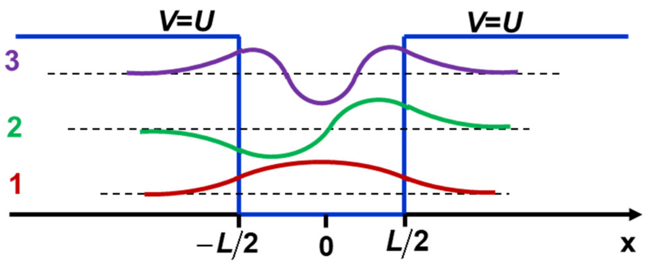
ECE 4060: Quantum Physics and Engineering

Fall 2020

Homework 11 **Due on Dec. 16, 2020 by 5:00 PM (via email)**

Problem 11.1: (Identical electrons in a potential well)

Consider a finite potential well:



The bound quantum energy eigenstates of the well are $|\phi_j\rangle$ where $j = 1,2,3$.

$$\hat{H}|\phi_j\rangle = E_j|\phi_j\rangle$$

Here, $\hat{H}$ is the Hamiltonian,

$$\hat{H} = \frac{\hat{p}^2}{2m} + V(\hat{x})$$

Now if we have two particles, A and B, in the well then the Hamiltonian for both can be written as,

$$\hat{H} = \hat{H}_A + \hat{H}_B = \left[\frac{\hat{p}_A^2}{2m} + V(\hat{x}_A)\right] + \left[\frac{\hat{p}_B^2}{2m} + V(\hat{x}_B)\right]$$

a) Suppose we have two electrons, A and B, and we can choose their spins to be either up or down as desired. Write the quantum state $|\psi\rangle$ corresponding to the lowest energy state that can be formed using these two electrons. Your state must obey the spin-statistics theorem for electrons.

b) Again suppose we have two electrons, A and B, and their quantum state is given as,

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left[|\phi_1\rangle_A \otimes |z\uparrow\rangle_A \otimes |\phi_2\rangle_B \otimes |z\downarrow\rangle_B - |\phi_2\rangle_A \otimes |z\downarrow\rangle_A \otimes |\phi_1\rangle_B \otimes |z\uparrow\rangle_B\right]$$

If a measurement is made to locate the two particles, what is the a-priori probability of finding the two particles at the same location?

c) Again suppose we have two electrons, A and B, and their quantum state is given as,

$$|\psi\rangle = \frac{1}{\sqrt{2}}\Big[|\phi_1\rangle_A \otimes |z\uparrow\rangle_A \otimes |\phi_2\rangle_B \otimes |z\uparrow\rangle_B - |\phi_2\rangle_A \otimes |z\uparrow\rangle_A \otimes |\phi_1\rangle_B \otimes |z\uparrow\rangle_B\Big]$$

If a measurement is made to locate the two particles, what is the a-priori probability of finding the two particles at the same location?

d) Again suppose we have two electrons, A and B, and their quantum state is given as,

$$|\psi\rangle = \frac{1}{\sqrt{2}}\Big[|\phi_1\rangle_A \otimes |z\uparrow\rangle_A \otimes |\phi_2\rangle_B \otimes |z\uparrow\rangle_B - |\phi_2\rangle_A \otimes |z\uparrow\rangle_A \otimes |\phi_1\rangle_B \otimes |z\uparrow\rangle_B\Big]$$

If a measurement is made to measure the spin angular momentum of both the particles along the x-axis, what is the a-priori probability of finding both the particles with spin in the positive x-direction?

e) Find the mean value of the energy of the two particles, i.e. find $\langle\psi|\hat{H}|\psi\rangle$, if the quantum state is:

$$|\psi\rangle = \frac{1}{\sqrt{2}}\Big[|\phi_1\rangle_A \otimes |z\uparrow\rangle_A \otimes |\phi_2\rangle_B \otimes |z\downarrow\rangle_B - |\phi_2\rangle_A \otimes |z\downarrow\rangle_A \otimes |\phi_1\rangle_B \otimes |z\uparrow\rangle_B\Big]$$

f) Now suppose a magnetic field has been applied from outside in the z-direction and the total Hamiltonian $\hat{H}_{total}$ in the presence of the magnetic field, is,

$$\hat{H}_{total} = \hat{H}_A + \hat{H}_B + \frac{e}{m}\Big[\hat{\vec{S}}_A + \hat{\vec{S}}_B\Big].\vec{B}$$

$$= \hat{H}_A + \hat{H}_B + \frac{e}{m}\Big[\hat{S}_z^A + \hat{S}_z^B\Big]B_z$$

Find the mean value of the total energy, i.e. find $\langle\psi|\hat{H}_{total}|\psi\rangle$, if the quantum state is:

$$|\psi\rangle = \frac{1}{\sqrt{2}}\Big[|\phi_1\rangle_A \otimes |x\uparrow\rangle_A \otimes |\phi_2\rangle_B \otimes |x\downarrow\rangle_B - |\phi_2\rangle_A \otimes |x\downarrow\rangle_A \otimes |\phi_1\rangle_B \otimes |x\uparrow\rangle_B\Big]$$

Note: make sure you understand how operators act in their own respective Hilbert space.