the spin in up state}

$$= \left|\left({}_A\langle\vec{r}| \otimes {}_B\langle z\downarrow|\right)|\chi\rangle\right|^2 + \left|\left({}_A\langle\vec{r}| \otimes {}_B\langle z\uparrow|\right)|\chi\rangle\right|^2 = \sum_j \left|\left({}_A\langle\vec{r}| \otimes {}_B\langle j|\right)|\chi\rangle\right|^2$$

(Sum j is over all orthogonal states of B)

$$= \frac{|tA|^2}{2}\left[\left|\langle\vec{r}|\phi_1\rangle_A\right|^2 + \left|\langle\vec{r}|\phi_2\rangle_A\right|^2\right]$$

$$= \frac{|tA|^2}{2r^2}\left[\left|e^{ik(r-d\cos\theta)}\right|^2 + \left|e^{ik(r-d\cos\theta)}\right|^2\right]$$

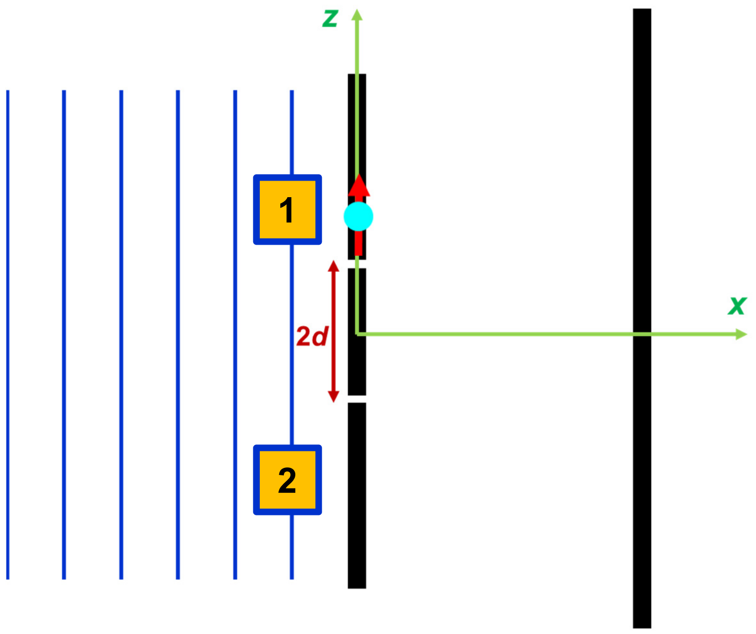
$$= \frac{|tA|^2}{2r^2}[1+1]$$

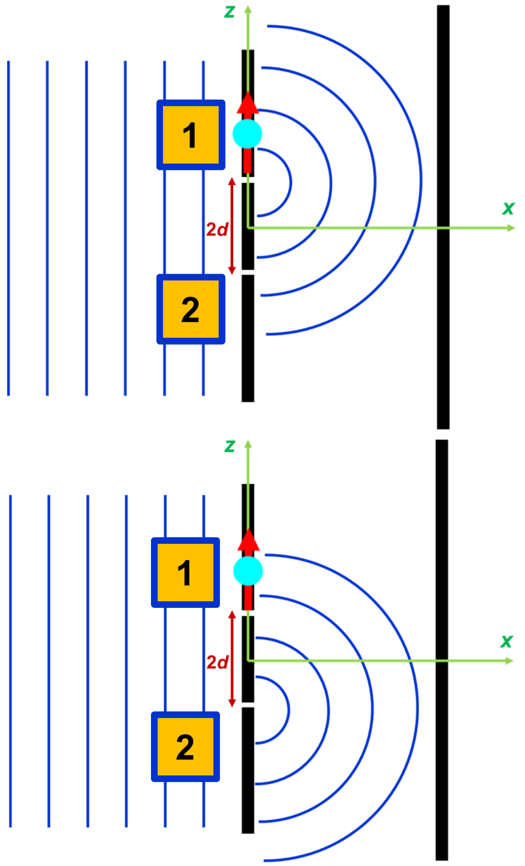
$$= \frac{|tA|^2}{r^2}$$

→ **No interference fringes!**

Entanglement of the electron with the spin “observer” results in the destruction of the interference pattern in the same way as when the quantum superposition was collapsed by the observation made by a conscious observer

What if the Unconscious Observer in Slit 1 Failed to "See" the Electron?





Output will be a superposition of these two outcomes

For x < 0:

$$\left|\chi\right\rangle = \left|\psi\right\rangle_A \otimes \left|z\uparrow\right\rangle_B$$

$$\psi_A(\vec{r}) = \left\langle \vec{r}\,\middle|\,\psi\right\rangle_A = Ae^{ikx}$$

$$+\ \text{reflected wave}$$

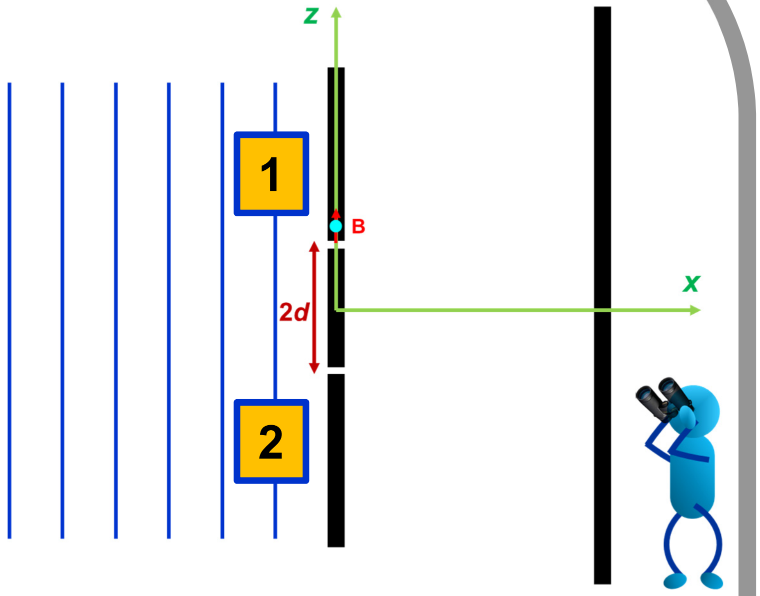
For x > 0:

$$\left|\chi\right\rangle \approx \frac{tA}{\sqrt{2}}\left[\left|\phi_1\right\rangle_A \otimes \left|z\uparrow\right\rangle_B + \left|\phi_2\right\rangle_A \otimes \left|z\uparrow\right\rangle_B\right]$$

$$= \frac{tA}{\sqrt{2}}\left[\left|\phi_1\right\rangle_A + \left|\phi_2\right\rangle_A\right] \otimes \left|z\uparrow\right\rangle_B$$

Unentangled state!!!

What if the Unconscious Observer in Slit 1 Failed to "See" the Electron?



$$|\chi\rangle \approx \frac{tA}{\sqrt{2}}\left[|\phi_1\rangle_A + |\phi_2\rangle_A\right]\otimes|z\uparrow\rangle_B$$

Probability of finding the electron at location $\vec{r}$ beyond the screen:

$$= \left|\left({}_A\langle\vec{r}|\otimes{}_B\langle z\downarrow|\right)|\chi\rangle\right|^2 + \left|\left({}_A\langle\vec{r}|\otimes{}_B\langle z\uparrow|\right)|\chi\rangle\right|^2$$

$$= \frac{|tA|^2}{2}\left[\left|\langle\vec{r}|\phi_1\rangle_A + \langle\vec{r}|\phi_2\rangle_B\right|^2 + 0\right]$$

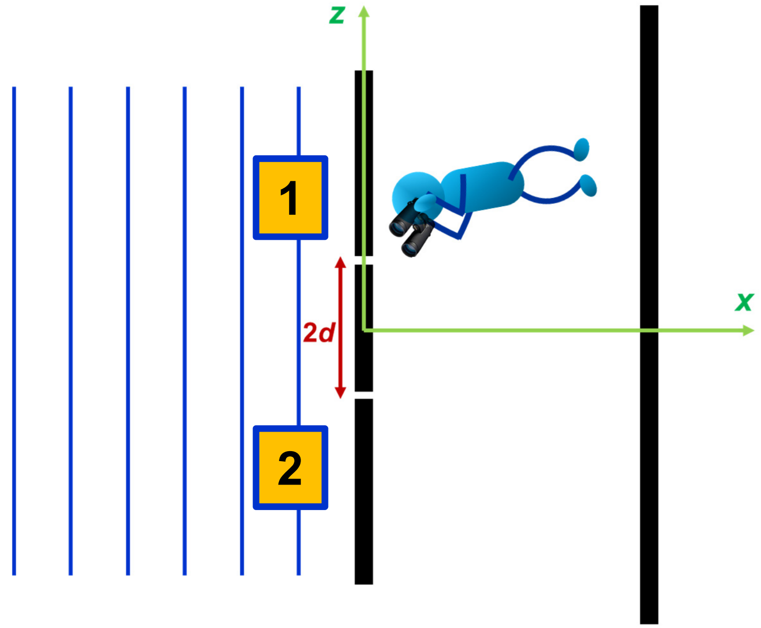
$$= \frac{|tA|^2}{2r^2}\left[1+1+\underbrace{e^{i2kd\cos\theta} + e^{-i2kd\cos\theta}}_{} + 0\right]$$

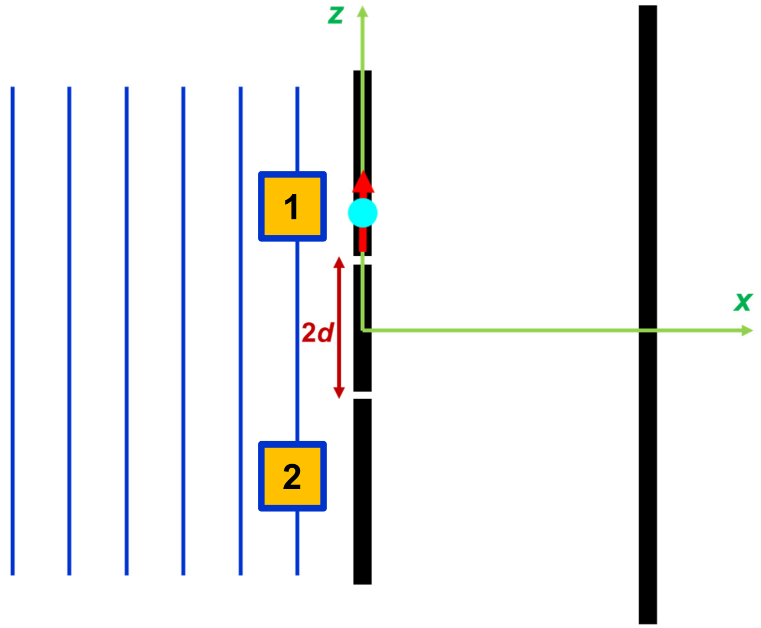
Interference terms!

$$= \frac{|tA|^2}{r^2}\left[1+\cos\left(2kd\cos\theta\right)\right]$$

Again we get the interference pattern!!

The Quantum Double Slit Experiment: Lessons



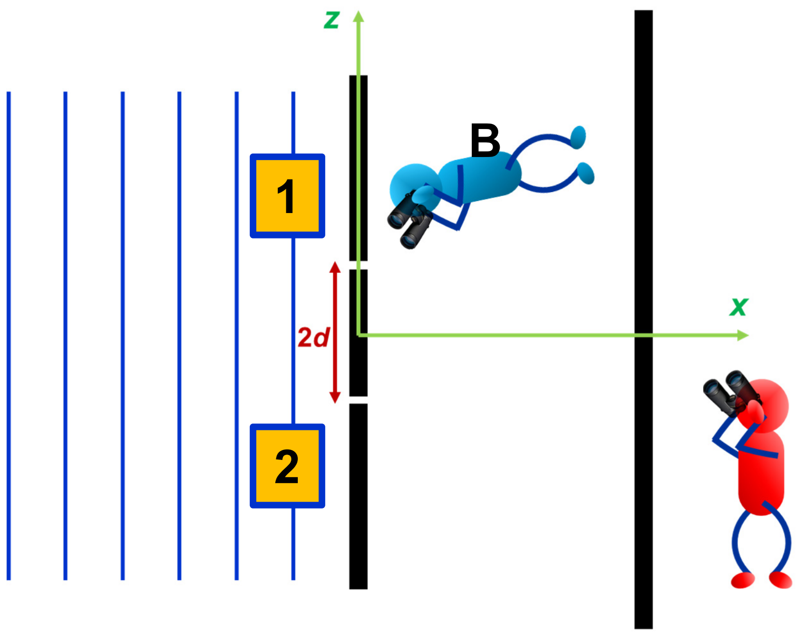


Whether the "which path" information is acquired by a conscious observer, or recorded by an unconscious observer, the interference pattern, which is a technique used here for detecting superpositions, disappears and, therefore, we may conclude that both these processes destroy superpositions

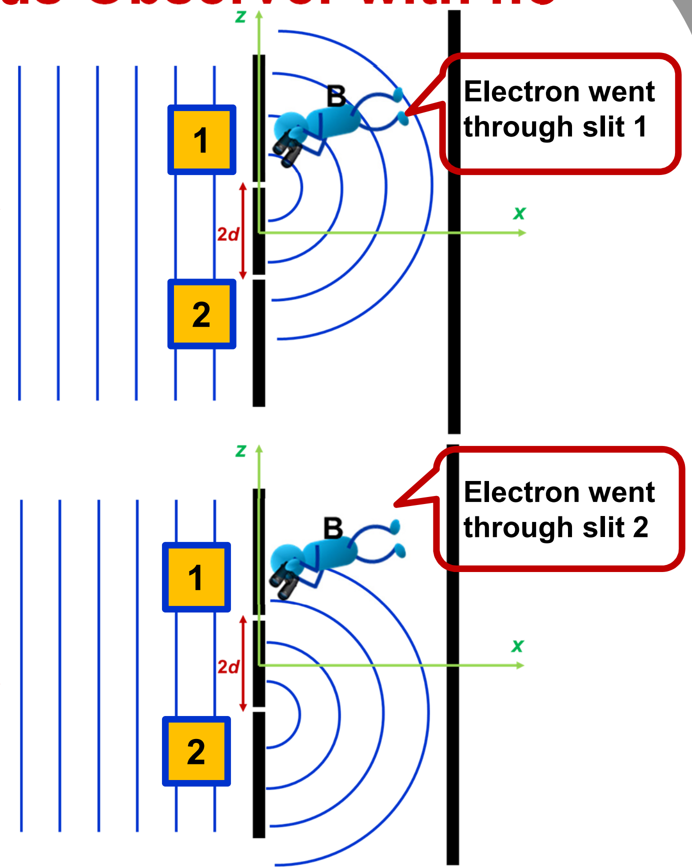
PS: Any other measurement, besides recording the interference patterns, done at the screen on the traveling electron alone, that aims to detect superpositions will fail to detect any superposition in both the above cases

But wait a minute ....

“Which Path” Measurement by a Conscious Observer with no State Collapse



Output will be a superposition of these two outcomes

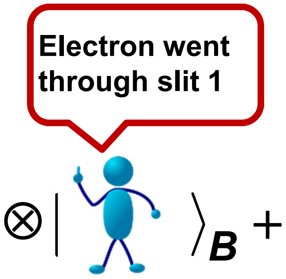


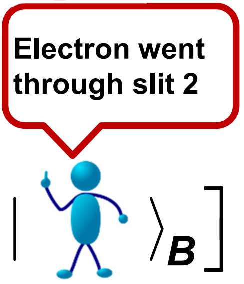
For x < 0:

$$\psi_A(\vec{r}) = \langle \vec{r}|\psi\rangle = Ae^{ikx}$$

$+$ reflected wave

For x > 0:



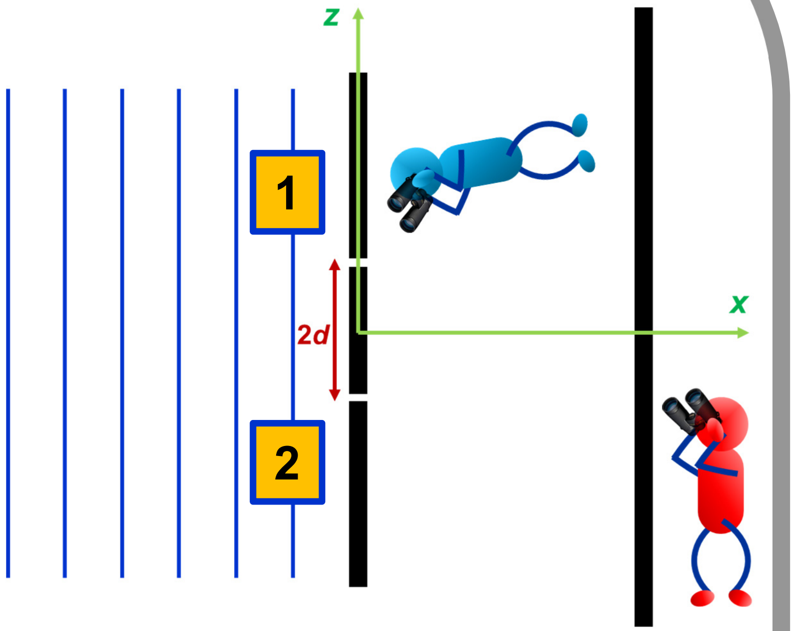


$$|\chi\rangle \approx \frac{tA}{\sqrt{2}}\left[|\phi_1\rangle_A \otimes |\quad\rangle_B + |\phi_2\rangle_A \otimes |\quad\rangle_B\right]$$

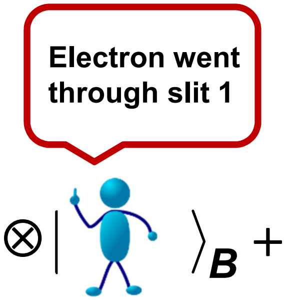
Entangled state!!!

The state of the electron and of the mind of the “conscious” observer have become entangled

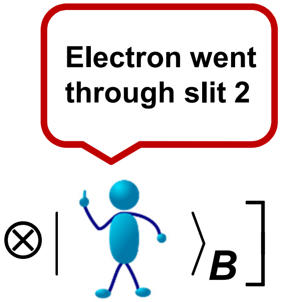
“Which Path” Measurement by a Conscious Observer with no State Collapse



$$|\chi\rangle \approx \frac{tA}{\sqrt{2}}\Big[|\phi_1\rangle_A \otimes |$$



$$\rangle_B + |\phi_2\rangle_A \otimes |$$



$$\rangle_B\Big]$$

Probability of finding the electron at location $\vec{r}$ beyond the screen:

$$= \Big|\Big({}_A\langle\vec{r}|\otimes{}_B\langle$$

$$|\Big)|\chi\rangle\Big|^2 + \Big|\Big({}_A\langle\vec{r}|\otimes{}_B\langle$$

$$|\Big)|\chi\rangle\Big|^2$$

$$= \frac{|tA|^2}{2}\Big[\big|\langle\vec{r}|\phi_1\rangle_A\big|^2 + \big|\langle\vec{r}|\phi_2\rangle_A\big|^2\Big]$$

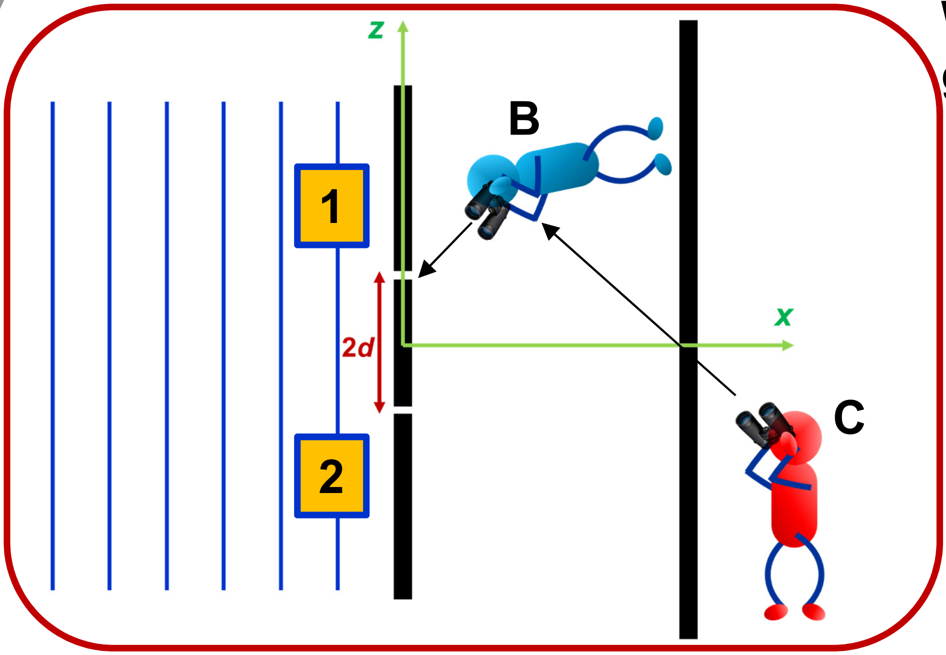
$$= \frac{|tA|^2}{2r^2}\Big[\big|e^{ik(r-d\cos\theta)}\big|^2 + \big|e^{ik(r-d\cos\theta)}\big|^2\Big]$$

$$= \frac{|tA|^2}{r^2}$$

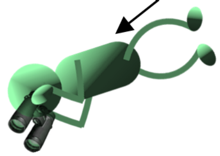
Now again we get no interference pattern!

Do conscious observers really collapse quantum states or do they just get entangled with it upon making a measurement ………..

"Which Path" Measurements: Propagation of Knowledge - I



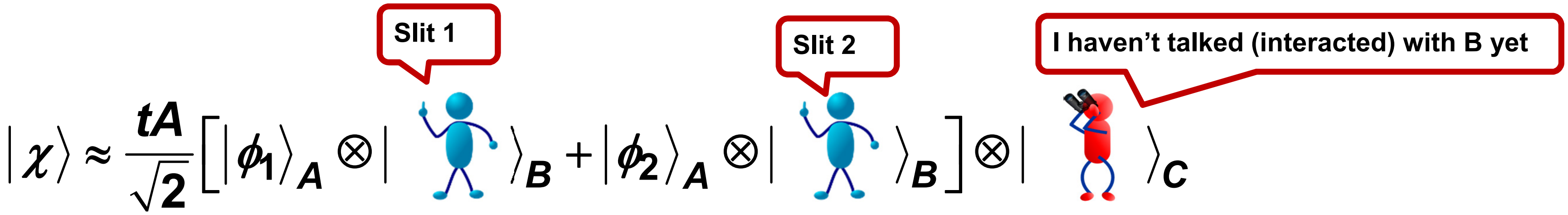
What does this observer think is going on?



Lets pursue the previous line of thought a bit more ………

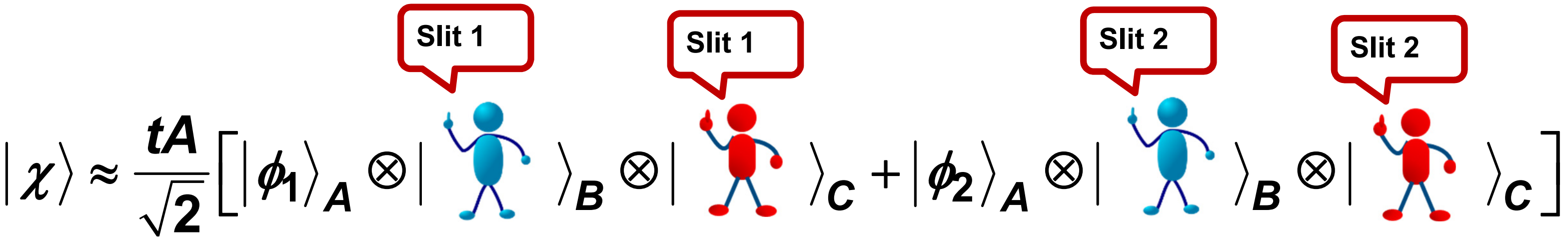
The Many Worlds Interpretation:
(Hugh Everett and Bryce DeWitt)

After observer B has made the measurement:



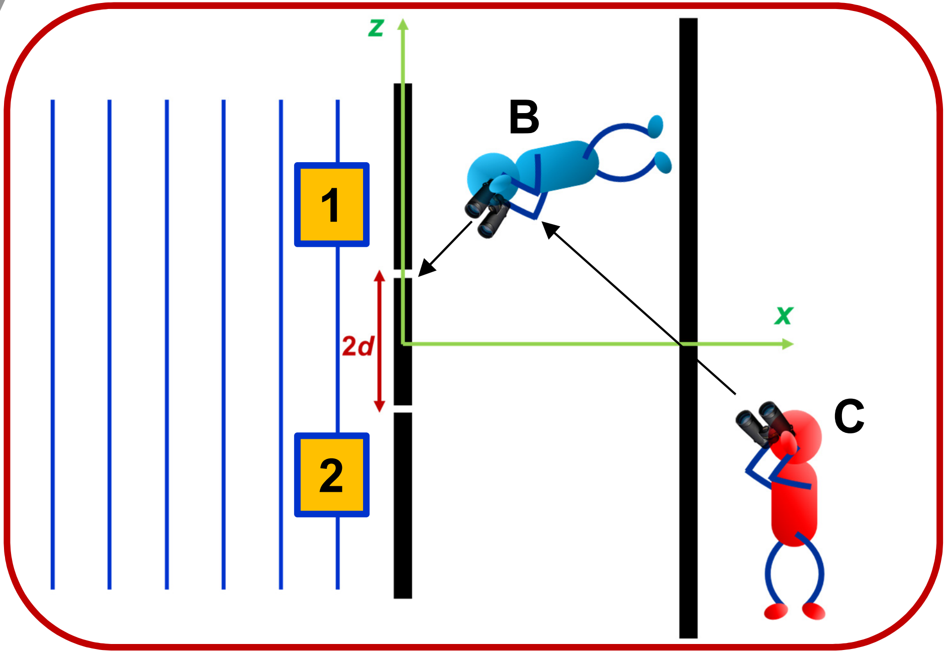
$$|\chi\rangle \approx \frac{tA}{\sqrt{2}}\left[|\phi_1\rangle_A \otimes |\quad\rangle_B + |\phi_2\rangle_A \otimes |\quad\rangle_B\right] \otimes |\quad\rangle_C$$

After observer C has "talked" (i.e. interacted) with observer B:



$$|\chi\rangle \approx \frac{tA}{\sqrt{2}}\left[|\phi_1\rangle_A \otimes |\quad\rangle_B \otimes |\quad\rangle_C + |\phi_2\rangle_A \otimes |\quad\rangle_B \otimes |\quad\rangle_C\right]$$

“Which Path” Measurements: Propagation of Knowledge - II



The Copenhagen Interpretation:
(after Niels Bohr’s Institute in Copenhagen)

1) The observer B collapsed the electron state when he observed it and that is the end

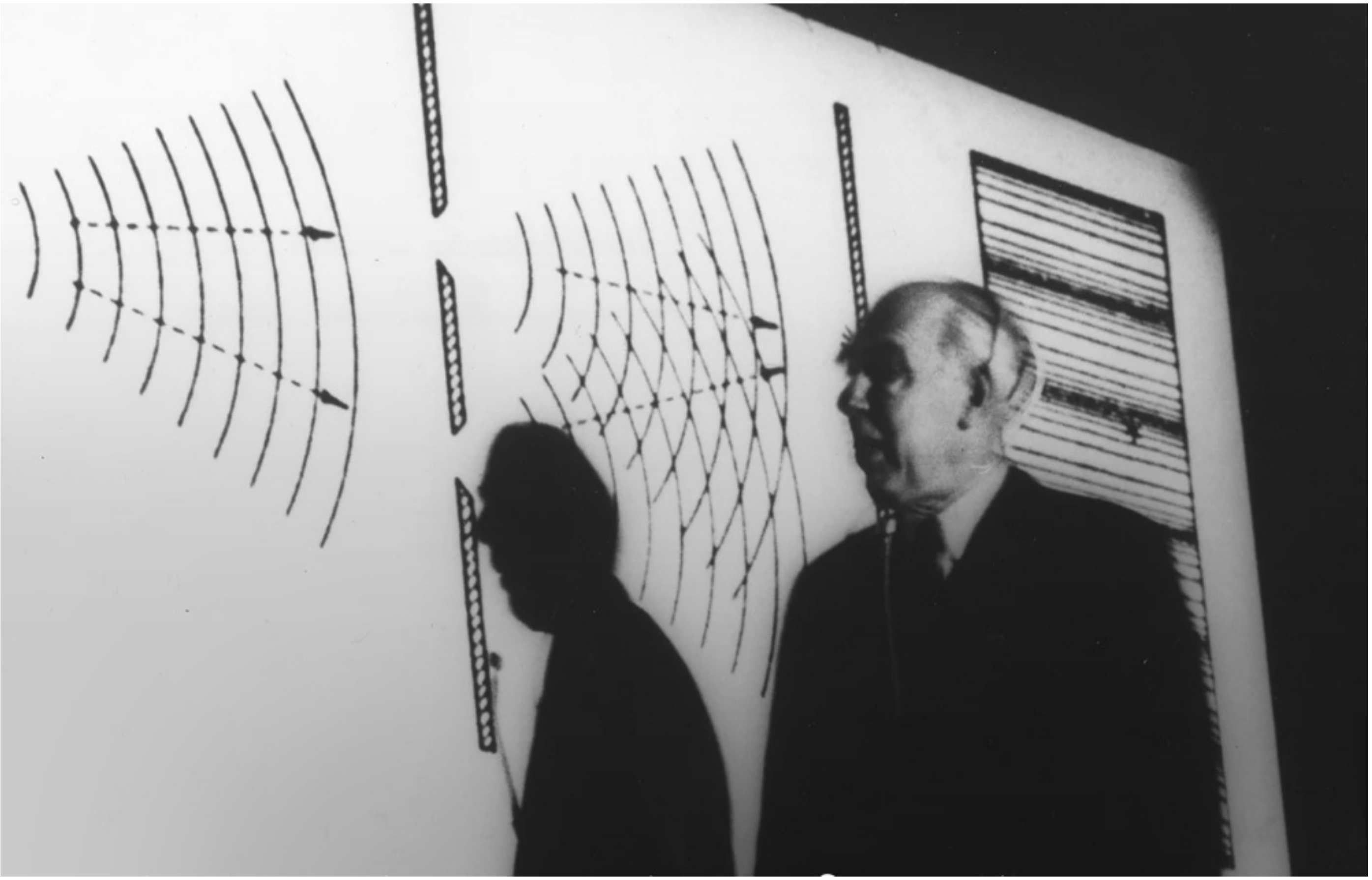
2) Follow the rules of quantum mechanics (the postulates from handout 11) and then just “shut up, and calculate” the desired probabilities

After observer B has made the measurement then for $x > 0$:

Measurement by B

$$\begin{tikzcd} & \left|\psi\right\rangle \approx tA\left|\phi_1\right\rangle \\ \left|\psi\right\rangle \approx \frac{tA}{\sqrt{2}}\left[\left|\phi_1\right\rangle + \left|\phi_2\right\rangle\right] \arrow[ur, "1/2"] \arrow[dr, "1/2"'] & \\ & \left|\psi\right\rangle \approx tA\left|\phi_2\right\rangle \end{tikzcd}$$

The Copenhagen Interpretation: “Shut Up, and Calculate”



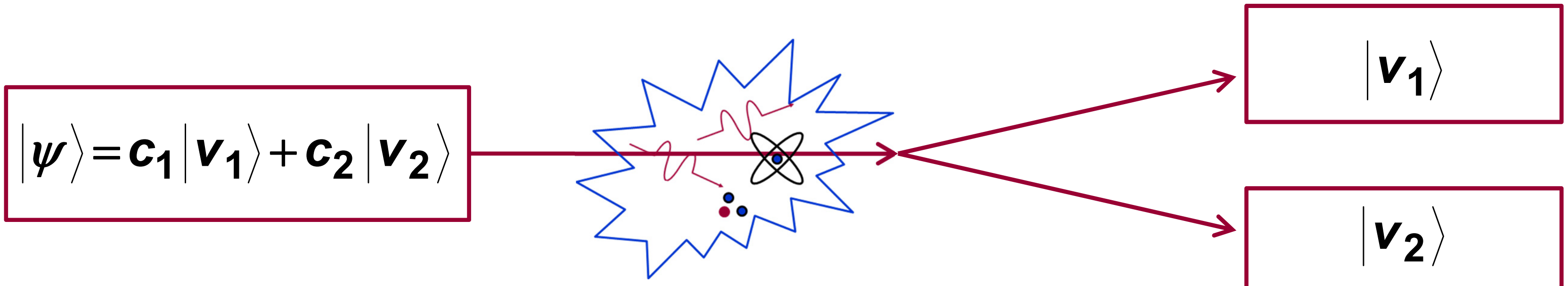
Niels Bohr explaining the double-slit experiment according to his interpretation

Entanglement and Decoherence

There is an intimate connection between entanglement and decoherence

First view point:

Decoherence



$|\psi\rangle = c_1|v_1\rangle + c_2|v_2\rangle$

$|v_1\rangle$

$|v_2\rangle$

Environment

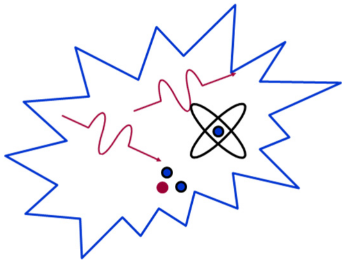
Environment makes a "measurement" on the system and collapses the quantum superposition.

The collapsed state depends on the information gained by the environment in the measurement.

A conscious observer can later look at the environment and acquire this same information

Entanglement and Decoherence

Second view point:



Environment

First, we need to make a model of the environment

Suppose the (mutually orthogonal) environment states are:

$$|E_0\rangle \quad |E_1\rangle \quad |E_2\rangle$$

The initial quantum state of the system is:

$$|\psi\rangle = c_1|v_1\rangle + c_2|v_2\rangle$$

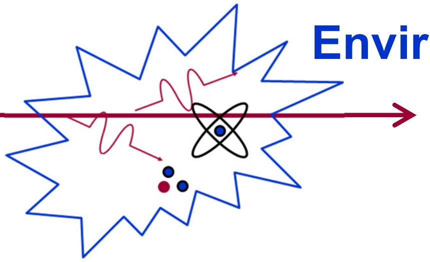
The initial joint state of the "system + environment" is:

$$|\phi(t=0)\rangle = |\psi\rangle \otimes |E_0\rangle$$

$$= c_1|v_1\rangle \otimes |E_0\rangle + c_2|v_2\rangle \otimes |E_0\rangle$$

Unentangled state

$$\boxed{|\psi\rangle = c_1|v_1\rangle + c_2|v_2\rangle}$$



Environment

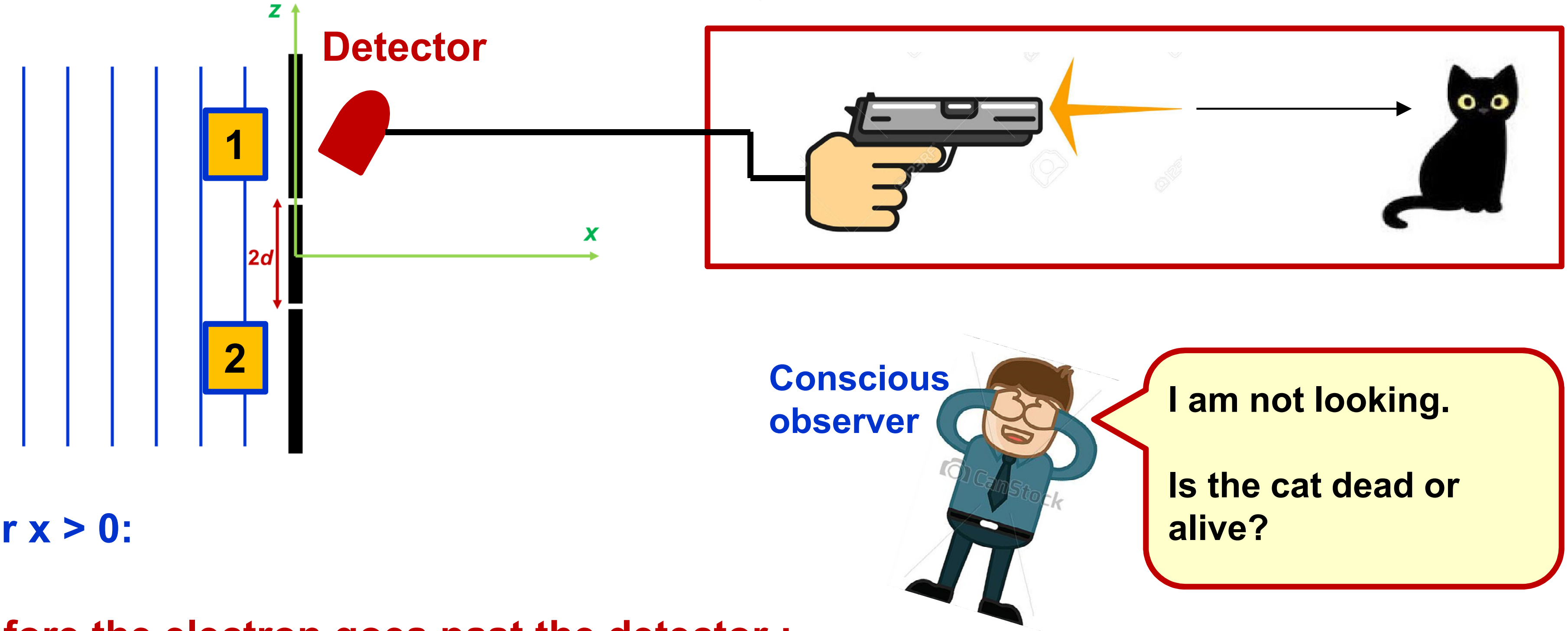
The final joint state of the "system + environment" is:

$$|\phi(t)\rangle = c_1|v_1\rangle \otimes |E_1\rangle + c_2|v_2\rangle \otimes |E_2\rangle$$

Entangled state →

Any subsequent measurement on the system alone will not be able to detect the superposition present in the initial state of the system

The Schrödinger's Cat Paradox



For x > 0:

Before the electron goes past the detector :

$|\chi\rangle \approx \frac{tA}{\sqrt{2}}\left[|\phi_1\rangle_A + |\phi_2\rangle_A\right]\otimes|$  $\rangle_B$

After the electron goes past the detector :

$|\chi\rangle \approx \frac{tA}{\sqrt{2}}\left[|\phi_1\rangle_A \otimes|$  $\rangle_B + |\phi_2\rangle_A \otimes|$  $\rangle_B\right]$