Lecture on
MOS (Metal Oxide Semiconductor) Structure

In this lecture you will learn:

• The fundamental set of equations governing the behavior of NMOS structure

• Accumulation, Flatband, Depletion, and Inversion Regimes

• Large signal and small signal models of the NMOS capacitor
A N-MOS (or NMOS) Capacitor Structure

N+ Si Gate or Metal Gate

SiO₂

P-Si Substrate (or Bulk)

Doping: \( N_a \)

Gate metal contact

Metal contact

A NMOS Capacitor in 3D

N+ Si Gate or Metal Gate

SiO₂

P-Si substrate (or bulk)

Doping: \( N_a \)
A NMOS Capacitor

Assumptions:

1) The potential in the metal gate is \( \phi_M \)
   If the gate is N+ Si then \( \phi_M = \phi_n \)

2) The potential deep in the P-Si substrate is \( \phi_p \)

3) The oxide (SiO₂) is insulating (zero conductivity; no free electrons and holes) and is completely free of any charges

4) There cannot be any volume charge density inside the metal gate (it is very conductive). But there can be a surface charge density on the surface of the metal gate

5) Dielectric constants:
   \[ \varepsilon_{\text{ox}} = 3.9 \varepsilon_0 \quad \varepsilon_s = 11.7 \varepsilon_0 \]

A NMOS Capacitor in Equilibrium

Potential Plot:
\[ \phi(x) \]

We need to find the potential in equilibrium everywhere
A NMOS Capacitor in Equilibrium: Depletion Region

Step 1: Charges Flow

- $V_{GB} = 0$

Step 2: Depletion region is created in the substrate near the oxide interface, and a surface or sheet charge density is created on the metal gate

Positive surface charge density (C/cm$^2$)

$Q_G = +qN_a \rho_{do}$

Negative depletion charge density (C/cm$^3$)

$\rho = -qN_a$

A NMOS Capacitor in Equilibrium: Charge Densities

Charge density plot:

$Q_G = +qN_a \rho_{do}$

Total charge per unit area in the semiconductor (C/cm$^2$)

$Q_B = -qN_a \rho_{do}$
A NMOS Capacitor in Equilibrium: Electric Field

**Electric field in the semiconductor:**

\[
\frac{dE_x}{dx} = \frac{\rho}{\varepsilon_s} = -\frac{qN_d}{\varepsilon_s}
\]

Boundary condition:

\[
E_x(x = x_{do}) = 0
\]

\[E_x(x) = \frac{qN_d}{\varepsilon_s} (x_{do} - x)\]

- Linearly varying

Some Electrostatics

Consider an interface between media of different dielectric constants:

\[\varepsilon_1 \quad \vec{E}_1 \quad \varepsilon_2 \quad \vec{E}_2\]

Suppose you know \(\vec{E}_1\), can you find \(\vec{E}_2\) ???

**Use the principle:** The product of the dielectric constant and the normal component of the electric field on both sides of an interface are related as follows:

\[\varepsilon_2 \vec{E}_2 - \varepsilon_1 \vec{E}_1 = Q_l = \text{Interface sheet charge density (C/cm}^2)\]

- Note that \(\vec{E}_1\) is the electric field JUST to the left of the interface and \(\vec{E}_2\) is the electric field JUST to right of the interface
A NMOS Capacitor in Equilibrium: Electric Field

Electric field in the oxide:
\[
\frac{dE_x}{dx} = \frac{\rho}{\varepsilon_{ox}} = 0
\]
\(\Rightarrow E_x(x) = \text{constant} \)
\(\Rightarrow E_x(x) = \frac{qN_a x_{do}}{\varepsilon_{ox}} \)

Boundary condition:
\[
E(x = 0^+) = \frac{qN_a x_{do}}{\varepsilon_s}
\]
\[
E(x = 0^-) = \frac{qN_a x_{do}}{\varepsilon_{ox}}
\]

A NMOS Capacitor in Equilibrium: Potential

Potential in the semiconductor:
\[
\frac{d\phi(x)}{dx} = -E_x(x) = -\frac{qN_a}{\varepsilon_s} (x_{do} - x)
\]
\[
\phi(x) = \phi_p + \frac{qN_a}{2\varepsilon_s} (x_{do} - x)^2
\]

Boundary condition:
\[
\phi(x = x_{do}) = \phi_p
\]
A NMOS Capacitor in Equilibrium: Potential

\[ \frac{d\phi(x)}{dx} = -E_x(x) = -\frac{qN_a x_d}{\varepsilon_{ox}} \]

Boundary condition:
\[ \phi(x = 0) = \phi_p + \frac{qN_a x_d^2}{2\varepsilon_s} \]

Potential in the oxide:
\[ \phi(x) = \phi_p + \frac{qN_a x_d^2}{2\varepsilon_s} - \frac{qN_a x_d}{\varepsilon_{ox}} x \]

Must have:
\[ \phi(x = -t_{ox}) = \phi_p + \frac{qN_a x_d^2}{2\varepsilon_s} + \frac{qN_a x_d}{\varepsilon_{ox}} t_{ox} = \phi_M \]

Therefore:
\[ x_d = -\frac{\varepsilon_s}{C_{ox}} + \left( \frac{\varepsilon_s}{C_{ox}} \right)^2 + \frac{2\varepsilon_s}{qN_a} \phi_B \]

Oxide capacitance (per unit area):
\[ C_{ox} = \frac{\varepsilon_{ox}}{t_{ox}} \]
### A NMOS Capacitor in Equilibrium: Potential

- **Potential drop in the oxide**
- **Potential drop in the semiconductor**

\[ \phi_B = \frac{qN_a x_{do}}{\epsilon_{ox}} + \frac{qN_s x_{do}^2}{2 \epsilon_s} + \frac{qN_s x_{do}^2}{2 \epsilon_s} \]

\[ \phi_B = V_{OX} + V_S \]

- **Oxide capacitance (per unit area)**

\[ C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} \]

### A Biased NMOS Capacitor: \( V_{GB} > 0 \)

- **All of the applied bias falls across the depletion region and the oxide**

\[ \phi_B + V_{GB} = \phi_B + V_{GB} = V_{OX} + V_S \]

\[ \phi_B + V_{GB} = \frac{qN_a x_d}{C_{ox}} + \frac{qN_s x_d^2}{2 \epsilon_s} \]

- **The depletion region widens and the oxide field increases when \( V_{GB} \) is positive**
A Biased NMOS Capacitor: $V_{GB} < 0$

All of the applied bias falls across the depletion region and the oxide.

The depletion region shortens and the oxide field decreases when $V_{GB}$ is negative.

Potential drop in the oxide

Potential drop in the semiconductor

The depletion region shrinks and the oxide field also decreases for $V_{GB} < 0$.

$$x_d = \frac{\epsilon_s}{C_{ox}} + \left( \frac{\epsilon_s}{C_{ox}} \right)^2 \left( \frac{2\epsilon_s}{qN_a} \right) (\phi_B + V_{GB})$$

$$E_{ox} = \frac{qN_a x_d}{\epsilon_{ox}}$$
**A Biased NMOS Capacitor: Flatband Condition**

When $V_{gb}$ is sufficiently negative, the depletion region thickness shrinks to zero. This value of $V_{gb}$ is called the flatband voltage $V_{fb}$.

The potential in the flatband condition can be described by:

$$x_d = \epsilon_s \frac{1}{C_{ox}} + \left( \epsilon_s \frac{1}{C_{ox}} \right)^2 + \left( \frac{2\epsilon_s}{qN_A} \right) (\Phi_B + V_{FB}) = 0$$

$$\Rightarrow V_{FB} = -\Phi_B = -\left( \Phi_M - \Phi_p \right)$$

**A Biased NMOS Capacitor: Accumulation ($V_{gb} < V_{FB}$)**

The entire potential drop for $V_{gb} < V_{FB}$ falls across the oxide:

$$\Phi_B + V_{GB} = V_{OX}$$

$$\Rightarrow V_{GB} - V_{FB} = V_{OX}$$

The semiconductor accumulation charge can be expressed as:

$$Q_p = -C_{ox} (V_{GB} - V_{FB})$$
A Biased NMOS Capacitor: Accumulation ($V_{GB} < V_{FB}$)

Potential:
- Charge accumulation (due to holes) on the semiconductor surface

Charge Density:
- Total charge per unit area in the hole accumulation layer
  \[ Q_P = -C_{ox}(V_{GB} - V_{FB}) \]
- Charge accumulation (due to holes) on the semiconductor surface
  \[ Q_G = C_{ox}(V_{GB} - V_{FB}) \]

A Biased NMOS Capacitor: Charges

- Depletion Region Charge ($C/cm^2$)
  \[ Q_B = -qN_a x_d \]
  \[ x_d = -\frac{\varepsilon_S}{C_{ox}} + \frac{\varepsilon_S^2}{C_{ox}^2} \left( \frac{2\varepsilon_S}{qN_a} \phi_B + V_{GB} \right) \]

- Accumulation Layer Charge ($C/cm^2$)
  \[ Q_P = -C_{ox}(V_{GB} - V_{FB}) \]
A Biased NMOS Capacitor: $V_{GB} > 0$

All of the applied bias falls across the depletion region and the oxide.

Potential drop in the oxide

Potential drop in the semiconductor

The depletion region widens and the oxide field increases with $V_{GB}$ for $V_{GB} > V_{FB}$

$\phi_B + V_{GB} = \phi_{OX} + \phi_S$

$= \frac{qN_A x_d}{C_{OX}} + \frac{qN_A x_d^2}{2\varepsilon_s}$

$\phi_B + V_{GB} = V_{GB} - V_{FB}$

$V_{GB} - V_{FB} = \phi_S - \phi_p + \frac{qN_A (\phi_s - \phi_p)}{2\varepsilon_s}$

$\Rightarrow V_{GB} - V_{FB} = \phi_s - \phi_p + \frac{qN_A x_d^2}{2\varepsilon_s}$

A Biased NMOS Capacitor: Depletion ($V_{GB} > V_{FB}$)

Surface potential $\phi_s$

Potential drop in the semiconductor

Potential drop in the oxide
A Biased NMOS Capacitor: Surface Potential

\[ \phi_s = \phi_p \quad V_{GB} \leq V_{FB} \]

\[ V_{GB} - V_{FB} = \phi_s - \phi_p + \frac{2 \varepsilon_s q N_s (\phi_s - \phi_p)}{C_{ox}} \quad V_{FB} \leq V_{GB} \]

A Biased NMOS Capacitor

\[ V_{GB} - V_{FB} = V_{OX} + V_s \]

\[ V_{GB} - V_{FB} = \frac{q N_s x_d}{C_{ox}} + \frac{q N_s x_d^2}{2 \varepsilon_s} + V_{OX} + V_s \]

\[ V_{GB} - V_{FB} = \phi_s - \phi_p + \frac{2 \varepsilon_s q N_s (\phi_s - \phi_p)}{C_{ox}} \]

Same equation written in 3 different ways valid for:

\[ V_{FB} < V_{GB} < V_{TN} \]
A Biased NMOS Capacitor: Electron Density ($V_{GB} > V_{FB}$)

- As $V_{GB}$ is increased, $\phi_S$ also increases.
- The electron density in the semiconductor depends on the potential as:

$$n(x) = n_e e^{-\frac{q\phi(x)}{kT}} = n_e e^{-\frac{q\phi_0}{kT}} e^{-\frac{q\phi(x) + \phi_p}{kT}} \approx N_a e^{-\frac{q\phi(x) + \phi_p}{kT}}$$

Electron density is the largest right at the surface of the semiconductor where the potential is the highest:

$$n(x = 0) \approx N_a e^{-\frac{q\phi_0 + \phi_p}{kT}}$$

A Biased NMOS Capacitor: Threshold Condition

- When $V_{GB}$ is increased and the surface potential $\phi_S$ reaches $-\phi_P$, the electron density at the surface becomes comparable to the hole density in the substrate and cannot be ignored.
- The gate voltage $V_{GB}$ at which $\phi_S$ equals $-\phi_P$ is called the threshold voltage $V_{TN}$:

$$V_{TN} - V_{FB} = -2\phi_P + \frac{\sqrt{2\epsilon_s qN_a (-2\phi_P)}}{C_{ox}}$$
A Biased NMOS Capacitor: Inversion ($V_{GB} > V_{TN}$)

- When the gate voltage $V_{GB}$ is increased above $V_{TN}$ the electron density right at the surface increases (exponentially with the surface potential $\phi_S$)

- This surface electron density is called the inversion layer (assumed to be of zero thickness in this course)

\[
Q_G = +qN_a x_d - Q_N
\]

\[
Q_N = \text{Inversion layer charge density (C/cm}^2\text{)}
\]

\[
\phi(x = 0) = \phi_S = -\phi_p
\]

- When the gate voltage $V_{GB}$ is increased above $V_{TN}$ the inversion layer charge increases so rapidly that the extra applied potential drops entirely across the oxide, and the surface potential $\phi_S$ remains close to $-\phi_p$

- Consequently, the depletion region thickness (and the depletion region charge) does not increase when the gate voltage $V_{GB}$ is increased above $V_{TN}$

\[
\phi_S - \phi_p = \frac{qN_a x_d^2}{2 \varepsilon_S} \Rightarrow -2\phi_p = \frac{qN_a x_{d_{max}}^2}{2 \varepsilon_S}
\]

\[
\Rightarrow \quad V_{TN} = V_{FB} + \frac{qN_a x_{d_{max}}^2}{C_{ox}} + \frac{qN_a x_{d_{max}}^2}{2 \varepsilon_S}
\]
A Biased NMOS Capacitor: Surface Potential

\[
\phi_s = +\phi_p \quad \text{for} \quad V_{GB} \leq V_{FB}
\]
\[
V_{GB} - V_{FB} = \phi_s - \phi_p + \frac{2 \varepsilon_s q N_s (\phi_s - \phi_p)}{C_{ox}} \quad \text{for} \quad V_{FB} \leq V_{GB} \leq V_{TN}
\]
\[
\phi_s = -\phi_p \quad \text{for} \quad V_{GB} \geq V_{TN}
\]

A Biased NMOS Capacitor: Inversion \((V_{GB} > V_{TN})\)

How to calculate the inversion layer charge \(Q_N\) when \(V_{GB} > V_{TN}\)?

Start from:

\[
V_{GB} - V_{FB} = V_{ox} + V_S
\]

\[
= E_{ox} t_{ox} + \frac{q N_s x_{d, \text{max}}^2}{2 \varepsilon_s} \quad \text{for} \quad V_{S} = \frac{q N_a x_{d, \text{max}}^2}{2 \varepsilon_s}
\]

By Gauss' law:

\[-\varepsilon_{ox} E_{ox} = Q_N - q N_s x_{d, \text{max}}\]

Therefore:

\[
V_{GB} - V_{FB} = \frac{Q_N}{C_{ox}} + \frac{q N_s x_{d, \text{max}}}{2 \varepsilon_s} \quad \text{for} \quad \frac{Q_N}{C_{ox}} + \frac{q N_s x_{d, \text{max}}}{2 \varepsilon_s}
\]

\[
\Rightarrow V_{GB} = -\frac{Q_N}{C_{ox}} + V_{TN}
\]

\[
\Rightarrow Q_N = -C_{ox} (V_{GB} - V_{TN})
\]

Inversion layer charge increases linearly with the gate voltage above threshold.
A Biased NMOS Capacitor: Charges

Depletion Region Charge (C/cm²)

\[ Q_B = -qN_a x_d \quad \text{and} \quad Q_B = -qN_a x_{d,\text{max}} \]

Inversion Layer Charge (C/cm²)

\[ Q_N = -C_{ox}(V_{GB} - V_{TN}) \]

Accumulation Layer Charge (C/cm²)

\[ Q_P = -C_{ox}(V_{GB} - V_{FB}) \]

A N-MOS (or NMOS) Capacitor Structure

Gate metal contact

N+ Si Gate or Metal Gate

SiO₂

P-Si Substrate (or Bulk) Doping: \( N_a \)

Metal contact

\[ x = 0 \]
A Biased NMOS Capacitor: Charges

Gate Charge ($C/$cm$^2$) (Must be equal and opposite to the total semiconductor charge)

Gate Charge ($Q_G$)

$V_{FB}$

$V_{TN}$

$V_{GB}$

Capacitance of a NMOS Capacitor:

$C = \frac{dQ_G}{dV_{GB}}$

Accumulation

Depletion

Inversion

The Small Signal Capacitance of a NMOS Capacitor

- The small signal capacitance (per unit area) of the MOS capacitor is defined as:

$$C = \frac{dQ_G}{dV_{GB}}$$

where $Q_G$ is the charge density (units: C/cm$^2$) on the gate

(1) Accumulation ($V_{GB} < V_{FB}$):

$Q_G = C_{ox} (V_{GB} - V_{FB})$

$\Rightarrow C = C_{ox}$
The Small Signal Capacitance of a NMOS Capacitor

(2) Depletion \((V_{TN} > V_{GB} > V_{FB})\):

\[
Q_G = qN_a x_d \\
C = \frac{dQ_G}{dV_{GB}} = qN_a \frac{dx_d}{dV_{GB}}
\]

Differentiate the equation (derived earlier):

\[
\frac{qN_a x_d^2}{2 \varepsilon_s} + \frac{qN_a x_d}{\varepsilon_s} = V_{GB} - V_{FB}
\]

To get:

\[
\left[ x_d + \frac{1}{\varepsilon_s} \right] \frac{qN_a}{C_{ox}} \ dx_d = dV_{GB}
\]

Define:

\[
C_b = \frac{\varepsilon_s}{x_d}
\]

Finally:

\[
\frac{1}{C} = \frac{1}{C_{ox}} + \frac{1}{C_b}
\]

The Small Signal Capacitance of a NMOS Capacitor

(3) Inversion \((V_{GB} > V_{TN})\):

\[
Q_G = qN_a x_{d_{max}} - Q_N \\
C = \frac{dQ_G}{dV_{GB}} = -\frac{dQ_N}{dV_{GB}} = C_{ox}
\]

\[
Q_N = -C_{ox} (V_{GB} - V_{TN})
\]

\(x_{d_{max}}\) does not change with \(V_{GB}\) above threshold
The Small Signal Capacitance of a NMOS Capacitor

A NMOS Capacitor with a Channel Contact

- In the presence of an inversion layer, the additional contacts allow one to directly change the potential of the inversion layer channel w.r.t. to the bulk (substrate)
A Biased NMOS Capacitor: Inversion with \( V_{CB} \neq 0 \)

- We had said that the surface potential \( \phi_S \) remains fixed at \(-\phi_p\) when \( V_GB \) is increased beyond \( V_{TN} \).
- But with a non-zero \( V_{CB} \), the surface potential \( \phi_S \) in inversion can be changed to \((-\phi_p + V_{CB})\).
- The new value of the depletion region width is:

\[
\phi_S - \phi_p = \frac{qN_s x_d^2}{2\varepsilon_s} \Rightarrow -2\phi_p + V_{CB} = \frac{qN_s x_{d_{max}}^2}{2\varepsilon_s}
\]

**Question:** How do we now find the inversion layer charge \( Q_N \) when \( V_{CB} \) is not zero?

**A Biased NMOS Capacitor: Surface Potential**

\[
\phi_s = +\phi_p \\
V_{GB} - V_{FB} = \phi_s - \phi_p + \frac{2 \varepsilon_s q N_s (\phi_s - \phi_p)}{C_{ox}} \quad V_{FB} \leq V_{GB} \leq V_{TN} \\
\phi_s = -\phi_p + V_{CB} \quad V_{GB} \geq V_{TN}
\]
A Biased NMOS Capacitor: Inversion with $V_{CB} \neq 0$

How to calculate the inversion layer charge $Q_N$? Same way as before……

Start from: $V_{GB} - V_{FB} = V_{ox} + V_S$

$$= E_{ox} t_{ox} + \frac{qN_s x_{d_{max}}^2}{2\varepsilon_s}$$

$$V_S = \frac{qN_s x_{d_{max}}^2}{2\varepsilon_s}$$

By Gauss’ law: $-\varepsilon_{ox} E_{ox} = Q_N - qN_s x_{d_{max}}$

Therefore: $V_{GB} - V_{FB} = \frac{Q_N}{C_{ox}} + \frac{qN_s x_{d_{max}}}{C_{ox}} + \frac{qN_s x_{d_{max}}^2}{2\varepsilon_s}$

$$\Rightarrow V_{GB} = \frac{Q_N}{C_{ox}} + V_{FB} + \frac{qN_s x_{d_{max}}}{C_{ox}} + \frac{qN_s x_{d_{max}}^2}{2\varepsilon_s}$$

A Biased NMOS Capacitor: Inversion with $V_{CB} \neq 0$

$$V_{GB} = -\frac{Q_N}{C_{ox}} + V_{FB} + \frac{qN_s x_{d_{max}}}{C_{ox}} + \frac{qN_s x_{d_{max}}^2}{2\varepsilon_s}$$

$$V_{TN} = V_{FB} + \frac{qN_s x_{d_{max}}}{C_{ox}} + \frac{qN_s x_{d_{max}}^2}{2\varepsilon_s}$$

$$= V_{FB} - 2\phi_p + V_{CB} + \frac{2\varepsilon_s qN_s [2\phi_p + V_{CB}]}{C_{ox}}$$

$$\Rightarrow Q_N = -C_{ox} (V_{GB} - V_{TN}) \quad \text{Same as before but now } V_{TN} \text{ depends on } V_{CB}$$
NMOS Capacitor: Effect of $V_{CB}$ ($V_{GB} > V_{TN}$)

$V_{CB} > 0$
- Inversion charge decreases
- Depletion region expands

$V_{CB} < 0$
- Inversion charge increases
- Depletion region shrinks