Lecture 7
Large and Small Signal Modelling of PN Junction Diodes

In this lecture you will learn:

- Circuit models of PN junction diodes
- Small signal modeling of nonlinear circuit elements
- Small signal models of PN junction diodes
- Junction resistance and capacitances

Current Flow in a PN Junction Diode

\[ J_T = qn^2 \left( \frac{D_n}{N_d L_n} \coth\left( \frac{W_p}{L_n} \right) + \frac{D_p}{N_d L_p} \coth\left( \frac{W_n}{L_p} \right) \right) \left( \frac{qV_D}{kT} - 1 \right) \]

\[ I_D = A J_T = I_0 \left( \frac{qV_D}{kT} - 1 \right) \]

\[ I_0 = qn^2 A \left( \frac{D_n}{N_d L_n} \coth\left( \frac{W_p}{L_n} \right) + \frac{D_p}{N_d L_p} \coth\left( \frac{W_n}{L_p} \right) \right) \]

Reverse bias
Forward bias
Junction breakdown

\[ V_D \]
\[ V_{on} \]
Simplest Circuit Model for a PN Junction Diode

\[ V = I_D R + V_D \]
\[ \Rightarrow I_D = \frac{V - V_D}{R} \]

Load Line:
\[ V = V_D + I_D R \]

For most diodes:
\[ 0.4 \leq V_{ON} \leq 0.8 \]

Solution for current:

If \( V \leq V_{ON} \):
\[ I_D \approx 0 \]
\[ V_D = V \]

If \( V > V_{ON} \):
\[ I_D = \frac{V - V_{ON}}{R} \]
\[ V_D = V_{ON} \]

Better Circuit Model a PN Junction Diode

\[ \frac{dI_D}{dV_D} = g_d \]
Circuit Example for a PN Junction Diode

Load Line:

\[ V = I_D R + V_D \]
\[ \Rightarrow I_D = \frac{V - V_D}{R} \]

Solution for current:

If \( V \leq V_{ON} \):
\[ V_D = V \]
\[ I_D \approx 0 \]

If \( V > V_{ON} \):
\[ V_D = \frac{V}{1 + g_d R} + V_{ON} \]
\[ I_D = \frac{g_d R}{1 + g_d R} \]

Linear Circuit Elements

Ohm's Law:

\[ I = \frac{V}{R} = GV \]

Ohm's law implies a LINEAR relationship between current and voltage.

The current-voltage relationship of resistors is linear.
Nonlinear Circuit Elements

Nonlinear Element:

Current is a function of the voltage (but the current-voltage relationship is not linear)

For example:

\[ I(V) = A\sqrt{|V|} + Be^{CV} \]

The current-voltage relationship of most devices is not linear!

Small Signal Modeling of Nonlinear Circuit Elements

Bias point

\[ I_{BIAS} = I(V_{BIAS}) \]

\[ I_{BIAS} + i_{ac}(t) = I(V_{BIAS} + v_{ac}(t)) \]
Small Signal Modeling of Nonlinear Circuit Elements

Taylor expand the current-voltage relation around the bias voltage:

\[
I_{\text{BIAS}} + i_{\text{ac}}(t) = I(V_{\text{BIAS}} + v_{\text{ac}}(t))
= I(V_{\text{BIAS}}) + \left. \frac{dl}{dV} \right|_{V=V_{\text{BIAS}}} v_{\text{ac}}(t) + \frac{1}{2} \left. \frac{d^2l}{dV^2} \right|_{V=V_{\text{BIAS}}} v_{\text{ac}}^2(t) + \cdots
\]

\[
I_{\text{BIAS}} + i_{\text{ac}}(t) \approx I(V_{\text{BIAS}}) + g v_{\text{ac}}(t)
\Rightarrow i_{\text{ac}}(t) \approx g v_{\text{ac}}(t)
\]

Complete circuit is:

In small signal models, nonlinear circuit elements are replaced by their linearized models that are valid over a limited range of excursion around the bias point.
Small Signal Modeling of Nonlinear Circuit Elements

A difficult problem:

\[ V_{BIAS} \]

\[ i_{BIAS} + i_{ac}(t) \]

\[ V_{DC} \]

\[ v_{ac}(t) \]

DC Load Line:

\[ V_{DC} = I_{BIAS} R + V_{BIAS} \]

\[ \Rightarrow I_{BIAS} = \frac{V_{DC} - V_{BIAS}}{R} \]

A simpler problem:

\[ \frac{1}{T} = g = \frac{di}{dV} \text{ at } V_{BIAS} \]

Small Signal Model of a PN Junction Diode: Junction Conductance

P-doped

N-doped

\[ I_D = I_o \left( \frac{qV_D}{e^{KT}} - 1 \right) \]

\[ \Rightarrow I_d + i_d = I_o \left( \frac{q(V_D + V_d)}{e^{KT}} - 1 \right) \approx I_D + \frac{\partial I_D}{\partial V_D} V_d + \ldots \ldots = I_D + g_d V_d \]

\[ g_d = \frac{1}{r_d} = \frac{\partial I_D}{\partial V_D} = \frac{qI_o}{KT} e^{KT} = \frac{q(I_D + I_o)}{KT} = \frac{qI_D}{KT} \]

\[ f_o \]

Reverse bias

Forward bias

In strong forward bias

Differential resistance

Differential conductance
Small Signal Model of a PN Junction Diode: Junction Conductance

At high frequencies, part of the current $i_d$ flows through the junction but part of it also charges up the junction capacitance

$$i_d \approx g_d v_d + C_J \frac{dv_d}{dt}$$

$$C_J = \frac{\varepsilon_s A}{(x_p + x_n)}$$
There is also charge stored in the quasi-neutral regions that changes as the junction voltage is varied (negative and positive charge stored at the same location!!)

Charge stored:

\[ Q_d = qA \left[ \int_{x_n}^{x_p} p'(x) dx + qA \int_{-W_p-x_p}^{-x_p} x dx \right] \]

Diffusion Capacitance:

\[ C_d = \frac{q^2 A \sqrt{qV_D}}{KT} \left[ \frac{n_p^2}{N_a} \frac{\cosh \left( \frac{W_p}{L_n} \right)}{\sinh \left( \frac{W_p}{L_n} \right)} + \frac{n_n^2}{N_d} \frac{\cosh \left( \frac{W_n}{L_p} \right)}{\sinh \left( \frac{W_n}{L_p} \right)} \right] \]

Increases exponentially with bias!
At high frequencies, part of the current $i_d$ flows through the junction but part of it also charges up the junction capacitance and the diffusion capacitance.

$$i_d = g_d v_d + (C_j + C_d) \frac{dv_d}{dt}$$

Small Signal Model of a PN Junction Diode: Total Capacitance

Capacitances of a PN Junction Diode

Total Capacitance: $C = C_j + C_d$
Small Signal Model of a PN Junction Diode in Reverse Bias

\[ g_d = \frac{1}{r_d} = \frac{q(I_D + I_o)}{K T} \approx 0 \]

\[ C_d \approx 0 \]

Breadboard Wiring: Good Wiring
Breadboard Wiring: Bad Wiring