Lecture 11

The Postulates of Quantum Physics

In this lecture you will learn:

- **The postulates of quantum physics**

Postulates of Quantum Physics

There are several postulates of quantum mechanics

These are postulates since they cannot be derived from some other deeper theory

They are known only from experiments and inductive reasonings (of the kind made in this course till now)

Postulates 1, 2, and 3:

1) The state of any physical system (e.g a particle) at time 't' is described by a vector (or a ket), denoted by $|\psi(t)\rangle$, that belongs to a Hilbert space

2) Every measurable quantity A of this physical system (i.e. an observable, like position or momentum of a particle) is described by a Hermitian operator $\hat{A}$ that acts in this Hilbert space

3)The only possible outcome of a measurement of the observable A is one of the eigenvalues of the operator $\hat{A}$ described by:

$$\hat{A}|v_k\rangle = \lambda_k |v_k\rangle$$

$$\langle v_k | v_j \rangle = \delta_{kj} \qquad \sum_j |v_j\rangle\langle v_j| = \hat{1}$$

Postulates of Quantum Physics

Postulate 4:

$$\hat{A}\left|v_j\right\rangle = \lambda_j \left|v_j\right\rangle$$

4) We cannot know for certain the result of a measurement of an observable A before it is made, but the a-priori probability of getting λ_k as a result when observable *A* is measured is given by:

$$\left|\left\langle v_k \middle| \psi\right\rangle\right|^2$$

Discussion: we can write:

$$\left|\psi\right\rangle = \sum_k a_k \left|v_k\right\rangle$$

→ The state as written is a linear superposition of the eigenvectors of *A*

The a-priori probability of getting λ_k as result is then: $\left|a_k\right|^2 = \left|\left\langle v_k \middle| \psi\right\rangle\right|^2$

The a-priori probabilities must all add up to unity:

$$1 = \left\langle \psi \middle| \psi \right\rangle = \left\langle \psi \right| \hat{1} \left| \psi \right\rangle = \left\langle \psi \right| \sum_k \left| v_k \right\rangle \left\langle v_k \right| \left| \psi \right\rangle$$

$$= \sum_k \left\langle \psi \middle| v_k \right\rangle \left\langle v_k \middle| \psi \right\rangle = \sum_k \left| a_k \right|^2$$

Postulates of Quantum Physics

Postulate 5:

5) Immediately after the measurement of an observable *A*, the quantum state collapses into the eigensubspace corresponding to the measured eigenvalue

Discussion:

Suppose the quantum state of a system is: $|\psi\rangle$

Suppose a physical observable *A* is measured:

$$\hat{A}|v_k\rangle = \lambda_k |v_k\rangle \qquad\qquad |\psi\rangle = \sum_k a_k |v_k\rangle$$

And the result is: λ_j

Question: **What is the state of the system just after the measurement?**

- i) **The quantum state has information about all that is knowable about the system**
- ii) **The quantum state post-measurement must reflect the result of the measurement**
- iii) **If second measurement of *A* is made immediately after the first measurement, the result λ_j must be obtained with probability 1**

Answer: **The state immediately after the first measurement must be:** $|v_j\rangle$

$$|\psi\rangle = \sum_k a_k |v_k\rangle \xrightarrow[\text{Result: } \lambda_j]{\text{Measurement}} |v_j\rangle$$

Postulates of Quantum Physics

Postulate 6:

The time evolution of a quantum state $|\psi(t)\rangle$ is given by the Schrödinger's equation:

$$i\hbar\frac{\partial}{\partial t}|\psi(t)\rangle = \hat{H}|\psi(t)\rangle$$

If the Hamiltonian is time-independent then the formal solution, subject to the boundary condition that at time *t*=0 the state is $|\psi(t=0)\rangle$, is:

$$|\psi(t)\rangle = e^{-i\frac{\hat{H}}{\hbar}t}|\psi(t=0)\rangle = \left(1+\left(-\frac{i}{\hbar}\hat{H}t\right)+\frac{1}{2!}\left(-\frac{i}{\hbar}\hat{H}t\right)^2+.....\right)|\psi(t=0)\rangle$$

Stationary states: Eigenstates of the Hamiltonian are called stationary states

Suppose:

$$\hat{H}|e_k\rangle = \varepsilon_k|e_k\rangle \qquad |\psi(t=0)\rangle = \sum_k c_k|e_k\rangle$$

Then:

$$|\psi(t)\rangle = e^{-i\frac{\hat{H}}{\hbar}t}\sum_k c_k|e_k\rangle = \sum_k c_k\, e^{-i\frac{\varepsilon_k t}{\hbar}}|e_k\rangle$$

The probability of being in a particular energy eigenstate does not change with time:

$$|\langle e_j|\psi(t)\rangle|^2 = |\langle e_j|\psi(t=0)\rangle|^2 = |c_j|^2$$