In this lecture you will learn:

- Negative Feedback and Stability
- High Frequency Behavior of Amplifier Circuits
- Gain Margin, Phase Margin, and Stability
- Frequency Compensation

Stability problems: The output will be sensitive to the temperature and/or the power supply voltage.

As the temperature or the power supply fluctuates, the output is going to fluctuate.
Negative Feedback and Stability

Open loop system:

\[ v_{in}(\omega) \quad A(\omega) \quad v_{out}(\omega) = A(\omega)v_{in}(\omega) \]

Open gain: \( A(\omega) \)

Stability problems: suppose the open loop gain is sensitive to the temperature or the power supply voltage. As the temperature or the power supply fluctuates, the output is going to fluctuate.

Closed loop system with negative feedback:

\[ v_{in}(\omega) \quad A(\omega) \quad v_{out}(\omega) = \frac{A(\omega)}{1 + KA(\omega)}v_{in}(\omega) \]

Closed loop gain: \( \frac{A(\omega)}{1 + KA(\omega)} \)

Stability problems resolved: Now as the temperature or the power supply fluctuates, the output is going to be much more stable (because it is almost independent of \( A(\omega) \)).

If for small frequencies, \( |A(\omega - 0)| >> 1 \) and \( |KA(\omega - 0)| >> 1 \):

Closed loop gain: \( \frac{A(\omega - 0)}{1 + KA(\omega - 0)} \approx \frac{1}{K} \)

Stability problems resolved: Now as the temperature or the power supply fluctuates, the output is going to be much more stable (because it is almost independent of \( A(\omega) \)).

Negative feedback improves stability at the expense of gain

A positive feedback can lead to instability and/or oscillations!
Differential Amplifiers, Negative Feedback, and Stability

A high-gain differential amplifier is almost always operated using a negative feedback:

\[
v_{out}(\omega) = A(\omega)[v_{+}(\omega) - v_{-}(\omega)]
\]

\[
v_{out}(\omega) = -v_{in}(\omega) \frac{R_2}{R_1 + A(\omega) + \frac{R_2}{R_1}} = -v_{in}(\omega) \frac{R_2}{1 + \frac{R_2}{R_1} 1 + KA(\omega)}
\]

\[
K = \frac{1}{1 + \frac{R_2}{R_1}}
\]

If \(|A(\omega - 0)| >> 1\), then:

\[
v_{out}(\omega) \approx -v_{in}(\omega) \frac{R_2}{R_1}
\]

Negative feedback improves stability at the expense of gain.

A positive feedback can lead to instability and/or oscillations!

Amplifier Gain: Frequency Response

Consider a differential amplifier:

The amplifier gain can be expressed (most generally) as:

\[
A(\omega) = A_0 \frac{(1 + j\omega \tau_1)(1 + j\omega \tau_2)(1 + j\omega \tau_3) \ldots \ldots \ldots \ldots}{(1 + j\omega \tau_1)(1 + j\omega \tau_2)(1 + j\omega \tau_3) \ldots \ldots \ldots \ldots}
\]

Multiple zeros

Multiple poles

Suppose, for simplicity, the amplifier gain can be expressed as:

\[
A(\omega) = \frac{A_0}{(1 + j\omega \tau_1)(1 + j\omega \tau_2)(1 + j\omega \tau_3) \ldots \ldots \ldots \ldots}
\]

Multiple poles
Magnitude and Phase of Response Functions: Single Pole Case

\[ A(\omega) = \frac{A_0}{1 + j\omega \tau} \]

\[
10 \log_{10} |A(\omega)|^2
\]

\[
10 \log_{10} |A_0|^2
\]

\[ \angle A(\omega) \]

\[ \frac{1}{\tau_1} \]

-20 dB/dec

-90

-180

-270

Magnitude and Phase of Response Functions: Multiple Pole Case

\[ A(\omega) = \frac{A_0}{(1 + j\omega \tau_1)(1 + j\omega \tau_2)} \]

\[
10 \log_{10} |A(\omega)|^2
\]

\[
10 \log_{10} |A_0|^2
\]

\[ \angle A(\omega) \]

\[ \frac{1}{\tau_1}, \frac{1}{\tau_2}, \frac{1}{\tau_3} \]

-20 dB/dec

-40 dB/dec

-90

-180

-270
Negative Feedback and Positive Feedback

Closed loop system with negative feedback:

\[ v_{out}(\omega) = \frac{A(\omega)}{1 + KA(\omega)} v_{in}(\omega) \]

Closed loop gain:

\[ A(\omega) = \frac{1}{1 + KA(\omega)} \]

At frequencies between \(1/\tau_2\) and \(1/\tau_3\), \(\angle A(\omega)\) is 180-degrees

\(\Rightarrow\) The feedback is positive, not negative!!!

Phase Response and Amplifier Stability

Consider a differential amplifier operated using a negative feedback:

\[ v_{out}(\omega) = A(\omega)[v_+(\omega) - v_-(\omega)] \]

\[ v_{out}(\omega) = -v_{in}(\omega) \frac{R_2}{R_1} A(\omega) + \frac{R_2}{R_1} \]

\[ K = \frac{1}{1 + \frac{R_2}{R_1}} \]

A positive feedback can happen at high frequencies when:

\[ \angle A(\omega) \rightarrow -180^\circ \]

Denominator can become very small or zero!
Phase Response and Amplifier Stability

Closed loop system with feedback:

\[ v_{in}(\omega) \rightarrow A(\omega) \rightarrow v_{out}(\omega) = \frac{A(\omega)}{1 + KA(\omega)} v_{in}(\omega) \]

How to avoid unwanted output oscillations at the frequency \( \omega \) at which \( \angle A(\omega) = -180^\circ \)?

\[ v_{out}(\omega) = \frac{A(\omega)}{1 + KA(\omega)} v_{in}(\omega) = \frac{-|A(\omega)|}{1 - K|A(\omega)|} v_{in}(\omega) \]

Output will be non-zero, even if the input is zero, if:

\[ 1 - K|A(\omega)| = 0 \]
\[ \Rightarrow |A(\omega)| = \frac{1}{K} > 1 \]

Solution:
To avoid this positive feedback from causing oscillations, the magnitude \( |A(\omega)| \) of the gain must get much less than unity before \( \angle A(\omega) = -180^\circ \).
Gain Margin

\[ A(\omega) = \frac{A_0}{(1 + j\omega \tau_1)(1 + j\omega \tau_2)(1 + j\omega \tau_3)} \]

-30 dB
-40 dB/dec
-60 dB/dec

Gain margin = 30 dB

How small is the gain compared to unity (0 dB) when the phase of the response becomes -180°.

Phase Margin

\[ A(\omega) = \frac{A_0}{(1 + j\omega \tau_1)(1 + j\omega \tau_2)(1 + j\omega \tau_3)} \]

Phase margin = 25°

Phase margin = 23°

How small is the phase compared to -180° when the amplitude of the response becomes unity (0 dB).

Phase margin = 157°

Phase margin = 23°
Consider a differential amplifier operated using a negative feedback:

\[ v_{in}(\omega) \rightarrow A(\omega) \rightarrow v_{out}(\omega) \]

Very often, almost always in fact, when you are done designing the amplifier you figure out that you don't have enough gain and phase margins!!

How to solve this problem?

**Frequency Compensation:**
Add a low frequency pole inside \( A(\omega) \) (by adding extra capacitors, for example), sacrifice bandwidth, but regain stability (PTO...)

\[
\begin{align*}
10\log_{10}|A(\omega)|^2 &\quad \text{Problem!} \\
10\log_{10}|A_{0}|^2 &\quad \text{Added pole} \\
\end{align*}
\]
The frequency compensation capacitor is generally placed in the Miller position.

Texas Instruments LM 741 Operational Amplifier
Texas Instruments LM 741 Operational Amplifier

Frequency compensation capacitor

It is weren’t in the Miller position, it would need to be much larger!!