Department of Electrical and Computer Engineering, Cornell University

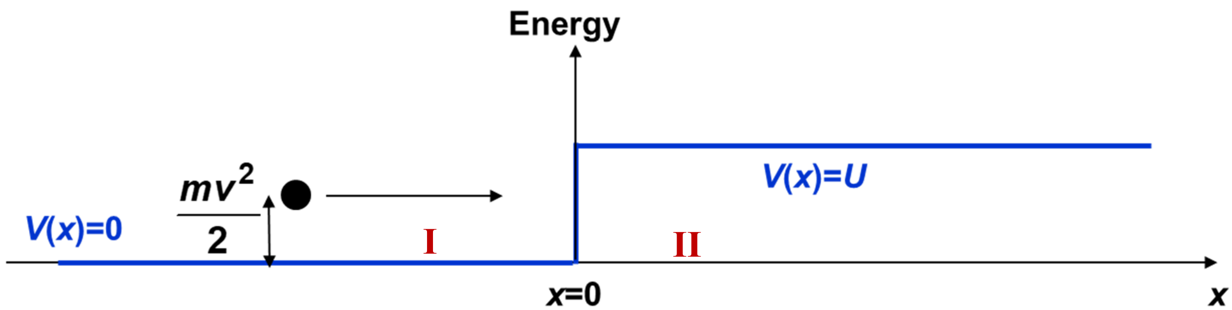
ECE 4060: Quantum Physics and Engineering

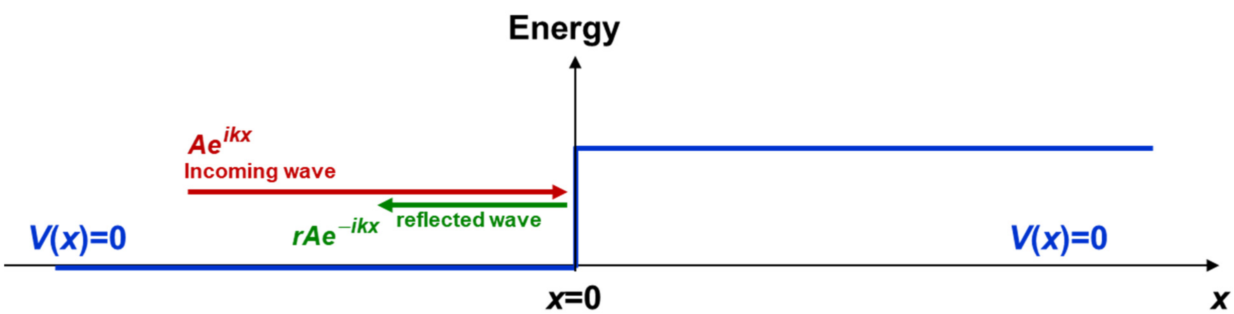
Fall 2020

Homework 2 **Due on Sep. 23, 2020 by 5:00 PM (via email)**

Problem 2.1: (A potential step)

Consider the following potential step.





We consider the situation where the kinetic energy of the incident particle is smaller than the height of the potential step (as shown above). Suppose the energy E of the incident particle satisfies,

$$E = \frac{1}{2}mv^2 = \frac{\hbar^2 k^2}{2m} < U$$

In region I, the solution would be a superposition of an incident wave and a reflected wave, as shown in the figure above. The appropriate solution for $\phi(x)$ in region II is,

$$\phi(x) = Be^{-\gamma x} + Ce^{+\gamma x}$$

a) Find the value of γ in terms of the incident particle energy E and the potential step height U.

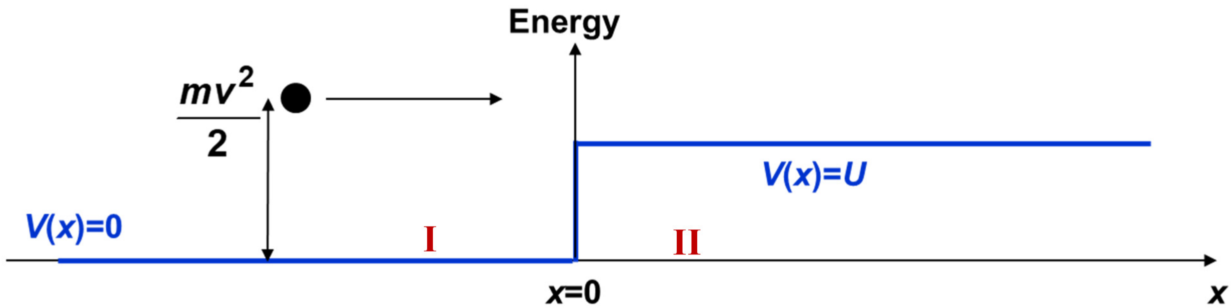
b) Argue on physical grounds that the coefficient C should be zero.

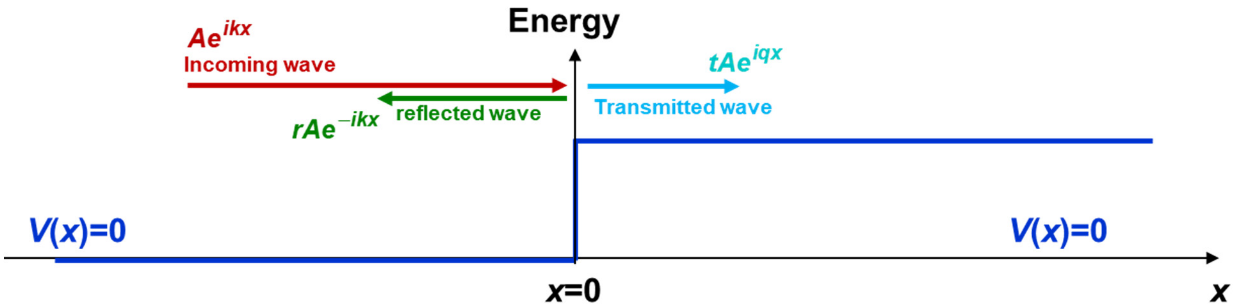
c) Using all the boundary conditions at your disposal, find the reflection coefficient r as a function of the given parameters.

d) Find the reflection probability $|r|^2$.

Problem 2.2: (A potential step again)

Consider the following potential step.





We now consider the situation where the kinetic energy of the incident particle is larger than the height of the potential step (as shown above). Suppose the energy E of the incident particle satisfies

$$E = \frac{1}{2}mv^2 = \frac{\hbar^2 k^2}{2m} > U$$

In region I, as before, the solution would be a superposition of an incident wave and a reflected wave, as shown in the figure above. The appropriate solution for $\phi(x)$ in region II is a transmitted wave,

$$\phi(x) = tAe^{iqx}$$

a) Find the value of q in terms of the incident particle energy E and the potential step height U.

b) Using all the boundary conditions at your disposal, find the reflection coefficient r and the transmission coefficient t as a function of the given parameters.

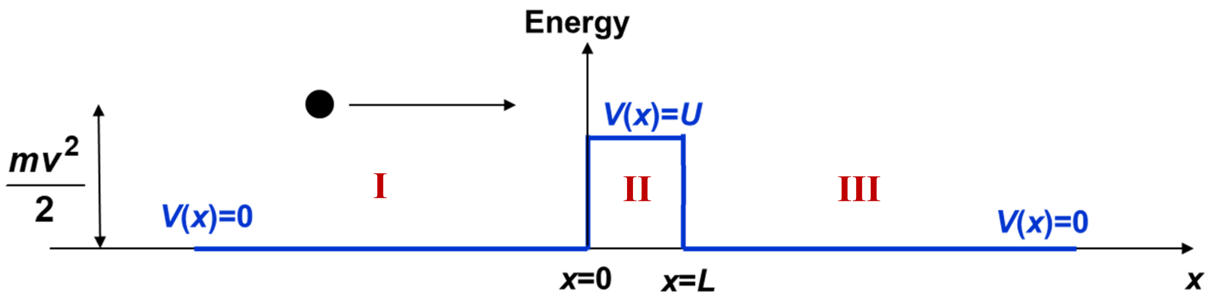
c) Do you think that the relation,

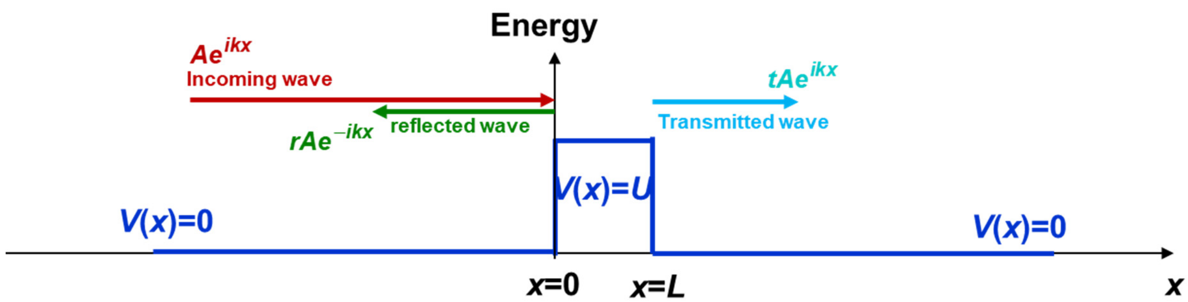
$$1 + |r|^2 = |t|^2$$

will hold true in this case? Be careful. Think conservation of probability currents.

Problem 2.3: (A potential barrier)

In the lectures we considered the behavior of a particle incident on a potential barrier and the kinetic energy of the particle was less than the height of the potential barrier. Here we consider the situation where the kinetic energy of the incident particle is larger than the height of the potential barrier (as shown below). According to classical physics, the particle would just cross over to the with unit probability. But quantum physics, as you will see, will have different things to say.





In regions I and III, the solutions would look the same as in the lecture handouts for the case when the kinetic energy of the incident particle was less than U. Suppose that the energy of the incident particle is $E = \frac{1}{2}mv^2 = \frac{\hbar^2 k^2}{2m} > U$.

a) The appropriate solution for $\phi(x)$ in region II is,

$$\phi(x) = Be^{iqx} + Ce^{-iqx}$$

Find the magnitude of the wavevector q in terms of the incident particle energy E and the potential barrier height U.

b) Using all the boundary conditions at your disposal, find the reflection coefficient r and the transmission coefficient t as a function of the given parameters.

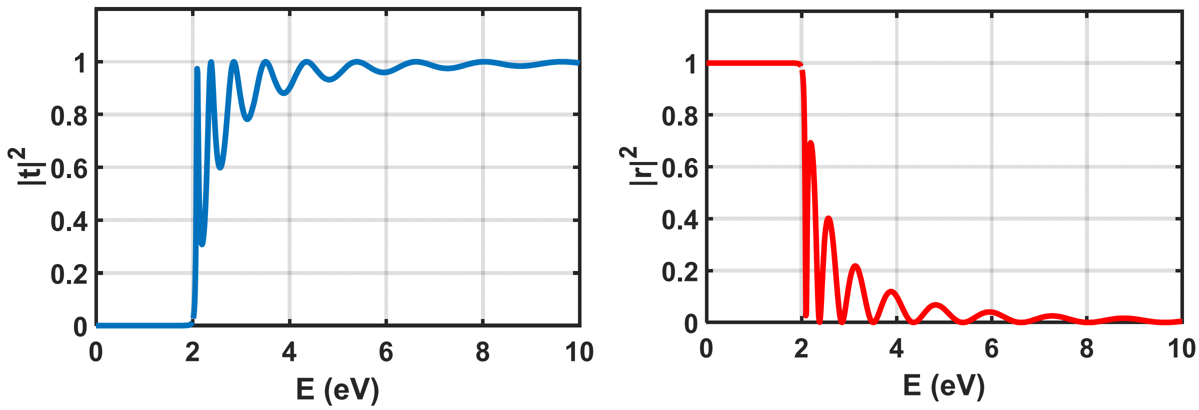
c) In this part you will need to use some plotting software (like matlab or maple). Assume the following values:

$$L = 20 \text{ Angstroms}$$

$$m = 9.1\times 10^{-31} \text{ kg}$$

$$U = 2.0 \text{ eV}$$

Plot (not sketch) the probability of transmission $|t|^2$ and the probability of reflection $|r|^2$ as a function of the electron energy in the range from 2 eV to 10 eV. Your plots should look as follows:



d) Why do you think the probability of transmission $|t|^2$ and the probability of reflection $|r|^2$ are not simply 1 and 0, respectively, for energies E larger than U, as is the case in classical physics?

e) In the transmission probability plot you will see some sharp resonances as a function of the electron energy. Can you figure out why these resonances are there and can you figure out an analytical expression for the energies at which these resonances appear?

Problem 2.4: (Linear algebra)

a) Consider the following matrix:

$$\hat{A} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

in some basis defined by vectors $|v_1\rangle$ and $|v_2\rangle$. Now we define a new basis set given by the vectors $|u_1\rangle$ and $|u_2\rangle$ where, $|u_1\rangle = (|v_1\rangle + i|v_2\rangle)/\sqrt{2}$ and $|u_2\rangle = (|v_1\rangle - i|v_2\rangle)/\sqrt{2}$. Find the 2x2 matrix $\hat{A}$ in the new basis.