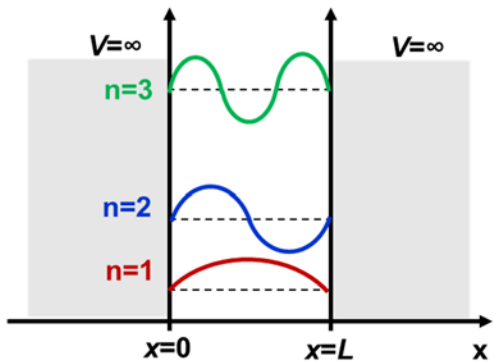
ECE 4060: Quantum Physics and Engineering

Fall 2020

Homework 8 **Due on Nov. 11, 2020 by 5:00 PM (via email)**

Problem 8.1: (Infinite Potential Well: A Time-Dependent Problem)

Consider the following infinite potential well:



We will consider only the first two energy levels in this problem. The Hamiltonian is,

$$\hat{H}_o = \sum_{j=1,2} E_j \left|e_j\right\rangle\left\langle e_j\right|$$

An electron in the potential well is subjected to a time-dependent electric field in the x-direction so that the Hamiltonian gets an extra term,

$$\hat{H} = \hat{H}_o - qE_x \sin(\omega t)\left(\hat{x} - \frac{L}{2}\right)$$

a) Find an expression for the dipole matrix $d = d_{12} = d_{21}$ element between the first and the second energy level. There should be no unevaluated integrals in your answer.

b) Write the Hamiltonian $\hat{H}$ of the two-level system in terms of the Pauli spin matrices.

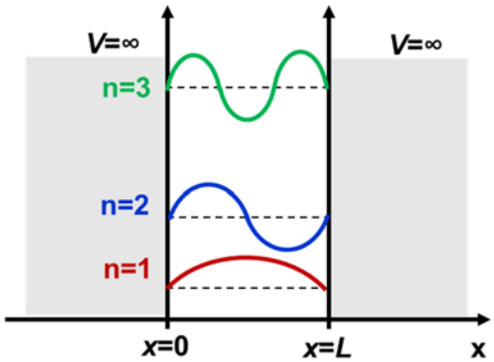
c) Assuming that the frequency of the electric field is tuned such that $\hbar\omega = E_2 - E_1$, and that at time *t*=0 the electron was sitting in the lower energy states (i.e. $\left|\psi(t=0)\right\rangle = \left|e_1\right\rangle$), find the state of the electron at a later time *t*, i.e. find $\left|\psi(t)\right\rangle$.

d) **More difficult:** Assuming that the frequency of the electric field is tuned such that $\hbar\omega = E_2 - E_1 + \hbar\delta\omega$, and that at time *t*=0 the electron was sitting in the lower energy states (i.e. $\left|\psi(t=0)\right\rangle = \left|e_1\right\rangle$), find the state of the electron at a later time *t*, i.e. find $\left|\psi(t)\right\rangle$. Note that $\hbar\delta\omega$

represents the detuning of the electric field frequency from the energy separation of the first two energy levels of the potential well.

Problem 8.2: (Infinite Potential Well: A Time-Independent Perturbation Problem)

Consider the following infinite potential well again:



The Hamiltonian is,

$$\hat{H}_o = \sum_j E_j |e_j\rangle\langle e_j|$$

An electron in the potential well is subjected to a time-independent DC electric field in the x-direction so that the Hamiltonian gets an extra term,

$$\hat{H} = \hat{H}_o - qE_x\left(\hat{x} - \frac{L}{2}\right)$$

a) Using the time independent perturbation theory, find the first order changes in the energies of the first two energy levels (i.e. find $\Delta E_1^{(1)}$ and $\Delta E_2^{(1)}$).

b) Using the time independent perturbation theory, find the second order changes in the energies of the first two energy levels (i.e. find $\Delta E_1^{(2)}$ and $\Delta E_2^{(2)}$). As a simplification, you may ignore matrix element of the perturbation between any one of the first two energy levels and any one of the higher energy levels.

c) Does the energy of the **first** eigenstate decrease or increase when a DC electric field is applied? Does the energy of the **second** eigenstate decrease or increase when a DC electric field is applied? These are the **Start shifts** that you just calculated.

d) Using the time independent perturbation theory, find the first order changes in the first two eigenstates, i.e. suppose one writes,

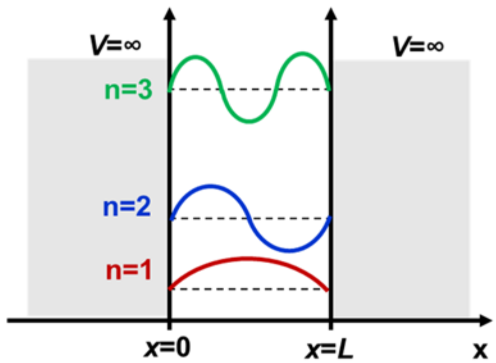
$$\left|e_1^{new}\right\rangle \approx |e_1\rangle + \sum_{m\neq 1} c_m^{(1)} |e_m\rangle$$

$$\left|e_2^{new}\right\rangle \approx |e_2\rangle + \sum_{m\neq 2} b_m^{(1)} |e_m\rangle$$

You need to find the coefficients, $c_m^{(1)}$ and $b_m^{(1)}$. As a simplification, you may ignore matrix element of the perturbation between any one of the first two energy levels and any one of the higher energy levels.

Problem 8.3: (Infinite Potential Well: A Finite Basis Expansion Problem)

Consider the following infinite potential well yet again:



The Hamiltonian is,

$$\hat{H}_o = \sum_j E_j \left| e_j \right\rangle \left\langle e_j \right|$$

An electron in the potential well is subjected to a time-independent DC electric field in the x-direction so that the Hamiltonian gets an extra term,

$$\hat{H} = \hat{H}_o - qE_x \left(\hat{x} - \frac{L}{2} \right)$$

We now truncate the Hilbert space and assume that the Hilbert space consists of only the first two states, and write the Hamiltonian as,

$$\hat{H} = \sum_{j=1,2} E_j \left| e_j \right\rangle \left\langle e_j \right| - qE_x \left(\hat{x} - \frac{L}{2} \right) \qquad \left\{ \sum_{j=1,2} \left| e_j \right\rangle \left\langle e_j \right| = \hat{1} \right.$$

a) Using the dipole matrix element d, write the Hamiltonian as a 2x2 matrix,

$$\hat{H} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

Now the Hamiltonian in part (a) has off-diagonal elements. In order to find the eigenstates and eigenvalues of the Hamiltonian, we need to diagonalize the Hamiltonian. This diagonalization is the same as a “change of basis” or a “rotation of basis”. In a linear algebra class, this would amount to finding the eigenvalues and eigenvectors of a 2x2 matrix.

b) Find the eigenenergies and eigenvectors of the Hamiltonian matrix in part (a).

c) Compare your answers in part (b) above to your answers in 8.2 (a) and 8.2 (b). Do they agree to the first and second power of the applied electric field strength?