Lecture 4
Electrons and Holes in Semiconductors

In this lecture you will learn:

• Generation-recombination in semiconductors in more detail
• The basic set of equations governing the behavior of electrons and holes in semiconductors
• Shockley Equations
• Quasi-neutrality in conductive materials

Majority and Minority Carriers

In N-doped Semiconductors:
Electrons are the majority carriers
Holes are the minority carriers

In P-doped Semiconductors:
Holes are the majority carriers
Electrons are the minority carriers

Golden Rule of Thumb:
When trying to understand semiconductor devices, always first see what the minority carriers are doing
Generation and Recombination in Semiconductors

Generation process:

Rate = \( G \)

Recombination process:

Rate = \( R \)

We can write the following equations for the carrier densities:

\[
\frac{\partial n(t)}{\partial t} = G - R \\
\frac{\partial p(t)}{\partial t} = G - R
\]

These equations tell how the electron and hole densities change in time as a result of recombination and generation processes.

Generation and Recombination in Thermal Equilibrium

• From the first lecture, in thermal equilibrium:
  The recombination rate = \( R_o = k n_o p_o \)
  equals the generation rate = \( G_o \)

  i.e. \( G_o = R_o \)

• Then in thermal equilibrium:

  \[
  \frac{\partial n_o(t)}{\partial t} = G_o - R_o = 0 \\
  \frac{\partial p_o(t)}{\partial t} = G_o - R_o = 0
  \]
Light Absorption in Semiconductors

Generation and Recombination Out of Thermal Equilibrium

1) Consider a P-doped slab of Silicon:

Electron-hole recombination rate in thermal equilibrium = $R_o = k n_o p_o$
equals the generation rate = $G_o = k n_i^2$

2) Now turn light on at time $t = 0$:

- Light breaks the Si-Si covalent bonds and generates excess electron-hole pairs
- The net generation rate now becomes: $G = G_o + G_L$

3) Mathematical model of the above situation:

\[ n = n_o + n'(t) \]
\[ p = p_o + p'(t) \]

- $n'(t)$ and $p'(t)$ are the excess electron and hole densities
- It must be that: $n'(t) = p'(t)$
- We also assume that: $n'(t), p'(t) << p_o$
Generation and Recombination Out of Thermal Equilibrium

- We can use the equations:
  \[
  \frac{\partial n(t)}{\partial t} = G - R \quad \frac{\partial p(t)}{\partial t} = G - R
  \]
  \[
  n = n_o + n'(t) \quad p = p_o + p'(t)
  \]

- Generation rate:
  \[
  G = G_o + G_L
  \]

- Recombination rate:
  \[
  R = k n p \approx k (n_o + n')(p_o + p') = k n_o p_o + k n' p_o
  \]
  \[
  \frac{1}{\tau_n} = k p_o \Rightarrow \tau_n \text{ is the lifetime of the minority carriers (i.e. electrons)}
  \]

- The equation for excess minority carriers (i.e. electrons) becomes:
  \[
  \frac{\partial n(t)}{\partial t} = G - R \quad \frac{\partial n'(t)}{\partial t} = G_L - \frac{n'(t)}{\tau_n}
  \]

- Solution with the boundary condition, \( n'(t = 0) = 0 \), is:
  \[
  n'(t) = G_L \tau_n \left( 1 - e^{-\frac{t}{\tau_n}} \right)
  \]

- Excess hole density is, of course:
  \[
  p'(t) = n'(t)
  \]

- As \( t \to \infty \) the excess electron and hole densities reach a steady state value:
  \[
  n'(t \to \infty) = G_L \tau_n \quad p'(t \to \infty) = G_L \tau_n
  \]

  \[
  n(t \to \infty) = n_o + G_L \tau_n \quad p(t \to \infty) = p_o + G_L \tau_n
  \]
Generation and Recombination Out of Thermal Equilibrium

Now suppose that light had been turned-on for a very very long time and it was turned-off at time $t = 0$

At time $t = 0$:

\[ n' = G_L \tau_n \]  \quad \text{and} \quad \[ n = n_o + G_L \tau_n \]

\[ p' = G_L \tau_n \]  \quad \text{and} \quad \[ p = p_o + G_L \tau_n \]

Since $n \neq n_o$, and $p \neq p_o$, the carrier densities are not equal to their thermal equilibrium values. Thermal equilibrium must get restored since the light has been turned-off.

**Question:** How does thermal equilibrium get restored??

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**Generation and Recombination Out of Thermal Equilibrium**

- We can use the equations:

\[ \frac{\partial n(t)}{\partial t} = G - R \]

\[ \frac{\partial p(t)}{\partial t} = G - R \]

\[ n = n_o + n'(t) \]

\[ p = p_o + p'(t) \]

- **Generation rate:**

\[ G = G_o \]

- **Recombination rate:**

\[ R = k \ n \ p \]

\[ = k \ (n_o + n') \ (p_o + p') \]

\[ \approx k \ (n_o + n') \ p_o \]

\[ = k \ n_o \ p_o + k \ n' \ p_o \]

\[ = R_o + \frac{n'}{\tau_n} \]

\[ \boxed{\text{Assumptions: } n', \ p' \ll \ p_o} \]

The excess recombination rate is proportional to the excess MINORITY carrier density.

- **The equation for excess minority carriers** (i.e. electrons) becomes:

\[ \frac{\partial n(t)}{\partial t} = G - R \]

\[ \frac{\partial n'(t)}{\partial t} = -\frac{n'(t)}{\tau_n} \]
Solution is:

\[
\frac{\partial n'(t)}{\partial t} = \frac{n'(t)}{\tau_n}
\]

\[n'(t) = n'(t = 0) e^{-\frac{t}{\tau_n}} \Rightarrow \text{Excess electron density decays exponentially to zero from its initial value}
\]

The excess carrier densities decay with time and thermal equilibrium values for carrier densities are restored.

The excess hole density will also decay in the same way:

\[p'(t) = p'(t = 0) e^{-\frac{t}{\tau_p}} \]

As \( t \to \infty \), the electron and hole densities reach their equilibrium values:

\[n'(t \to \infty) = 0 \quad \text{and} \quad n(t \to \infty) = n_o\]

\[p'(t \to \infty) = 0 \quad \text{and} \quad p(t \to \infty) = p_o\]

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Whenever you have to find an expression for \( R \) use the following recipe:

- If it is a p-doped semiconductor:

\[R = R_o + \frac{n'(x,t)}{\tau_n}\]

- If it is a n-doped semiconductor:

\[R = R_o + \frac{p'(x,t)}{\tau_p}\]

The excess recombination rate (i.e. \( R - R_o \)) is always proportional to the excess MINORITY carrier density.
Electron and Hole Current Density Equations

From last lecture............

\[ J_n(x) = q n(x) \mu_n E(x) + q D_n \frac{dn(x)}{dx} \]  \hspace{1cm} (1)

\[ J_p(x) = q p(x) \mu_p E(x) - q D_p \frac{dp(x)}{dx} \]  \hspace{1cm} (2)

These are two of Shockley’s equations!

William Shockley
John Bardeen
Walter Brattain

Shockley, Bardeen, and Brattain from Bell Labs were awarded the Nobel Prize for inventing the semiconductor transistor.

Electron and Hole Current Continuity Equations

• You have already seen the equations:

\[ \frac{\partial n(x, t)}{\partial t} = G - R \]
\[ \frac{\partial p(x, t)}{\partial t} = G - R \]

These equations tell how the electron and hole densities change in time as a result of recombination and generation processes.

• Carrier densities can also change in time if the current densities change in space !!!
Electron and Hole Current Continuity Equations

Consider the infinitesimal strip between \( x \) and \( x + \Delta x \)

\[ J_p(x) \quad \Delta x \quad J_p(x + \Delta x) \]

The difference in hole fluxes at \( x \) and \( x + \Delta x \) must result in piling up of holes in the infinitesimal strip ........

\[ J_p(x, t) - J_p(x + \Delta x, t) = q \frac{\partial p(x, t) \Delta x}{\partial t} \quad \text{Note that} \quad q p(x, t) \quad \text{is the hole charge density} \]

\[ \Rightarrow - \frac{\partial J_p(x, t)}{\partial x} = q \frac{\partial p(x, t)}{\partial t} \Rightarrow \frac{\partial p(x, t)}{\partial t} = - \frac{1}{q} \frac{\partial J_p(x, t)}{\partial x} \]

Now add recombination and generation to the above equation:

\[ \frac{\partial p(x, t)}{\partial t} = G - R \left( - \frac{1}{q} \frac{\partial J_p(x, t)}{\partial x} \right) \]

Electron and Hole Current Continuity Equations - III

One can do the same for electrons as well........

\[ J_n(x) \quad \Delta x \quad J_n(x + \Delta x) \]

\[ \frac{\partial n(x, t)}{\partial t} = G + R \left( \frac{1}{q} \frac{\partial J_n(x, t)}{\partial x} \right) \]

So now we have two new equations,

\[
\begin{align*}
\frac{\partial p(x, t)}{\partial t} &= G - R \left( \frac{1}{q} \frac{\partial J_p(x, t)}{\partial x} \right) \quad \text{(3)} \\
\frac{\partial n(x, t)}{\partial t} &= G + R \left( \frac{1}{q} \frac{\partial J_n(x, t)}{\partial x} \right) \quad \text{(4)}
\end{align*}
\]

These are two more of Shockley's equations!
**Gauss's Law and Electrostatics**

The net charge density in a semiconductor is,

\[ \rho(x, t) = q \left[ +N_d(x) - N_s(x) + p(x, t) - n(x, t) \right] \]

**Gauss's Law** in differential form:

\[ \frac{\partial E(x, t)}{\partial x} = \frac{\rho(x, t)}{\varepsilon_s} \]

\[ \frac{\partial E(x, t)}{\partial x} = \frac{q[+N_d(x) - N_s(x) + p(x, t) - n(x, t)]}{\varepsilon_s} \]  

This is the fifth and the last of the Shockley's equations!

**The Five Shockley Equations**

1. \[ J_n(x, t) = q n(x, t) \mu_n E(x, t) + q D_n \frac{d n(x, t)}{dx} \]  
2. \[ J_p(x, t) = q p(x, t) \mu_p E(x, t) - q D_p \frac{d p(x, t)}{dx} \]  
3. \[ \frac{\partial p(x, t)}{\partial t} = G - R - \frac{1}{q} \frac{\partial J_p(x, t)}{\partial x} \]  
4. \[ \frac{\partial n(x, t)}{\partial t} = G + R + \frac{1}{q} \frac{\partial J_n(x, t)}{\partial x} \]  
5. \[ \frac{\partial E(x, t)}{\partial x} = \frac{q[+N_d(x) - N_s(x) + p(x, t) - n(x, t)]}{\varepsilon_s} \]

Using these equations one can understand the behavior of semiconductor microelectronic devices!!

For Silicon: \( \varepsilon_s = 11.7 \varepsilon_o \)
Quasi-Neutrality

Materials with large conductivities are “quasi-neutral”

“Quasi-neutrality” implies that there cannot be large charge densities or electric fields inside a conductive material.

Let’s see why this is true…..and how deviations from quasi-neutrality disappear……..

Consider an infinite and conductive N-doped semiconductor with a net charge density at time \( t = 0 \):

\[
\rho \quad \varepsilon_0
\]

Charge density

The charge density will generate electric fields (by Gauss’ law):

\[
\nabla \cdot E = \frac{\rho}{\varepsilon_0}
\]

The electric field will generate electrical currents:

\[
J = \sigma E
\]

The electrical currents will pile electrons on top of the charge density and neutralize it and then there is no charge density left in the medium............

This whole process takes a time of the order of the dielectric relaxation time \( \tau_d \):

\[
\tau_d = \frac{\varepsilon_0}{\sigma} \sim 10^{-15} - 10^{-13} \text{ Seconds}
\]
Appendix: Restoration of Quasi-Neutrality

From Gauss' law:
\[ \nabla \cdot \vec{E}(\vec{r}, t) = \frac{\rho(\vec{r}, t)}{\varepsilon_0} \]

Current equation:
\[ J(\vec{r}, t) = \sigma \vec{E}(\vec{r}, t) \]

Use the continuity equation for charge:
\[ \frac{\partial \rho(\vec{r}, t)}{\partial t} = -\nabla \cdot \vec{J}(\vec{r}, t) = -\sigma \nabla \cdot \vec{E}(\vec{r}, t) = -\frac{\sigma}{\varepsilon_0} \rho(\vec{r}, t) \]
\[ \Rightarrow \frac{\partial \rho(\vec{r}, t)}{\partial t} + \frac{\rho(\vec{r}, t)}{\tau_d} = 0 \]
\[ \tau_d = \frac{\varepsilon_0}{\sigma} \]

Solution:
\[ \rho(\vec{r}, t) = \rho(\vec{r}, t = 0) e^{-\frac{t}{\tau_d}} \]

Charge density in a conductive medium disappears on a time scale of \( \tau_d \)