Bipolar Junction Transistors (BJTs)

In this lecture you will learn:

• The operation of bipolar junction transistors
• Forward and reverse active operations, saturation, cutoff
• Ebers-Moll model
• Small signal models
NPN Bipolar Junction Transistor

PNP Bipolar Junction Transistor
Suppose:

- The base-emitter junction is forward biased \( V_{BE} > 0 \)
- The base-collector junction is zero biased \( V_{CB} = 0 \)

This biasing scheme will put the device in the “forward active” operation (to be discussed fully later).

Consider the action in the base first (\( V_{BE} > 0 \) and \( V_{CB} = 0 \))

- The electrons diffuse from the emitter, cross the depletion region, and enter the base
- In the base, the electrons are the minority carriers
- In the base, the electrons diffuse towards the collector
- As soon as the electrons reach the base-collector depletion region they are immediately swept away into the collector by the strong electric fields in the depletion region
NPN BJT: Electron-Hole Populations

Consider the base first:

In the base, the electron population can be written as:

\[ n(x) = n_{po} + n'(x) \]

Equilibrium electron density  \( n_{po} = \frac{n_i^2}{N_{aB}} \)

In the base, the excess electron population satisfies the differential equation:

\[
\frac{\partial^2 n'(x)}{\partial x^2} - \frac{n'(x)}{L_n^2} = 0
\]

Boundary conditions:

\[
n'(x_p) = \frac{n_i^2}{N_{aB}} \left( \frac{qV_{BE}}{eKT} - 1 \right)
\]

\[
n'(x_p + W_B) = \frac{n_i^2}{N_{aB}} \left( \frac{qV_{BC}}{eKT} - 1 \right) = 0
\]

Solution is:

\[
n'(x) = n'(x_p) \left( 1 - \frac{x - x_p}{W_B} \right) = \frac{n_i^2}{N_{aB}} \left( \frac{qV_{BE}}{eKT} - 1 \right) \left( 1 - \frac{x - x_p}{W_B} \right)
\]

• Ignore carrier recombination (i.e. assume \( L_n = \infty \))

\[
\frac{\partial^2 n'(x)}{\partial x^2} = 0
\]

Boundary conditions:

\[
n'(x_p) = \frac{n_i^2}{N_{aB}} \left( \frac{qV_{BE}}{eKT} - 1 \right)
\]

\[
n'(x_p + W_B) = \frac{n_i^2}{N_{aB}} \left( \frac{qV_{BC}}{eKT} - 1 \right) = 0
\]
Consider the emitter now:

In the emitter, the hole population can be written as:

\[ p(x) = P_{no} + p'(x) \]

Equilibrium hole density \( P_{no} = \frac{n_i^2}{N_{dE}} \)

Excess hole density

In the emitter, the excess hole population satisfies the differential equation:

\[
\frac{\partial^2 p'(x)}{\partial x^2} - \frac{p'(x)}{L_p} = 0 \]

Boundary conditions \( p'(-x_n) = \frac{n_i^2}{N_{dE}} \left( \frac{qV_{BE}}{kT} - 1 \right) \)

\[ p'(-x_n - W_E) = 0 \]

\[
\frac{\partial^2 p'(x)}{\partial x^2} = 0 \]

Boundary conditions \( p'(-x_n) = \frac{n_i^2}{N_{dE}} \left( \frac{qV_{BE}}{kT} - 1 \right) \)

\[ p'(-x_n - W_E) = 0 \]

Solution is:

\[ p'(x) = p'(-x_n) \left( 1 + \frac{x + x_n}{W_E} \right) = \frac{n_i^2}{N_{dE}} \left( \frac{qV_{BE}}{kT} - 1 \right) \left( 1 + \frac{x + x_n}{W_E} \right) \]
**NPN BJT: Electron and Hole Current Densities**

In the base:
- The electron current is:
  \[ J_n(x) = q D_n \frac{\partial n(x)}{\partial x} = q n_i^2 \frac{D_n}{N_{aB}W_B} \left( \frac{q V_{BE}}{kT} - 1 \right) \]

In the emitter:
- The hole current is:
  \[ J_p(x) = -q D_p \frac{\partial p(x)}{\partial x} = q n_i^2 \frac{D_p}{N_{dE}W_E} \left( \frac{q V_{BE}}{kT} - 1 \right) \]

**NPN BJT: Terminal Currents**

Emitter current:
- The current flowing out of the emitter is the sum of the total electron and total hole currents in the emitter:
  \[ I_E = q n_i^2 A \left( \frac{D_p}{N_{dE}W_E} + \frac{D_n}{N_{aB}W_B} \left( \frac{q V_{BE}}{kT} - 1 \right) \right) \]
Collector Current:
- The current going into the collector is due to the electrons that got swept from the Base through the Base-Collector depletion region by the electric-fields:

\[ I_C = qn_i^2 A \left( \frac{D_n}{N_{ab} W_B} \right) \left( \frac{q V_{BE}}{e \cdot kT} - 1 \right) \]

Base Current:
- The current going into the Base is due to the holes that got injected from the base into the emitter:

\[ I_B = qn_i^2 A \left( \frac{D_p}{N_{de} W_E} \right) \left( \frac{q V_{BE}}{e \cdot kT} - 1 \right) \]

\[ I_E = I_B + I_C \]
NPN BJT: Circuit Level Parameters

Current gain $\beta_F$:
Current gain of the BJT in the forward active operation is defined as the ratio of the collector and base currents:

$$\beta_F = \frac{I_C}{I_B} = \frac{D_0 N_{de} W_E}{N_{ab} W_B} \Rightarrow I_C = \beta_F I_B$$

Typical values of $\beta_F$ are between 20-200 and:

$$N_{de} \gg N_{ab} > N_{dc}$$

$\alpha_F$:
In the forward active operation $\alpha_F$ is defined as the ratio of the collector and emitter currents:

$$\alpha_F = \frac{I_C}{I_E} = \frac{D_0}{D_p + D_0} \Rightarrow I_C = \alpha_F I_E$$

Transistor relation:
$\alpha_F$ and $\beta_F$ are related:

$$\beta_F = \frac{\alpha_F}{1 - \alpha_F}$$

NPN BJT: Ebers-Moll Model for Forward Active Operation

Suppose:

$$V_{BE} > 0, \quad V_{CB} = 0$$

The circuit level simplified model with an ideal diode and a current-controlled current source models the NPN transistor in the forward active operation.
NPN BJT: Forward Active Operation

\[ V_{BE} > 0 \]
\[ V_{CB} \geq 0 \]

Forward active operation

\[ \beta_F = \frac{I_C}{I_B} \]
\[ \alpha_F = \frac{I_C}{I_E} \]
\[ \beta_F = \frac{\alpha_F}{1 - \alpha_F} \]

In a well designed transistor: \( \beta_F \gg \beta_R \)

NPN BJT: Forward and Reverse Active Operations

\[ V_{BE} > 0 \]
\[ V_{CB} < 0 \]
\[ V_{BE} \leq 0 \]
\[ V_{BE} \leq 0 \]

Forward active operation

\[ \beta_F = \frac{I_C}{I_B} \]
\[ \alpha_F = \frac{I_C}{I_E} \]
\[ \beta_F = \frac{\alpha_F}{1 - \alpha_F} \]

Reverse active operation

\[ \beta_R = \frac{I_E}{I_B} \cdot \frac{D_n}{N_{ab}W_B} \cdot \frac{N_{bc}W_C}{D_p} \]
\[ \alpha_R = \frac{I_E}{I_C} \]
\[ \beta_R = \frac{\alpha_R}{1 - \alpha_R} \]
Suppose:
\[ V_{BC} > 0, \quad V_{BE} = 0 \]

\[
I_R = qn^2 A \left( \frac{D_p}{N_{dc}W_C} + \frac{D_n}{N_{ab}W_B} \right) \frac{qV_{BC}}{e^{qV_{BC}/kT} - 1} = I_{CS} \left( \frac{qV_{BC}}{e^{qV_{BC}/kT} - 1} \right)
\]

The circuit level simplified model with an ideal diode and a current-controlled current source models the NPN transistor in the reverse active operation.

Terminal currents:
\[
I_R = I_{CS} \left( \frac{qV_{BC}}{e^{qV_{BC}/kT} - 1} \right)
\]
\[
I_F = I_{ES} \left( \frac{qV_{BE}}{e^{qV_{BE}/kT} - 1} \right)
\]

And
\[
I_B = (1 - \alpha_F)I_F + (1 - \alpha_R)I_R
\]
\[
I_C = \alpha_F I_F - I_R
\]
\[
I_E = I_F - \alpha_R I_R
\]
NPN BJT: Regimes of Operation - I

**In forward active operation:**

\[ I_B > 0 \quad V_{BE} > 0 \quad V_{CB} \geq 0 \]

Since: \[ V_{CE} = V_{CB} + V_{BE} \]

⇒ In forward active operation: \[ V_{CE} \geq V_{BE} \]

\[ I_C = qn^2A \left( \frac{D_n}{N_{ab}W_B} \right) \left( \frac{qV_{BE}}{KT} - 1 \right) = \beta_F I_B \]

⇒ Independent of \( V_{CE} \)

**Forward active:**
- Base-emitter junction forward biased
- Base-collector junction reversed biased

\[ I_B > 0 \quad V_{BE} > 0 \quad V_{CB} \geq 0 \]

**Saturation:**
- Base-emitter junction forward biased
- Base-collector junction forward biased

\[ I_B > 0 \quad V_{BE} > 0 \quad V_{CB} < 0 \]

**Carrier Densities in Different Regimes of Operation**

**Forward active:**
- \( V_{BE} > 0 \)
- \( V_{CB} \geq 0 \)

**Saturation:**
- \( V_{BE} > 0 \)
- \( V_{CB} < 0 \)

The forward biased base-collector junction reduces the collector current!
NPN-BJT: Regimes of Operation - II

**Forward active:**
Base-emitter junction forward biased
Base-collector junction reversed biased
- $I_B > 0$  $V_{BE} > 0$  $V_{CB} \geq 0$
- $I_C = \beta_F I_B$

**Saturation:**
Base-emitter junction forward biased
Base-collector junction forward biased
- $I_B > 0$  $V_{BE} > 0$  $V_{CB} < 0$

**Cutoff:**
Base current zero
- $I_B = 0$

**Reverse active:**
Base-emitter junction reverse biased
Base-collector junction forward biased
- $I_B > 0$  $V_{BE} \leq 0$  $V_{CB} < 0$
- $I_E = -\beta_R I_B$

NPN BJT: Different Regimes of Operation

- **Forward Active**
  - $V_{BE} > 0$
  - $V_{CB} \geq 0$

- **Saturation**
  - $V_{CB} < 0$
  - $V_{BE} > 0$

- **Reverse Active**
  - $V_{CB} < 0$
  - $V_{BE} \leq 0$
NPN BJT: A Simple Amplifier Circuit

Current gain (in forward active regime):
\[
\frac{I_{OUT}}{I_{IN}} = \frac{I_C}{I_B} = \beta_F
\]

Load line equation:
\[
V_{CE} = V_{DD} - I_C R
\implies I_C = \frac{V_{DD} - V_{CE}}{R}
\]

Lesson: Don't let the base-collector junction become forward biased.

NPN BJT

Better/easier definition:
In saturation, \( I_B > 0 \) and \( V_{CE} < V_{CE-SAT} \)

In forward active, \( I_B > 0 \) and \( V_{CE} > V_{CE-SAT} \)
Approximate analysis of transistor DC biasing:

- If: $V_{IN} < V_{BE-ON} \Rightarrow I_B = 0 \Rightarrow$ Transistor in cut-off
- If: $V_{IN} \geq V_{BE-ON}$
  
  \[ I_B = \frac{V_{IN} - V_{BE-ON}}{R_S} \]

Assume forward active operation ($V_{CE} > V_{CE-SAT}$):

\[ I_C = \beta F I_B \]

\[ V_{OUT} = V_{DD} - I_C R = V_{DD} - \beta F \left( \frac{V_{IN} - V_{BE-ON}}{R_S} \right) R \]

Final Step - confirm if the assumption of forward active operation was valid:

$V_{CE} \geq V_{CE-SAT}$

\[ \Rightarrow V_{CE} = V_{OUT} = V_{DD} - I_C R = V_{DD} - \beta F \left( \frac{V_{IN} - V_{BE-ON}}{R_S} \right) R \geq V_{CE-SAT} \]
**NPN BJT Common Emitter (CE) Voltage Amplifier**

\[ V_{BE-ON} \sim 0.6 \, \text{V} \]
\[ V_{CE-SAT} \sim 0.2 \, \text{V} \]

We need better techniques to calculate the voltage gain of such amplifier circuits.

We need small signal models of the BJTs!

**NPN BJT: Small Signal Circuit Model**

**Base current:**
\[ I_B = qn^2A \left( \frac{D_p}{N_{de}W_E} \right) \left( \frac{qV_{BE}}{kT} - 1 \right) \]
\[ = I_{BS} \left( \frac{qV_{BE}}{kT} - 1 \right) \]
\[ \Rightarrow I_B + I_b = I_{BS} \left( \frac{q(V_{BE} + V_\pi)}{kT} - 1 \right) \]
\[ \Rightarrow I_b = \frac{qI_B}{\partial V_{BE}} v_\pi = \frac{q(l_B + I_{BS})}{kT} v_\pi = \frac{qI_B}{kT} v_\pi = g_m v_\pi \]
\[ g_m = \frac{qI_B}{kT} \]

**Collector current:**
\[ I_c + i_c = \beta_F (I_B + I_b) \]
\[ \Rightarrow i_c = \beta_F I_b = \beta_F g_m v_\pi = g_m v_\pi \]
\[ g_m = \beta_F g_\pi = \beta_F \frac{qI_c}{kT} \]

Increases linearly with the collector current.
NPN BJT: Small Signal Circuit Model

\[ g_m = \beta F g_{\pi} = \frac{q I_C}{K T} \]

NPN BJT: Forward Active Current vs \( V_{CE} \)

As \( V_{CE} \) becomes more positive, the base-collector junction becomes more reverse biased, and the thickness of the depletion increases thereby reducing the thickness \( W_B \) of the base.

Consequently, the collector current increases.

The output conductance \( g_o \) is not zero!
NPN BJT: Output Conductance and the Early Voltage

The slope of the $I_C$ vs $V_{CE}$ curves are modeled using the early voltage $V_A$:

$$\frac{dI_C}{dV_{CE}} = g_o = \frac{I_C}{V_A} = \lambda n I_C$$

The early voltage is usually in the 50-200 V range

NPN BJT: Output Conductance

Output conductance:

$$g_o = \frac{1}{r_o} = \frac{\partial I_C}{\partial V_{CE}}$$
NPN BJT Common Emitter (CE) Voltage Amplifier

\[ v_\pi = v_s \frac{r_\pi}{r_\pi + R_s} \]
\[ i_c = g_m v_\pi + \frac{v_{out}}{r_o} \]
\[ v_{out} = -i_c R \]
\[ \Rightarrow v_{out} = -g_m (r_o || R) v_\pi \]
\[ \Rightarrow v_{out} = -g_m (r_o || R) \frac{r_\pi}{r_\pi + R_s} v_s \]

Voltage gain:
\[ A_v = \frac{v_{out}}{v_s} = -g_m (r_o || R) \frac{r_s}{r_\pi + R_s} \]

NPN BJT CE Amplifier: Limits of Output Voltage Swing

Minimum output voltage and maximum input voltage:
If the output voltage becomes too small (happens when the input voltage becomes too large), the BJT will go into the saturation region (in the saturation region the gain is small).

Maximum output voltage and minimum input voltage:
If the input voltage becomes too small the BJT will go into cut-off.