In this lecture you will learn:

- The operation of bipolar junction transistors
- Forward and reverse active operations, saturation, cutoff
- Ebers-Moll model
PNP Bipolar Junction Transistor

A Silicon PNP BJT
Suppose:
The base-emitter junction is forward biased
\( V_{BE} < 0 \)
The base-collector junction is zero biased
\( V_{CB} = 0 \)
This biasing scheme will put the device in the “forward active” operation (to be discussed fully later)

Consider the action in the base first (\( V_{BE} < 0 \) and \( V_{CB} = 0 \))
• The holes diffuse from the emitter, cross the depletion region, and enter the base
• In the base, the holes are the minority carriers
• In the base, the holes diffuse towards the collector
• As soon as the holes reach the base-collector depletion region they are immediately swept away into the collector by the strong electric fields in the depletion region
PNP BJT: Electron-Hole Populations

Consider the base first:

In the base, the hole population can be written as:

\[
p(x) = p_{\text{no}} + p'(x)\]

Equilibrium hole density
Excess hole density

In the base, the excess electron population satisfies the differential equation:

\[
\frac{\partial^2 p'(x)}{\partial x^2} \cdot \frac{p'(x)}{L_p} = 0
\]

Boundary conditions

\[
p'(x_n) = \frac{n_i^2}{N_{dB}} \left( e^{-\frac{qV_{BE}}{KT}} - 1 \right)
\]

\[
p'(x_n + W_B) = \frac{n_i^2}{N_{dB}} \left( e^{-\frac{qV_{BC}}{KT}} - 1 \right) = 0
\]

\[
p'(x) = p'(x_p) \left( 1 - \frac{x - x_n}{W_B} \right) = \frac{n_i^2}{N_{dB}} \left( e^{-\frac{qV_{BE}}{KT}} - 1 \right) \left( 1 - \frac{x - x_n}{W_B} \right)
\]

忽略载流子复合（即假设 \( L_p = \infty \))

\[
\frac{\partial^2 p'(x)}{\partial x^2} = 0
\]

Boundary conditions

\[
p'(x_n) = \frac{n_i^2}{N_{dB}} \left( e^{-\frac{qV_{BE}}{KT}} - 1 \right)
\]

\[
p'(x_n + W_B) = \frac{n_i^2}{N_{dB}} \left( e^{-\frac{qV_{BC}}{KT}} - 1 \right) = 0
\]

Solution is:
Consider the emitter now:

In the emitter, the electron population can be written as:

\[ n(x) = n_{po} + n'(x) \]

Equilibrium electron density Excess electron density

In the emitter, the excess electron population satisfies the differential equation:

\[ \frac{\partial^2 n'(x)}{\partial x^2} - \frac{n'(x)}{L_n^2} = 0 \]

Boundary conditions

\[ n'(-x_p) = \frac{n_i^2}{N_{aE}} \left( -\frac{qV_{BE}}{kT} - 1 \right) \]

\[ n'(-x_p - W_E) = 0 \]

Solution is:

\[ n'(x) = n'(-x_p) \left( 1 + \frac{x + x_p}{W_E} \right) = \frac{n_i^2}{N_{aE}} \left( -\frac{qV_{BE}}{kT} - 1 \right) \left( 1 + \frac{x + x_p}{W_E} \right) \]
In the base:
• The hole current is:
\[ J_p(x) = -q D_p \frac{\partial p(x)}{\partial x} = -q n_i^2 \frac{D_p}{N_{dB} W_B} \left( \frac{-q V_{BE}}{kT} - 1 \right) \]

In the emitter:
• The electron current is:
\[ J_n(x) = q D_n \frac{\partial n(x)}{\partial x} = -q n_i^2 \frac{D_n}{N_{aE} W_E} \left( \frac{-q V_{BE}}{kT} - 1 \right) \]

Emitter current:
• The current flowing out of the emitter is the sum of the total electron and total hole currents in the emitter:
\[ I_E = -q n_i^2 A \left( \frac{D_n}{N_{aE} W_E} + \frac{D_p}{N_{dB} W_B} \right) \left( \frac{-q V_{BE}}{kT} - 1 \right) \]
Collector Current:
• The current going into the collector is due to the holes that got swept from the Base through the Base-Collector depletion region by the electric-fields:

\[
I_C = -q n_t^2 A \left( \frac{D_p}{N_{db} W_B} \right) \left( e^{\frac{-q V_{BE}}{kT}} - 1 \right)
\]

Base Current:
• The current going into the Base is due to the electrons that got injected from the base into the emitter:

\[
I_B = -q n_t^2 A \left( \frac{D_n}{N_{ae} W_E} \right) \left( e^{\frac{-q V_{BE}}{kT}} - 1 \right)
\]

Collector Current:
\[
I_C = I_B + I_E
\]
PNP BJT: Circuit Level Parameters

Current gain $\beta_F$:
The current gain of the BJT in the forward active operation is defined as the ratio of the collector and base currents:

$$\beta_F = \frac{I_C}{I_B} = \frac{D_p}{N_{db}W_B} \frac{N_{ae}W_E}{D_n} \Rightarrow I_C = \beta_F I_B$$

Typical values of $\beta_F$ are between 20-200 and:

$$N_{ae} >> N_{db} > N_{ac}$$

$\alpha_F$:
In the forward active operation $\alpha_F$ is defined as the ratio of the collector and emitter currents:

$$\alpha_F = \frac{I_C}{I_E} = \frac{D_p}{D_n} \frac{N_{db}W_B}{D_p N_{ae}W_E + N_{db}W_B} \Rightarrow I_C = \alpha_F I_E$$

Transistor relation:
$\alpha_F$ and $\beta_F$ are related:

$$\beta_F = \frac{\alpha_F}{1 - \alpha_F}$$

PNP BJT: Ebers-Moll Model for Forward Active Operation

Suppose:

$$V_{BE} < 0 \quad V_{CB} = 0$$

The circuit level simplified model with an ideal diode and a current-controlled current source models the PNP transistor in the forward active operation.
PNP BJT: Forward and Reverse Active Operations

Forward active operation

\[
\beta_F = \frac{I_C}{I_B} \\
\alpha_F = \frac{I_C}{I_E} \\
\beta_F = \frac{\alpha_F}{1 - \alpha_F}
\]

Reverse active operation

\[
\beta_R = \frac{I_E}{I_B} \frac{D_p}{N_{acW_C}} \\
\alpha_R = \frac{I_E}{I_C} \\
\beta_R = \frac{\alpha_R}{1 - \alpha_R}
\]

In a well designed transistor: \( \beta_F \gg \beta_R \)

PNP BJT: Ebers-Moll Model for Reverse Active Operation

Suppose: \( V_{CB} > 0 \) \( V_{BE} = 0 \)

\[
I_R = qn^2A \frac{D_n}{N_{acW_C}} + D_p \frac{qV_{CB}}{N_{dBW_B} e^{KT} - 1} \\
= I_{CS} \left( \frac{qV_{CB}}{e^{KT} - 1} \right)
\]

The circuit level simplified model with an ideal diode and a current-controlled current source models the PNP transistor in the reverse active operation.
PNP BJT: Ebers-Moll Model and Terminal Currents

Terminal currents:

\[ I_R = I_{CS} \left( e^{\frac{qV_{CB}}{kT}} - 1 \right) = I_{CS} \left( e^{\frac{-qV_{BE}}{kT}} - 1 \right) \]
\[ I_F = I_{ES} \left( e^{\frac{-qV_{BE}}{kT}} - 1 \right) = I_{ES} \left( e^{\frac{qV_{EB}}{kT}} - 1 \right) \]

And

\[ I_B = (1 - \alpha_F)I_F + (1 - \alpha_R)I_R \]
\[ I_C = \alpha_F I_F - I_R \]
\[ I_E = I_F - \alpha_R I_R \]

PNP BJT: Different Regimes of Operation

- **Forward Active:**
  - \( V_{BE} > 0 \)
  - \( I_C \) Forward biased

- **Saturation:**
  - \( V_{CB} < 0 \)
  - \( V_{BE} > 0 \)
  - \( I_C \) Forward biased

- **Reverse Active:**
  - \( V_{CB} < 0 \)
  - \( V_{BE} < 0 \)
  - \( I_C \) Reversed biased
In forward active operation:
\[ I_B < 0 \quad V_{BE} < 0 \quad V_{CB} \leq 0 \]

Since: \[ V_{CE} = V_{CB} + V_{BE} \]

\[ \Rightarrow \] In forward active operation: \[ V_{CE} \leq V_{BE} \]

\[ I_C = -qn_i^2 A \left( \frac{D_p}{N_{db} W_B} \right) \left( e^{-\frac{q V_{BE}}{kT}} - 1 \right) = \beta_F I_B \]

\[ \Rightarrow \] Independent of \( V_{CE} \)

Forward active:
- Base-emitter junction forward biased
- Base-collector junction reversed biased

\[ I_B < 0 \quad V_{BE} < 0 \quad V_{CB} \leq 0 \]

Saturation:
- Base-emitter junction forward biased
- Base-collector junction forward biased

\[ I_B < 0 \quad V_{BE} < 0 \quad V_{CB} > 0 \]

Carrier Densities in Different Regimes of Operation

Forward active:
- \( \n^\prime(x) \) electrons
- \( p^\prime(x) \) holes
- \( N_{aE} \)
- \( W_E \)
- \( x_p \)
- \( x_n \)
- \( W_B \)
- \( W_C \)

Saturation:
- \( \n^\prime(x) \) electrons
- \( p^\prime(x) \) holes
- \( N_{aE} \Rightarrow N_{db} > N_{aC} \)
- \( W_E \)
- \( x_n \)
- \( x_p \)
- \( W_B \)
- \( W_C \)

The forward biased base-collector junction reduces the magnitude of the collector current!
PNP BJT: Regimes of Operation - II

**Forward active:**
Base-emitter junction forward biased
Base-collector junction reversed biased

\[ I_B < 0 \quad V_{BE} < 0 \quad V_{CB} \leq 0 \]

**Saturation:**
Base-emitter junction forward biased
Base-collector junction forward biased

\[ I_B < 0 \quad V_{BE} < 0 \quad V_{CB} > 0 \]

**Cutoff:**
Base current zero

\[ I_B = 0 \]

**Reverse active:**
Base-emitter junction reverse biased
Base-collector junction forward biased

\[ I_B < 0 \quad V_{BE} \geq 0 \quad V_{CB} > 0 \]

Curves for increasing \(|IB|\)

Forward active
\[ I_B < 0 \quad I_C = \beta_I I_B \]

\[ V_{CE} \leq V_{BE} \]

Saturation
\[ I_B < 0 \quad I_C > 0 \quad V_{CE} > V_{BE} \]

PNP BJT: A Simple Amplifier Circuit

**Current gain (in forward active regime):**

\[ \frac{I_C}{I_B} = \beta_F \]

**Load line equation:**

\[ V_{CE} = -I_C R - V_{DD} \]

\[ \Rightarrow I_C = -\frac{V_{DD} + V_{CE}}{R} \]

Curves for increasing \(|IB|\)

Forward active
\[ I_B < 0 \quad I_C = \beta_I I_B \]

\[ V_{CE} \leq V_{BE} \]

Saturation
\[ I_B < 0 \quad I_C > 0 \quad V_{CE} > V_{BE} \]

Lesson: Don’t let the base-collector junction become forward biased
A Silicon PNP BJT

Metal base contact  Metal emitter contact  Metal collector contact

Insulator (SiO₂)  Insulator (SiO₂)  Insulator (SiO₂)

N+ (contact)  P⁺ (emitter)  N (base)  P⁺ (contact layer)  P (collector)  P (substrate)