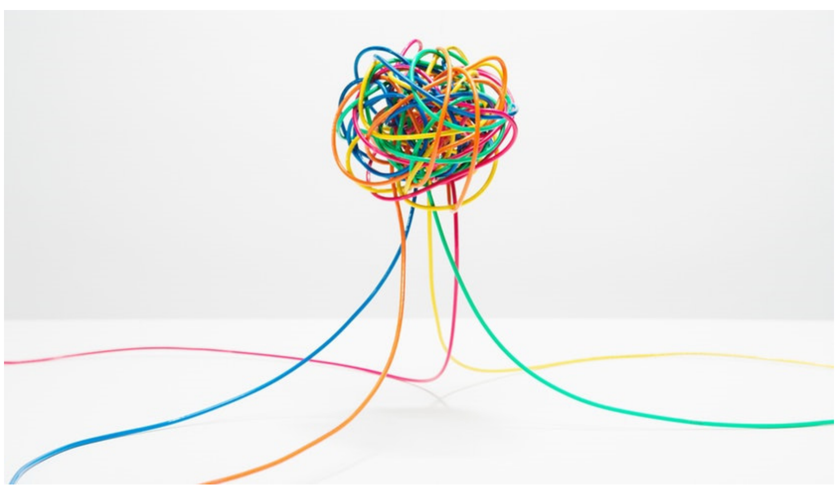
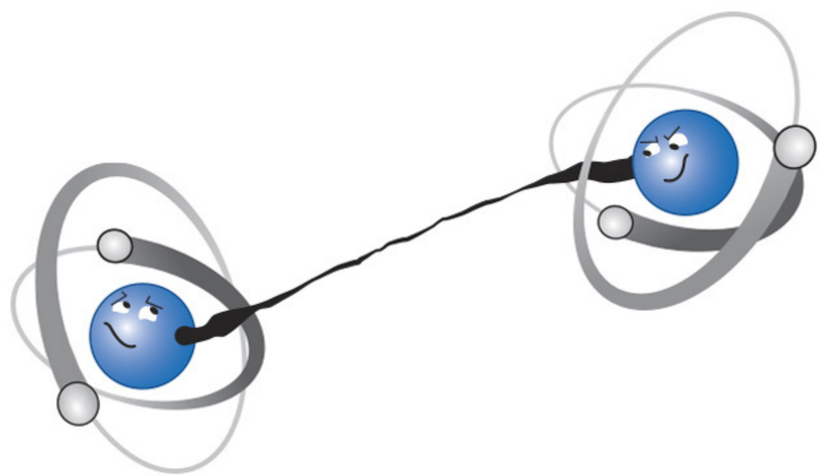
Lecture 16

Composite Quantum Systems, Quantum Entanglement, and the Death of Local Realism in Science

In this lecture you will learn:

- Quantum states of composite systems
- Joint Hilbert spaces
- Entangled quantum states
- EPR Paradox
- <u>End of local realism in physics</u>

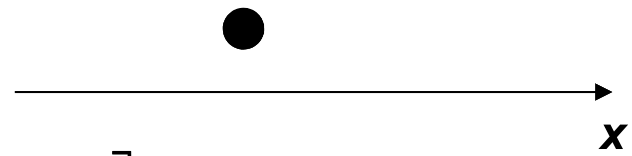




Handling Independent Degrees of Freedom: Enlarging the Hilbert Space

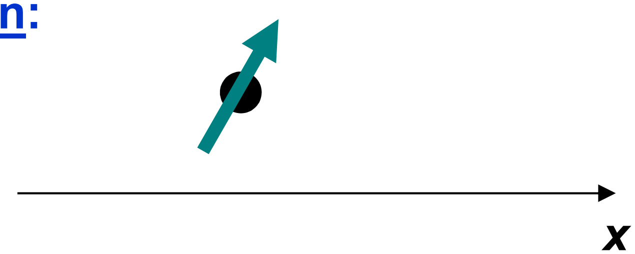
Consider the quantum state of a particle without spin:

$$|\psi\rangle = |\phi\rangle$$



$$\left[\hat{r}_k , \hat{p}_j\right] = i\hbar\delta_{k,j} \qquad \{k, j = x, y, z$$

Now consider the quantum state of a particle with spin:



Since spin represents a degree of freedom completely independent of particle's momentum and position:

$$\left[\hat{S}_k , \hat{p}_j\right] = \left[\hat{S}_k , \hat{r}_j\right] = 0$$

$$\{k, j = x, y, z$$

$$|\psi\rangle = |\phi\rangle \otimes |z\uparrow\rangle \qquad |\psi\rangle = |\phi\rangle \otimes |z\downarrow\rangle$$

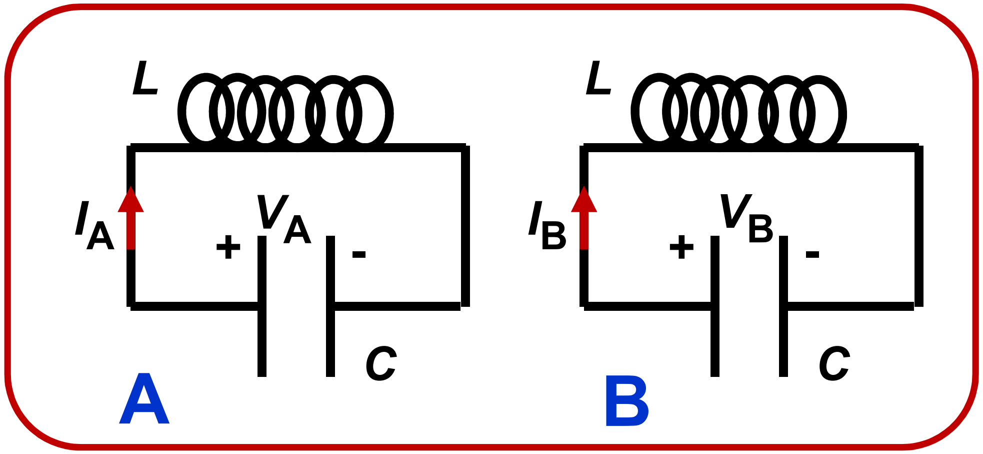
We just glued the states corresponding to the additional independent degree of freedom to the quantum state to get the complete quantum state of the particle

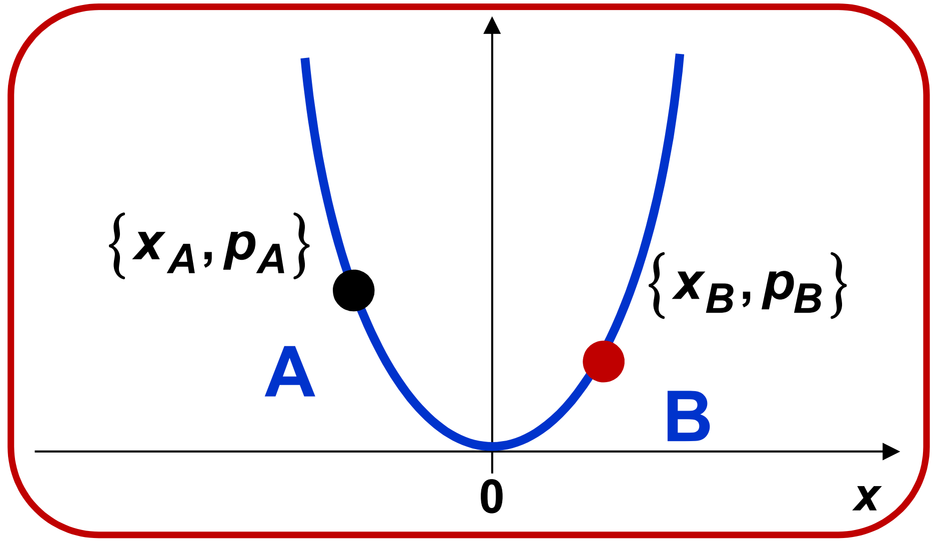
This gluing is basically enlarging the Hilbert space

Two Quantum Systems

Consider two separate and different quantum systems, A and B

These could be, for example, two different particles, or two different spins, or two different superconducting LC circuits, or two different electromagnetic modes in an optical cavity, …….





Spins of two different particles

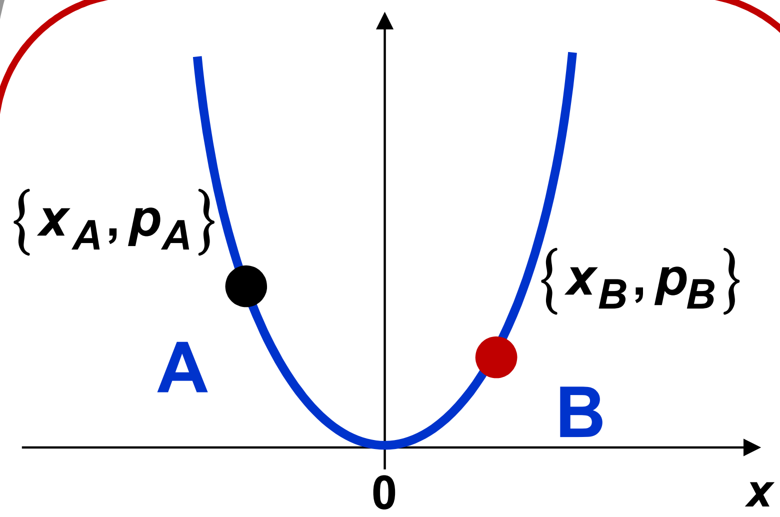
The observables of different particles/systems represent independent degrees of freedom:

$$[\hat{x}_A, \hat{x}_B] = [\hat{p}_A, \hat{p}_B] = 0$$

$$[\hat{x}_A, \hat{p}_B] = [\hat{x}_B, \hat{p}_A] = 0$$

$$[\hat{x}_A, \hat{p}_A] = [\hat{x}_B, \hat{p}_B] = i\hbar$$

Two Quantum Systems



$$\hat{x}_A |x\rangle_A = x|x\rangle_A$$

$$\hat{p}_A |p\rangle_A = p|p\rangle_A$$

$$\hat{H}_A |n\rangle_A = \hbar\omega_o\left(n+\frac{1}{2}\right)|n\rangle_A$$

$$\hat{x}_B |x\rangle_B = x|x\rangle_B$$

$$\hat{p}_B |p\rangle_B = p|p\rangle_B$$

$$\hat{H}_B |n\rangle_B = \hbar\omega_o\left(n+\frac{1}{2}\right)|n\rangle_B$$

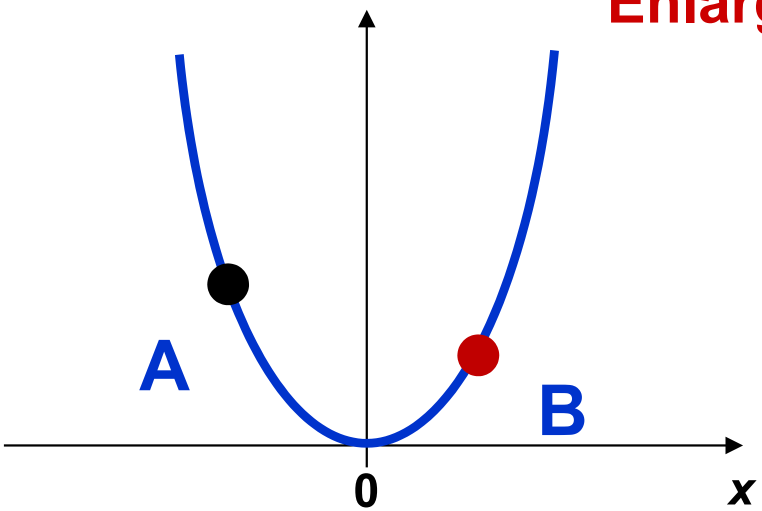
$$\hat{S}_z^A |z\uparrow\rangle_A = +\frac{\hbar}{2}|z\uparrow\rangle_A \qquad \hat{S}_z^B |z\uparrow\rangle_B = +\frac{\hbar}{2}|z\uparrow\rangle_B$$

$$\hat{S}_z^A |z\downarrow\rangle_A = -\frac{\hbar}{2}|z\downarrow\rangle_A \qquad \hat{S}_z^B |z\downarrow\rangle_B = -\frac{\hbar}{2}|z\downarrow\rangle_B$$

Each quantum system has its own Hilbert space and observables that are represented by operators which act in the Hilbert space of each quantum system

The observables of different particles/systems represent independent degrees of freedom

Joint Hilbert Space of a Composite Quantum System: Enlarging the Hilbert Space



Question: how do we describe the joint quantum state of two systems??

Answer: By "sticking" together the Hilbert spaces of the two systems, we create the <u>joint Hilbert space</u> of the combined system. So for example, a state $|\psi\rangle$ of the joint system in the joint Hilbert space can be written as:

$$|\psi\rangle = |\phi\rangle_A \otimes |\chi\rangle_B$$

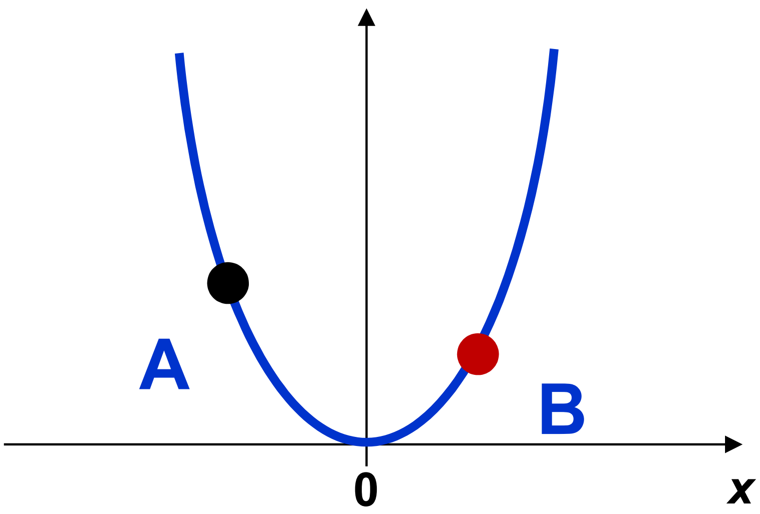
Tensor product

Where, $|\phi\rangle_A$ is a state in the Hilbert space of system A and $|\chi\rangle_B$ is a state in the Hilbert space of system B

With a little abuse of notation, we will often write the above state as:

$$|\psi\rangle = |\phi\rangle_A |\chi\rangle_B$$

Operators in the Joint Hilbert Space



Question: how do we describe observables and operators in the joint Hilbert space?

Answer: By "sticking" together operators of the two systems, we create an operator that acts in the joint Hilbert space.

Example: The operator for the observable O of system A and observable R of system B can be combined as follows:

$$\hat{O}_A \otimes \hat{R}_B$$

Tensor product

This operator acts on a state $|\psi\rangle$ of the joint Hilbert space as follows:

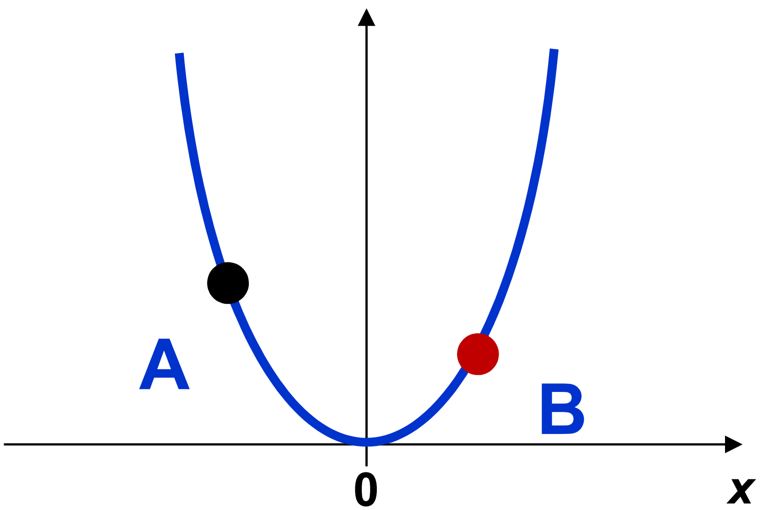
$$\left[\hat{O}_A \otimes \hat{R}_B\right]|\psi\rangle = \left[\hat{O}_A \otimes \hat{R}_B\right]|\phi\rangle_A \otimes |\chi\rangle_B = \left[\hat{O}_A |\phi\rangle_A\right] \otimes \left[\hat{R}_B |\chi\rangle_B\right]$$

Simply put, each operator in a tensor product acts in its own Hilbert space!

With a little abuse of notation, we write the operator as:

$$\hat{O}_A \otimes \hat{R}_B = \hat{O}_A \hat{R}_B$$

Inner Product in the Joint Hilbert Space



Question: how do we describe the inner product between two states in the joint Hilbert space?

Answer: Suppose:

$$|\psi_1\rangle = |\phi_1\rangle_A \otimes |\chi_1\rangle_B$$
$$|\psi_2\rangle = |\phi_2\rangle_A \otimes |\chi_2\rangle_B$$

Then:

$$\langle\psi_1|\psi_2\rangle = {}_A\langle\phi_2|\phi_1\rangle_A \cdot {}_B\langle\chi_2|\chi_1\rangle_B$$

Joint Hilbert Space of Two Spin 1/2 Particles: An Example

$$|\psi\rangle = |z\uparrow\rangle_A \otimes |z\downarrow\rangle_B$$

Consider a state of two different spin 1/2 particles (two systems): $|\psi\rangle = |z\uparrow\rangle_A \otimes |z\downarrow\rangle_B$

$$\begin{aligned}
\langle\psi|\hat{S}_z^A \otimes \hat{S}_z^B|\psi\rangle &= \left({}_B\langle z\downarrow| \otimes {}_A\langle z\uparrow|\right)\hat{S}_z^A \otimes \hat{S}_z^B\left(|z\uparrow\rangle_A \otimes |z\downarrow\rangle_B\right) \\
&= \left({}_B\langle z\downarrow|\,{}_A\langle z\uparrow|\right)\hat{S}_z^A\hat{S}_z^B\left(|z\uparrow\rangle_A |z\downarrow\rangle_B\right) \\
&= \left({}_B\langle z\downarrow|\,{}_A\langle z\uparrow|\right)\left(\hat{S}_z^A|z\uparrow\rangle_A\,\hat{S}_z^B|z\downarrow\rangle_B\right) \\
&= {}_B\langle z\downarrow|\hat{S}_z^B|z\downarrow\rangle_B \cdot {}_A\langle z\uparrow|\hat{S}_z^A|z\uparrow\rangle_A = \left(-\frac{\hbar}{2}\right)\left(+\frac{\hbar}{2}\right) = -\frac{\hbar^2}{4}
\end{aligned}$$

Joint Hilbert Space of Two Spin 1/2 Particles: An Example

$$|\psi\rangle = |z\uparrow\rangle_A \otimes |z\downarrow\rangle_B$$

The operator for one subsystem can also be written as:

$$\hat{S}_z^A = \hat{S}_z^A \otimes \hat{1}^B$$

$$\hat{S}_z^B = \hat{1}^A \otimes \hat{S}_z^B$$

Consider a state of two different spin 1/2 particles: $|\psi\rangle = |z\uparrow\rangle_A \otimes |z\downarrow\rangle_B$

$$\langle\psi|\hat{S}_z^A|\psi\rangle = \left[{}_B\langle z\downarrow| \otimes {}_A\langle z\uparrow|\right]\hat{S}_z^A\left[|z\uparrow\rangle_A \otimes |z\downarrow\rangle_B\right] = \left[{}_B\langle z\downarrow|\,{}_A\langle z\uparrow|\right]\left[\hat{S}_z^A|z\uparrow\rangle_A|z\downarrow\rangle_B\right]$$

$$= {}_B\langle z\downarrow|z\downarrow\rangle_B \cdot {}_A\langle z\uparrow|\hat{S}_z^A|z\uparrow\rangle_A = \frac{\hbar}{2}$$

$$\langle\psi|\hat{S}_z^B|\psi\rangle = \left[{}_B\langle z\downarrow| \otimes {}_A\langle z\uparrow|\right]\hat{S}_z^B\left[|z\uparrow\rangle_A \otimes |z\downarrow\rangle_B\right] = \left[{}_B\langle z\downarrow|\,{}_A\langle z\uparrow|\right]\left[|z\uparrow\rangle_A\hat{S}_z^B|z\downarrow\rangle_B\right]$$

$$= {}_A\langle z\uparrow|z\uparrow\rangle_A \cdot {}_B\langle z\downarrow|\hat{S}_z^B|z\downarrow\rangle_B = -\frac{\hbar}{2}$$

Measurements Performed on One Subsystem: A-Priori Probabilities

$$|\psi\rangle = |x\uparrow\rangle_A \otimes |x\downarrow\rangle_B = \frac{1}{\sqrt{2}}\Big[|z\uparrow\rangle_A + |z\downarrow\rangle_A\Big] \otimes \frac{1}{\sqrt{2}}\Big[|z\uparrow\rangle_B - |z\downarrow\rangle_B\Big]$$

$$= \frac{1}{2}\Big[|z\uparrow\rangle_A |z\uparrow\rangle_B - |z\uparrow\rangle_A |z\downarrow\rangle_B + |z\downarrow\rangle_A |z\uparrow\rangle_B - |z\downarrow\rangle_A |z\downarrow\rangle_B\Big]$$

Suppose spin of A is measured in the z-direction (spin of B is not measured)

What is the a-priori probability of measuring spin A to be in the +z-direction?

$$\left|\left({}_A\langle z\uparrow| \otimes {}_B\langle z\uparrow|\right)|\psi\rangle\right|^2 + \left|\left({}_A\langle z\uparrow| \otimes {}_B\langle z\downarrow|\right)|\psi\rangle\right|^2$$

$$= \frac{1}{2}$$

$$= \sum_{\substack{j = \text{All orthogonal states} \\ \text{of B that form a} \\ \text{complete set}}} \left|\left({}_A\langle z\uparrow| \otimes {}_B\langle j|\right)|\psi\rangle\right|^2$$

What is the a-priori probability of measuring spin A to be in the -z-direction?

$$\left|\left({}_A\langle z\downarrow| \otimes {}_B\langle z\uparrow|\right)|\psi\rangle\right|^2 + \left|\left({}_A\langle z\downarrow| \otimes {}_B\langle z\downarrow|\right)|\psi\rangle\right|^2$$

$$= \frac{1}{2}$$

$$= \sum_{\substack{j = \text{All orthogonal states} \\ \text{of B that form a} \\ \text{complete set}}} \left|\left({}_A\langle z\downarrow| \otimes {}_B\langle j|\right)|\psi\rangle\right|^2$$

Measurements Performed on One Subsystem: Collapsed State

$$|\psi\rangle = |x\uparrow\rangle_A \otimes |x\downarrow\rangle_B = \frac{1}{\sqrt{2}}\Big[|z\uparrow\rangle_A + |z\downarrow\rangle_A\Big] \otimes \frac{1}{\sqrt{2}}\Big[|z\uparrow\rangle_B - |z\downarrow\rangle_B\Big]$$

$$= \frac{1}{2}\Big[|z\uparrow\rangle_A |z\uparrow\rangle_B - |z\uparrow\rangle_A |z\downarrow\rangle_B + |z\downarrow\rangle_A |z\uparrow\rangle_B - |z\downarrow\rangle_A |z\downarrow\rangle_B\Big]$$

Suppose spin of A is measured in the z-direction (spin of B is not measured)

Suppose spin A is measured and the result was $+\hbar/2$ for $\hat{S}_z^A$:

$$\underbrace{\Big(|z\uparrow\rangle_A\, {}_A\langle z\uparrow| \otimes \hat{1}_B\Big)}_{\text{Projector}}|\psi\rangle$$

$$\xrightarrow{\text{normalize}} |\psi_c\rangle$$

$$|\psi_c\rangle = |z\uparrow\rangle_A \otimes \frac{1}{\sqrt{2}}\Big[|z\uparrow\rangle_B - |z\downarrow\rangle_B\Big]$$

$$= |z\uparrow\rangle_A \otimes |x\downarrow\rangle_B$$

Suppose spin A is measured and the result was $-\hbar/2$ for $\hat{S}_z^A$:

$$\underbrace{\Big(|z\downarrow\rangle_A\, {}_A\langle z\downarrow| \otimes \hat{1}_B\Big)}_{\text{Projector}}|\psi\rangle$$

$$\xrightarrow{\text{normalize}} |\psi_c\rangle$$

$$|\psi_c\rangle = |z\downarrow\rangle_A \otimes \frac{1}{\sqrt{2}}\Big[|z\uparrow\rangle_B - |z\downarrow\rangle_B\Big]$$

$$= |z\downarrow\rangle_A \otimes |x\downarrow\rangle_B$$

Entangled and Unentangled Quantum States of Composite Systems

Quantum states of composite systems come in two different varieties:

1) Unentangled states

2) Entangled states

Unentangled Quantum States of Bipartite Systems

Consider the following two states of the bipartite system:

$$|\psi_1\rangle = |z\uparrow\rangle_A \otimes |z\downarrow\rangle_B$$

$$|\psi_1\rangle = |z\uparrow\rangle_A \otimes |z\downarrow\rangle_B$$

$$|\psi_2\rangle = |x\uparrow\rangle_A \otimes |x\downarrow\rangle_B = \frac{1}{\sqrt{2}}\Big[|z\uparrow\rangle_A + |z\downarrow\rangle_A\Big] \otimes \frac{1}{\sqrt{2}}\Big[|z\uparrow\rangle_B - |z\downarrow\rangle_B\Big]$$

$$= \frac{1}{2}\Big[|z\uparrow\rangle_A |z\uparrow\rangle_B - |z\uparrow\rangle_A |z\downarrow\rangle_B + |z\downarrow\rangle_A |z\uparrow\rangle_B - |z\downarrow\rangle_A |z\downarrow\rangle_B\Big]$$

Both these states can be written in the form:

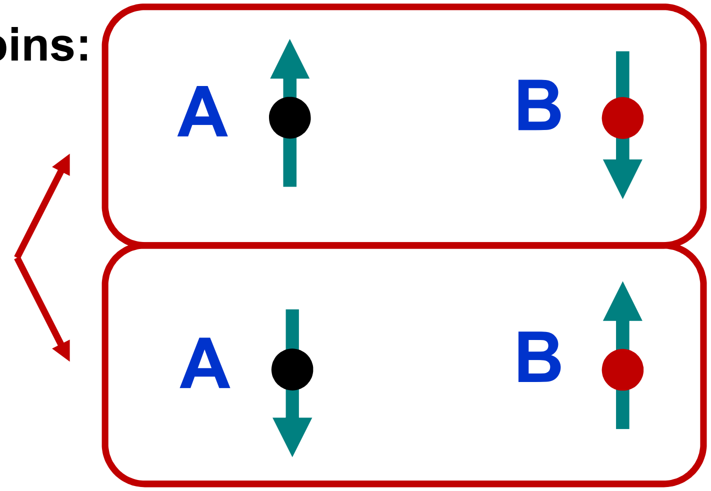
$$|\psi\rangle = |\phi\rangle_A \otimes |\chi\rangle_B = |\text{some state of A}\rangle_A \otimes |\text{some state of B}\rangle_B$$

In other words, the joint state of A and B can be separated out or factored out and written as a single tensor product. States which can be written this way are called unentangled states

Entangled Quantum States of Bipartite Systems

Consider the following quantum state of two spins:

$$|\psi\rangle = \frac{1}{\sqrt{2}}\Big[|z\uparrow\rangle_A \otimes |z\downarrow\rangle_B + |z\downarrow\rangle_A \otimes |z\uparrow\rangle_B\Big]$$



State is a superposition of these two composite realities

This state can not be written in the form:

$$|\psi\rangle = |\phi\rangle_A \otimes |\chi\rangle_B = |\text{some state of A}\rangle_A \otimes |\text{some state of B}\rangle_B$$

In other words, the joint state of A and B CANNOT be separated out or factored out and written as a single tensor product.

Such states are called entangled states

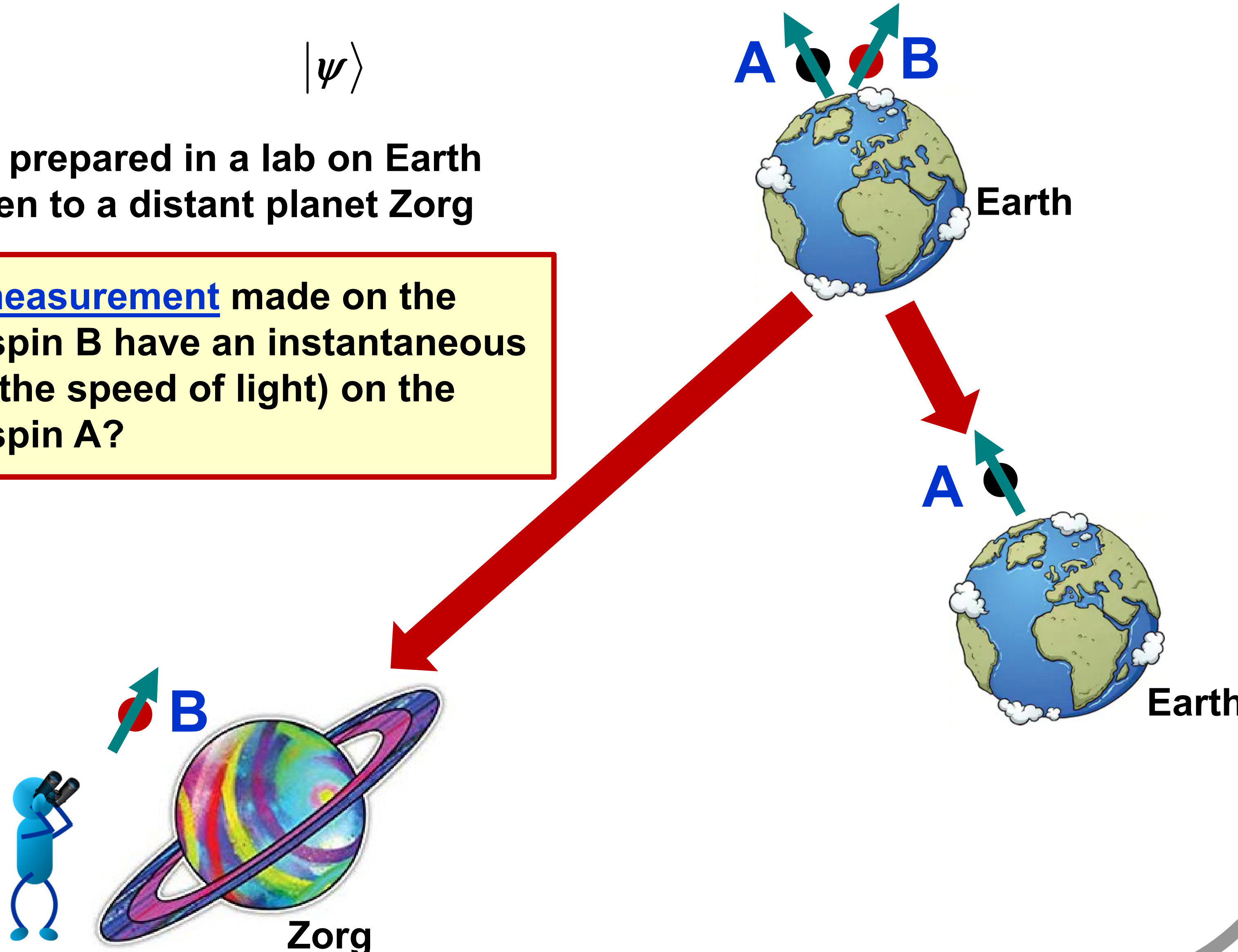
Local Measurements on Quantum States of Bipartite Systems

Consider again the following quantum state of two spins A and B:

$$|\psi\rangle$$

The above state is prepared in a lab on Earth
Then spin B is taken to a distant planet Zorg

> **Question:** **Can a measurement made on the quantum state of spin B have an instantaneous effect (faster than the speed of light) on the quantum state of spin A?**



Local Measurements on Unentangled Quantum States of Bipartite Systems - I

Consider the following unentangled quantum state of two spins:

$$|\psi\rangle = |x\uparrow\rangle_A \otimes |x\downarrow\rangle_B$$

$$= \frac{1}{\sqrt{2}}\left[|z\uparrow\rangle_A + |z\downarrow\rangle_A\right] \otimes \frac{1}{\sqrt{2}}\left[|z\uparrow\rangle_B - |z\downarrow\rangle_B\right]$$

$$= \frac{1}{2}\left[|z\uparrow\rangle_A |z\uparrow\rangle_B - |z\uparrow\rangle_A |z\downarrow\rangle_B + |z\downarrow\rangle_A |z\uparrow\rangle_B - |z\downarrow\rangle_A |z\downarrow\rangle_B\right]$$

$$|\psi\rangle = |x\uparrow\rangle_A \otimes |x\downarrow\rangle_B$$

Move B to planet Zorg

Measure spin of B

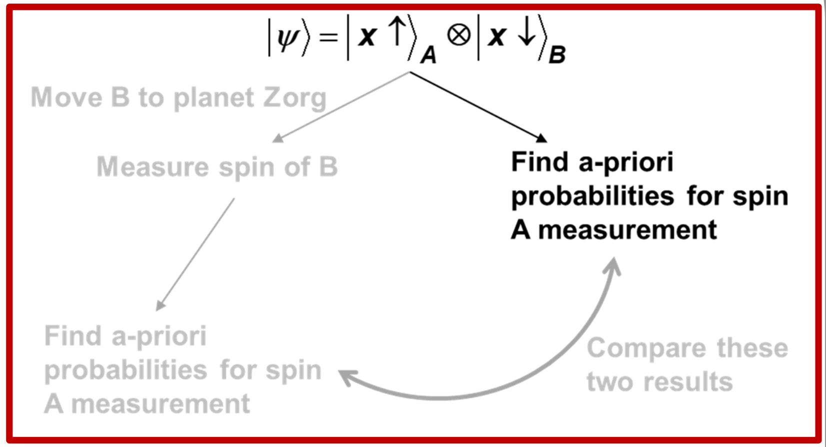
Find a-priori probabilities for spin A measurement

Find a-priori probabilities for spin A measurement

Compare these two results

Local Measurements on Unentangled Quantum States of Bipartite Systems - II

Consider the following unentangled quantum state:



$$|\psi\rangle = |x\uparrow\rangle_A \otimes |x\downarrow\rangle_B$$

$$= \frac{1}{\sqrt{2}}\Big[|z\uparrow\rangle_A + |z\downarrow\rangle_A\Big] \otimes \frac{1}{\sqrt{2}}\Big[|z\uparrow\rangle_B - |z\downarrow\rangle_B\Big]$$

$$= \frac{1}{2}\Big[|z\uparrow\rangle_A |z\uparrow\rangle_B - |z\uparrow\rangle_A |z\downarrow\rangle_B + |z\downarrow\rangle_A |z\uparrow\rangle_B - |z\downarrow\rangle_A |z\downarrow\rangle_B\Big]$$

Question: What are the a-priori probabilities of measuring spin-up or spin-down if spin of particle A is measured along the z-axis (i.e. if the observable S_z^A is measured)?

For spin-up A:

$$\Big|\big({}_A\langle z\uparrow| \otimes {}_B\langle z\uparrow|\big)|\psi\rangle\Big|^2 + \Big|\big({}_A\langle z\uparrow| \otimes {}_B\langle z\downarrow|\big)|\psi\rangle\Big|^2$$

$$= \frac{1}{2}$$

$$= \sum_{\substack{j = \text{All orthogonal states} \\ \text{of B that form a} \\ \text{complete set}}} \Big|\big({}_A\langle z\uparrow| \otimes {}_B\langle j|\big)|\psi\rangle\Big|^2$$

For spin-down A:

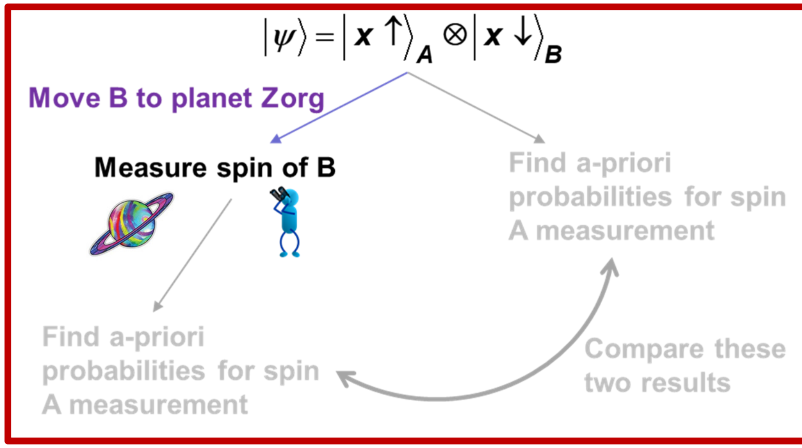
$$= \sum_{\substack{j = \text{All orthogonal} \\ \text{states of B that} \\ \text{form a} \\ \text{complete set}}} \Big|\big({}_A\langle z\downarrow| \otimes {}_B\langle j|\big)|\psi\rangle\Big|^2$$

$$= \frac{1}{2}$$

Local Measurements on Unentangled Quantum States of Bipartite Systems - III

Again consider the following unentangled quantum state:

$$|\psi\rangle = |x\uparrow\rangle_A \otimes |x\downarrow\rangle_B$$
$$= \frac{1}{\sqrt{2}}\Big[|z\uparrow\rangle_A + |z\downarrow\rangle_A\Big] \otimes \frac{1}{\sqrt{2}}\Big[|z\uparrow\rangle_B - |z\downarrow\rangle_B\Big]$$



Suppose the observable S_z^B is measured and a result of spin-down is obtained

$$\underbrace{\left(\hat{1}_A \otimes |z\downarrow\rangle_B\, {}_B\langle z\downarrow|\right)}_{\text{Projector}}|\psi\rangle \xrightarrow{\text{normalize}} |\psi_c\rangle$$

The resulting normalized collapsed state is:

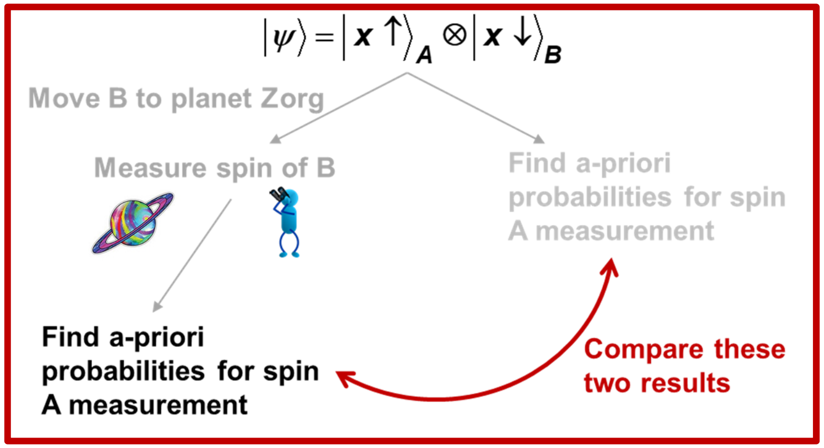
$$|\psi_c\rangle = -|x\uparrow\rangle_A \otimes |z\downarrow\rangle_B$$
$$= -\frac{1}{\sqrt{2}}\Big[|z\uparrow\rangle_A + |z\downarrow\rangle_A\Big] \otimes |z\downarrow\rangle_B = -\frac{1}{\sqrt{2}}\Big[|z\uparrow\rangle_A |z\downarrow\rangle_B + |z\downarrow\rangle_A |z\downarrow\rangle_B\Big]$$

Local Measurements on Unentangled Quantum States of Bipartite Systems - IV

The resulting collapsed state is:

$$\left|\psi_c\right\rangle = -\frac{1}{\sqrt{2}}\Big[\left|z\uparrow\right\rangle_A + \left|z\downarrow\right\rangle_A\Big]\otimes\left|z\downarrow\right\rangle_B$$

$$= -\frac{1}{\sqrt{2}}\Big[\left|z\uparrow\right\rangle_A\left|z\downarrow\right\rangle_B + \left|z\downarrow\right\rangle_A\left|z\downarrow\right\rangle_B\Big]$$



Question: **Given that the spin of particle B has been measured, we again ask what are now the a-priori probabilities of measuring spin-up or spin-down if spin of particle A is measured along the z-axis (i.e. if the observable S_z^A is measured)?**

For spin-up:

$$= \sum_{\substack{j = \text{All orthogonal states} \\ \text{of B that form a} \\ \text{complete set}}} \left|\Big({}_A\left\langle z\uparrow\right| \otimes {}_B\left\langle j\right|\Big)\left|\psi_c\right\rangle\right|^2$$

$$= \frac{1}{2} = \text{same as before!}$$

For spin-down:

$$= \sum_{\substack{j = \text{All orthogonal states} \\ \text{of B that form a} \\ \text{complete set}}} \left|\Big({}_A\left\langle z\downarrow\right| \otimes {}_B\left\langle j\right|\Big)\left|\psi_c\right\rangle\right|^2$$

$$= \frac{1}{2} = \text{same as before!}$$

In an unentangled state, the a-priori probabilities for measurements performed on system A do not change if measurements have been made on system B first

Local Measurements on Unentangled Quantum States of Bipartite Systems: Summary of Results

Consider the following unentangled quantum state of two spins:

$$\left|\psi\right\rangle = \left|x\uparrow\right\rangle_A \otimes \left|x\downarrow\right\rangle_B$$

$$= \frac{1}{\sqrt{2}}\left[\left|z\uparrow\right\rangle_A + \left|z\downarrow\right\rangle_A\right] \otimes \frac{1}{\sqrt{2}}\left[\left|z\uparrow\right\rangle_B - \left|z\downarrow\right\rangle_B\right]$$

$$= \frac{1}{2}\left[\left|z\uparrow\right\rangle_A\left|z\uparrow\right\rangle_B - \left|z\uparrow\right\rangle_A\left|z\downarrow\right\rangle_B + \left|z\downarrow\right\rangle_A\left|z\uparrow\right\rangle_B - \left|z\downarrow\right\rangle_A\left|z\downarrow\right\rangle_B\right]$$

$$\left|\psi\right\rangle = \left|x\uparrow\right\rangle_A \otimes \left|x\downarrow\right\rangle_B$$

Move B to planet Zorg

Measure spin of B

Find a-priori probabilities for spin A measurement

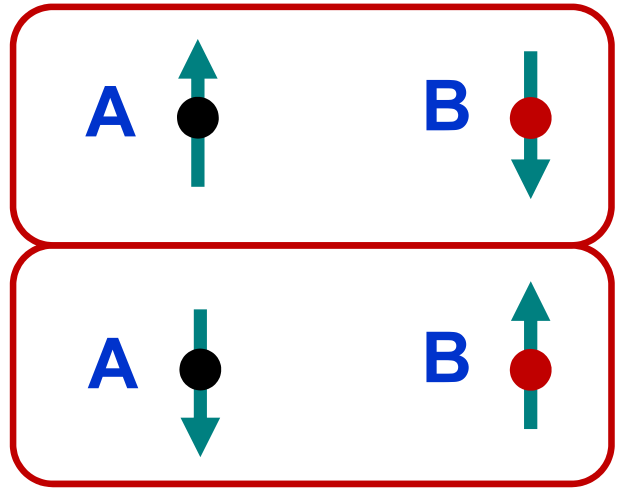
Find a-priori probabilities for spin A measurement

Compare these two results: THEY AGREE

Local Measurements on Entangled Quantum States of Bipartite Systems - I

Consider the following entangled quantum state:

$$|\psi\rangle = \frac{1}{\sqrt{2}}\Big[\,|z\uparrow\rangle_A \otimes |z\downarrow\rangle_B + |z\downarrow\rangle_A \otimes |z\uparrow\rangle_B\,\Big]$$



State is a superposition of these two composite realities

$$|\psi\rangle = \frac{1}{\sqrt{2}}\Big[\,|z\uparrow\rangle_A \otimes |z\downarrow\rangle_B + |z\downarrow\rangle_A \otimes |z\uparrow\rangle_B\,\Big]$$

Move B to planet Zorg

Measure spin of B

Find a-priori probabilities for spin A measurement

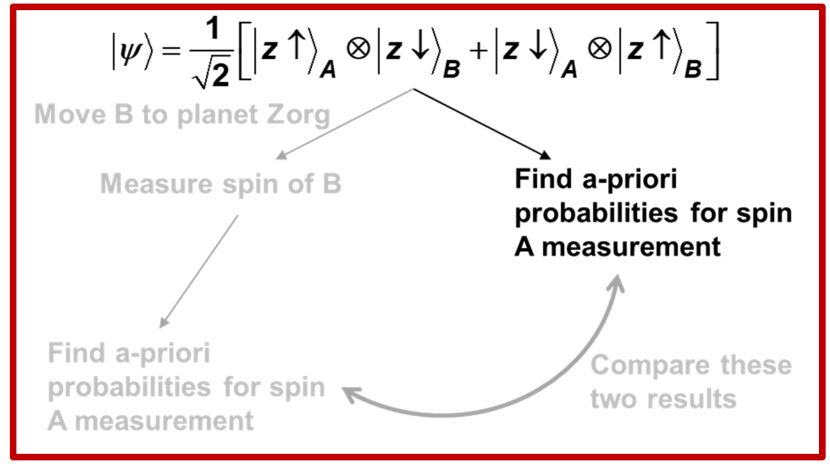
Find a-priori probabilities for spin A measurement

Compare these two results

Local Measurements on Entangled Quantum States of Bipartite Systems - II

Consider the following entangled quantum state:

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left[|z\uparrow\rangle_A \otimes |z\downarrow\rangle_B + |z\downarrow\rangle_A \otimes |z\uparrow\rangle_B\right]$$



Question: What are the a-priori probabilities of measuring spin-up or spin-down if spin of particle A is measured along the z-axis (i.e. if the observable S_z^A is measured)?

For spin-up:

$$= \sum_{\substack{j = \text{All orthogonal states} \\ \text{of B that form a} \\ \text{complete set}}} \left| \left({}_A\langle z\uparrow| \otimes {}_B\langle j| \right) |\psi_c\rangle \right|^2$$

$$= \frac{1}{2}$$

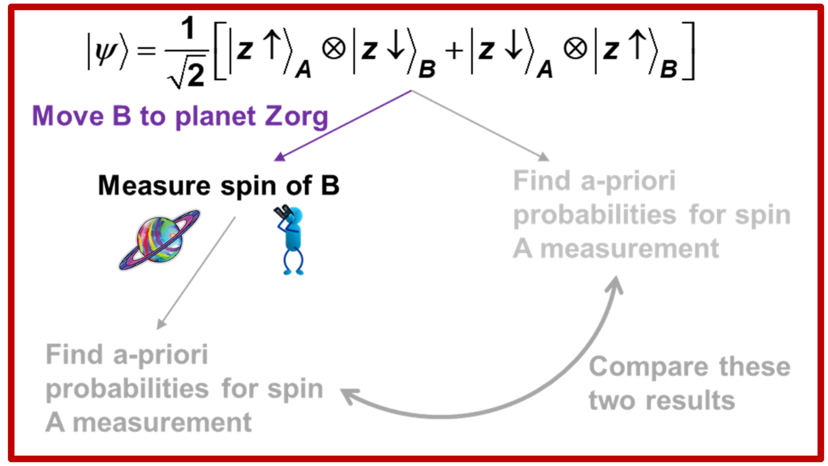
For spin-down:

$$= \sum_{\substack{j = \text{All orthogonal states} \\ \text{of B that form a} \\ \text{complete set}}} \left| \left({}_A\langle z\downarrow| \otimes {}_B\langle j| \right) |\psi_c\rangle \right|^2$$

$$= \frac{1}{2}$$

Local Measurements on Entangled Quantum States of Bipartite Systems - III

Consider again the following entangled quantum state:



$$|\psi\rangle = \frac{1}{\sqrt{2}}\left[|z\uparrow\rangle_A \otimes |z\downarrow\rangle_B + |z\downarrow\rangle_A \otimes |z\uparrow\rangle_B\right]$$

Suppose the observable S_z^B is measured and a result of spin-down is obtained:

$$\underbrace{\left(\hat{1}_A \otimes |z\downarrow\rangle_B \, {}_B\langle z\downarrow|\right)}_{\text{Projector}}|\psi\rangle \xrightarrow{\text{normalize}} |\psi_c\rangle$$

The resulting normalized collapsed state is:

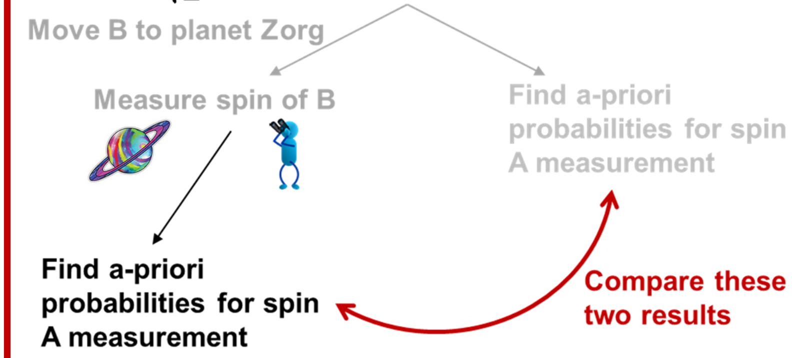
$$|\psi_c\rangle = |z\uparrow\rangle_A \otimes |z\downarrow\rangle_B$$

On planet Earth

On planet Zorg

Local Measurements on Unentangled Quantum States of Bipartite Systems - IV

$$|\psi\rangle = \frac{1}{\sqrt{2}}\Big[|z\uparrow\rangle_A \otimes |z\downarrow\rangle_B + |z\downarrow\rangle_A \otimes |z\uparrow\rangle_B\Big]$$



The resulting collapsed state is:

$$|\psi_c\rangle = |z\uparrow\rangle_A \otimes |z\downarrow\rangle_B$$

Question: Given that the spin of particle B has been measured, we again ask what are the a-priori probabilities of measuring spin-up or spin-down if spin of particle A is measured along the z-axis (i.e. if the observable S_z^A is measured)?

For spin-up:

$$= \sum_{\substack{j=\text{All orthogonal states}\\ \text{of B that form a}\\ \text{complete set}}} \Big| \big({}_A\langle z\uparrow| \otimes {}_B\langle j| \big) |\psi_c\rangle \Big|^2$$

$$= 1 = \text{ NOT the same as before!}$$

For spin-down:

$$= \sum_{\substack{j=\text{All orthogonal states}\\ \text{of B that form a}\\ \text{complete set}}} \Big| \big({}_A\langle z\downarrow| \otimes {}_B\langle j| \big) |\psi_c\rangle \Big|^2$$

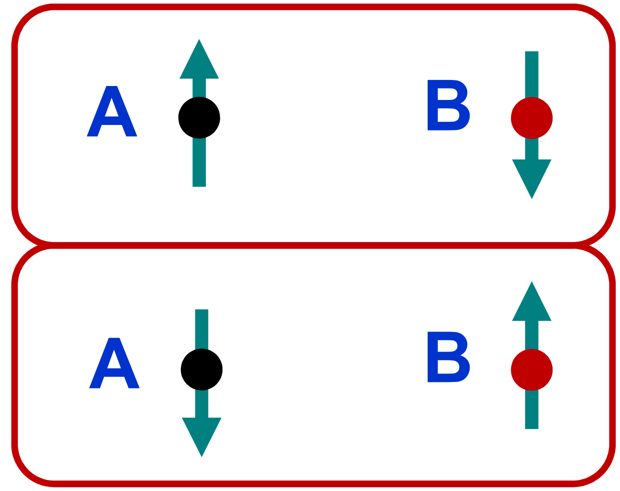
$$= 0 = \text{ NOT the same as before!}$$

In an entangled state, the a-priori probabilities for measurement results of system A change if measurements have been made on system B first

Local Measurements on Entangled Quantum States of Bipartite Systems: Summary of Results

Consider the following <u>entangled</u> quantum state:

$$|\psi\rangle = \frac{1}{\sqrt{2}}\Big[|z\uparrow\rangle_A \otimes |z\downarrow\rangle_B + |z\downarrow\rangle_A \otimes |z\uparrow\rangle_B\Big]$$



State is a superposition of these two composite realities

$$|\psi\rangle = \frac{1}{\sqrt{2}}\Big[|z\uparrow\rangle_A \otimes |z\downarrow\rangle_B + |z\downarrow\rangle_A \otimes |z\uparrow\rangle_B\Big]$$

Move B to planet Zorg

Measure spin of B

Find a-priori probabilities for spin A measurement

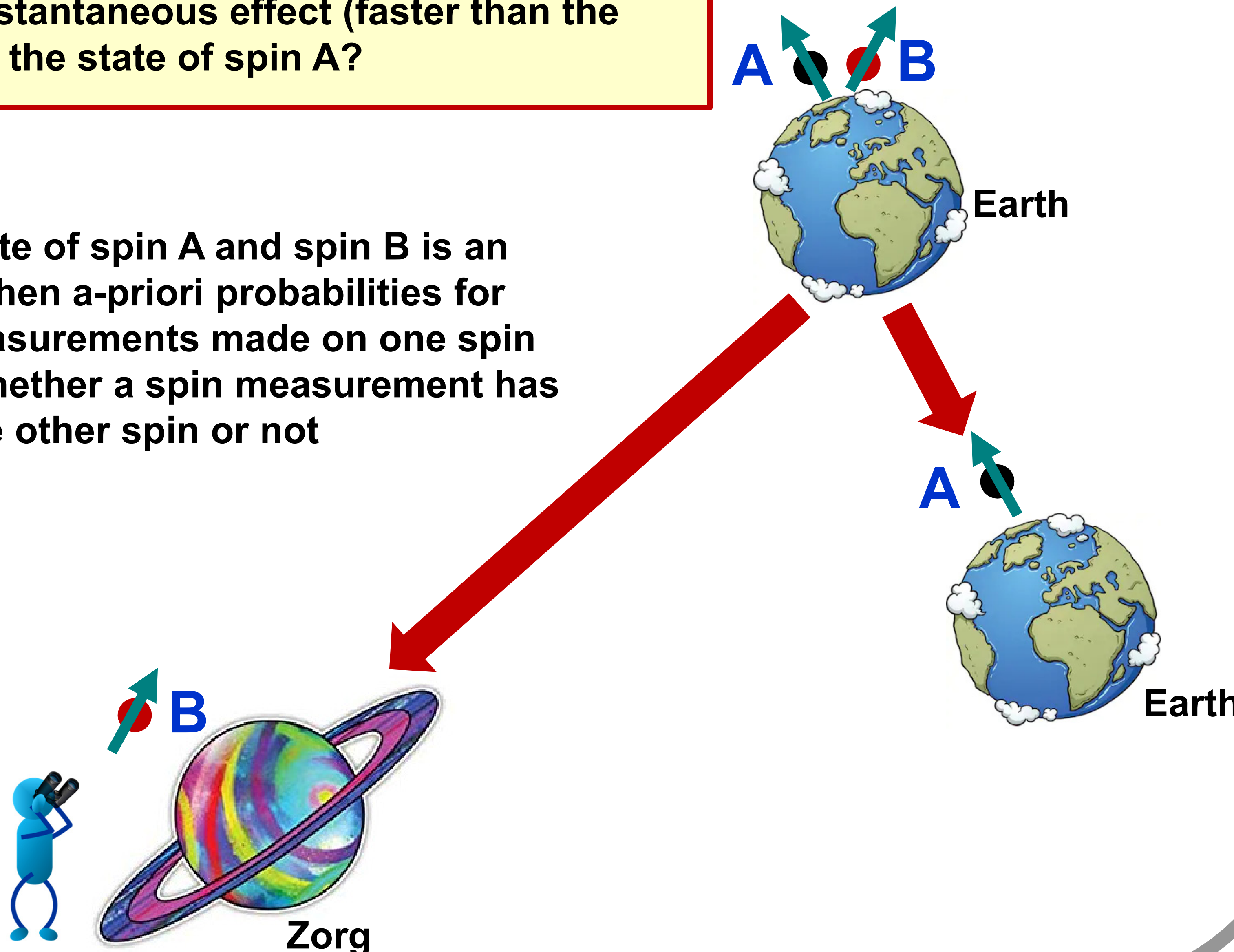
Find a-priori probabilities for spin A measurement

Compare these two results: <u>THEY DO NOT AGREE !!</u>

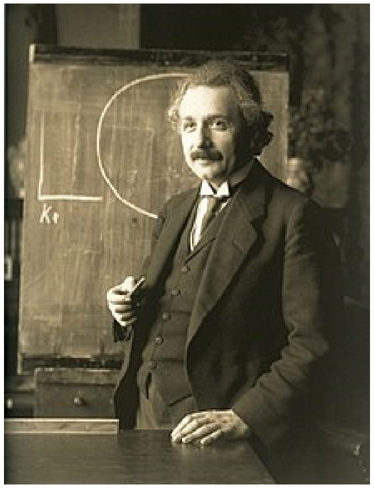
Unentangled and Entangled States: Summary of Results

Question: Can a measurement made on the state of spin B have an instantaneous effect (faster than the speed of light) on the state of spin A?

Answer: if the state of spin A and spin B is an entangled state, then a-priori probabilities for the results of measurements made on one spin will depend on whether a spin measurement has been made on the other spin or not



Entangled States and the Einstein-Podolsky-Rosen (EPR) Paradox



Einstein's philosophical postulate:

Any acceptable physical theory must fulfill "local realism“

Realism:
There must be an aspect of physical reality associated with a physical quantity, and this aspect should not depend on whether this physical quantity is measured or not

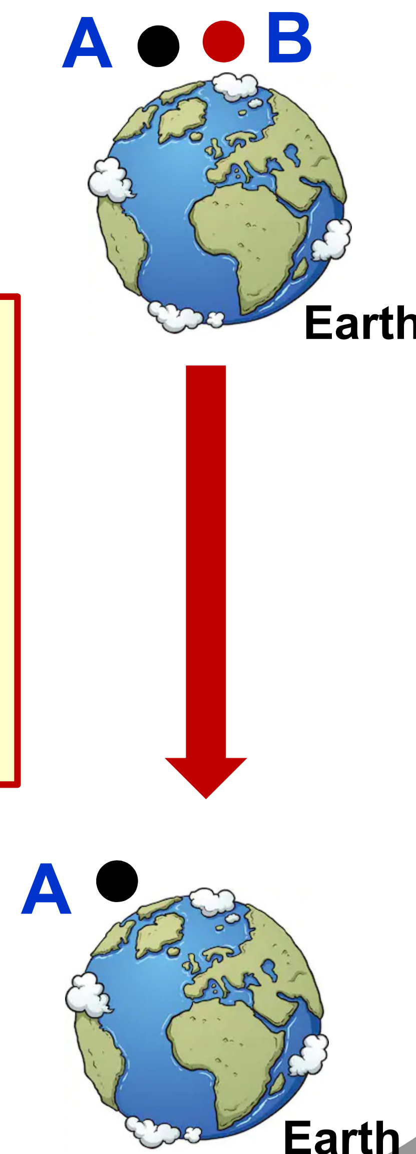
Locality:
Physical quantities should not instantaneously get affected or influenced by other physical quantities that are located far away

Local realism implies that real measurable attributes of a physical system cannot depend on things and happenings far away (that are outside the past light cone of that physical system)

Entangled States and the Einstein-Podolsky-Rosen (EPR) Paradox

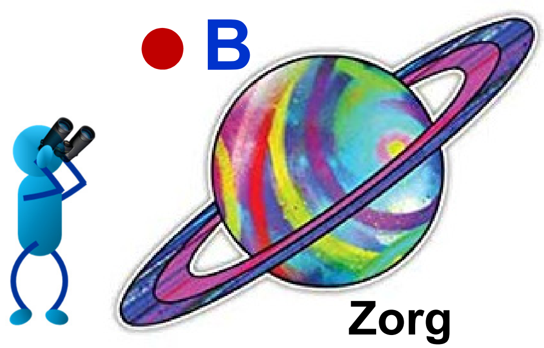
Consider again the following entangled quantum state of two particles A and B:

$$|\psi\rangle = \frac{1}{\sqrt{2}}\Big[|z\uparrow\rangle_A \otimes |z\downarrow\rangle_B + |z\downarrow\rangle_A \otimes |z\uparrow\rangle_B\Big]$$



The above state is prepared in a lab on Earth
Then particle B is taken to a distant planet

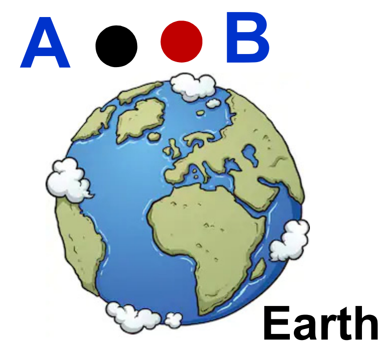
> **Question:** Can the choice of measurement made on the state of particle B have an instantaneous effect (faster than the speed of light) on the quantum state of particle A?
>
> If so, then how can one assign any element of reality that is local to the quantum state of A? In other words, the "real" physical state of particle A cannot depend on the choice of measurement on a far away particle B



Entangled States and the Einstein-Podolsky-Rosen (EPR) Paradox

Consider again the following entangled quantum state of two particles A and B:

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left[|z\uparrow\rangle_A \otimes |z\downarrow\rangle_B + |z\downarrow\rangle_A \otimes |z\uparrow\rangle_B\right]$$



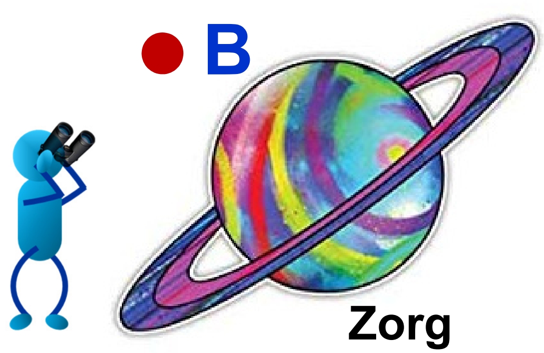
Suppose the spin of particle B is measured along the z-axis

1) If the result is spin down: collapsed state is:

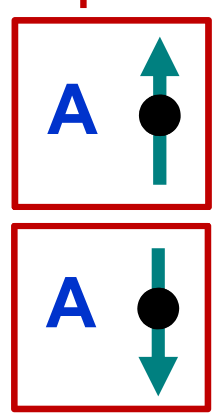
$$|\psi_c\rangle = |z\uparrow\rangle_A \otimes |z\downarrow\rangle_B$$

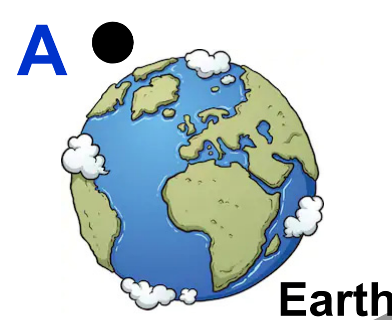
2) If the result is spin up: collapsed state is:

$$|\psi_c\rangle = |z\downarrow\rangle_A \otimes |z\uparrow\rangle_B$$



State of spin A in the collapsed state



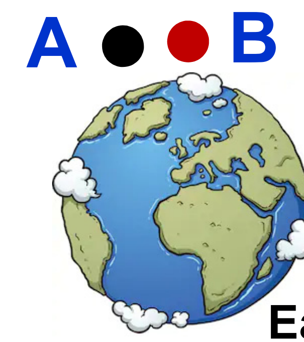


Entangled States and the Einstein-Podolsky-Rosen (EPR) Paradox

Consider yet again the following entangled quantum state of two particles A and B:

$$\left|\psi\right\rangle = \frac{1}{\sqrt{2}}\left[\left|z\uparrow\right\rangle_A \otimes \left|z\downarrow\right\rangle_B + \left|z\downarrow\right\rangle_A \otimes \left|z\uparrow\right\rangle_B\right]$$

$$= \frac{1}{\sqrt{2}}\left[\left|x\uparrow\right\rangle_A \otimes \left|x\uparrow\right\rangle_B - \left|x\downarrow\right\rangle_A \otimes \left|x\downarrow\right\rangle_B\right]$$



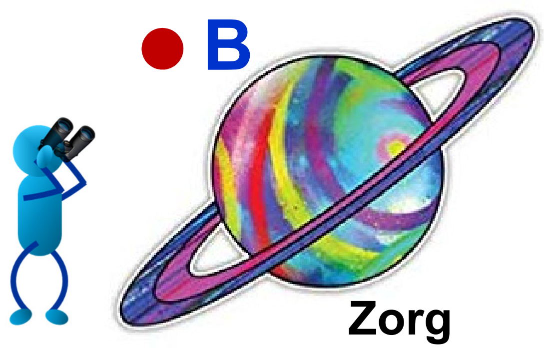
Suppose spin of particle B is now measured along the x-axis
(A different choice of measurement)

1) Result is spin down: collapsed state is:

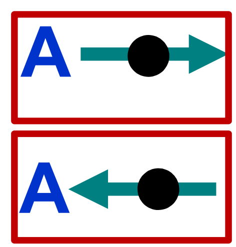
$$\left|\psi_c\right\rangle = -\left|x\downarrow\right\rangle_A \otimes \left|x\downarrow\right\rangle_B$$

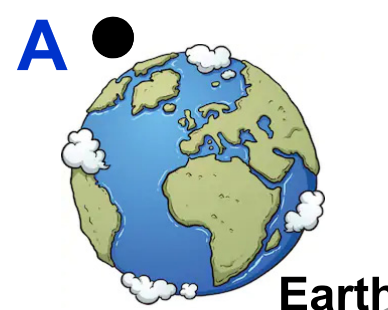
2) Result is spin up: collapsed state is:

$$\left|\psi_c\right\rangle = \left|x\uparrow\right\rangle_A \otimes \left|x\uparrow\right\rangle_B$$



State of spin A in the collapsed state

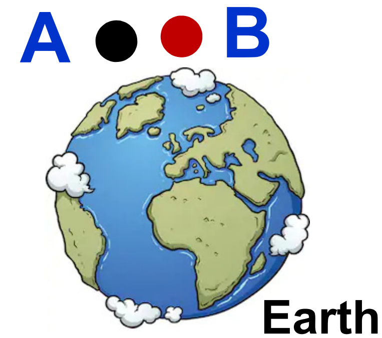




Entangled States and the Einstein-Podolsky-Rosen (EPR) Paradox: "Spooky Action at a Distance"

Consider again the following entangled quantum state of two particles A and B:

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left[|z\uparrow\rangle_A \otimes |z\downarrow\rangle_B + |z\downarrow\rangle_A \otimes |z\uparrow\rangle_B\right]$$



Einstein's Question: Can the choice of measurement made on the state of particle B have an instantaneous effect (faster than the speed of light) on the quantum state of particle A?

If so, then how can one assign any element of reality that is local to the quantum state of A?

Conclusion: No local realism exist in quantum physics!!