Lecture 16

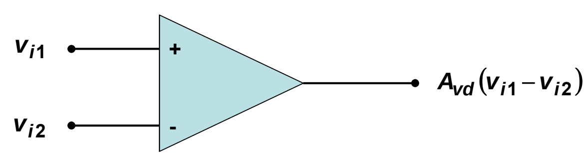
Differential Amplifiers – I
Basics

In this lecture you will learn:

- **Differential Amplifiers**
- **Differential FET Amplifiers**
- **Large Signal and Small Signal Analysis**
- **Half Circuit Techniques**

Ideal Differential Amplifiers

An ideal differential amplifier amplifies the difference signal between two inputs:



The need for differential amplifiers:
Differential amplifiers are used to remove unwanted signals that are common to both input signals. For example, in many cases useful information is carried by the difference between two signal sources, 1 and 2, and unwanted noise signals that add to both the 1 and 2 signals will be rejected by a differential amplifier which will amplify only the difference of these signals. Most high fidelity modern amplifiers are differential amplifiers.

These unwanted signals that add to both signals 1 and 2 could be a result of:

a) Variation in the power supply voltage as a function of time
b) Variation in the substrate potential of the entire chip
c) Variation in the temperature of the chip
d) Electromagnetic interference signals from the environment
e) etc

Difference Mode and Common Mode Signal Components

The difference-mode and the common-mode components of any two signals are defined as follows:

$$V_D = V_1 - V_2$$

$$V_C = \frac{V_1 + V_2}{2}$$

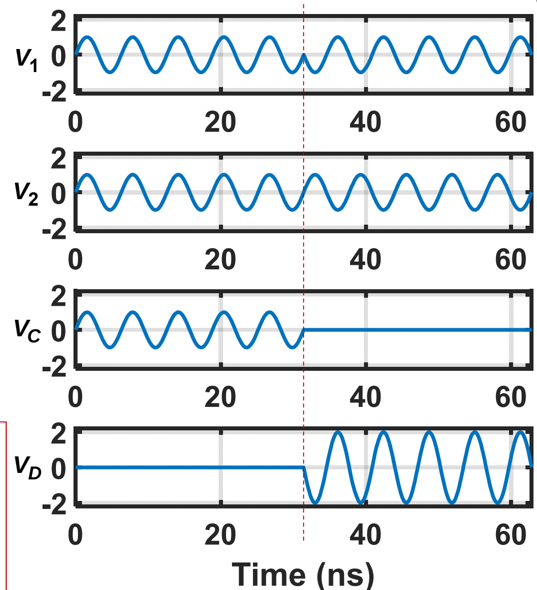
Any two signals can be written in terms of their difference-mode and common-mode components:

$$\Rightarrow V_1 = V_C + \frac{V_D}{2}$$

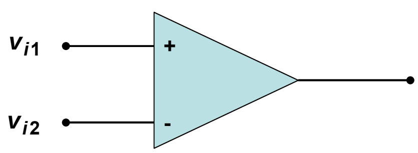
$$\Rightarrow V_2 = V_C - \frac{V_D}{2}$$

Out-of-phase signals have a large difference mode component and a small common mode component.

In-phase signals have a large common mode component and a small difference mode component.



Difference Mode and Common Mode Input Signal Components



The difference-mode and the common-mode components of two input signals are:

$$v_{id} = v_{i1} - v_{i2} \longrightarrow \text{Difference-mode component}$$

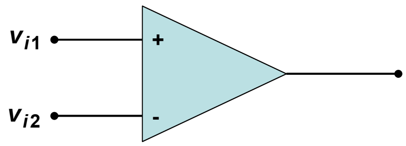
$$v_{ic} = \frac{v_{i1} + v_{i2}}{2} \longrightarrow \text{Common-mode component}$$

Since any two signals can be written in terms of their difference-mode and common-mode components:

$$\Rightarrow v_{i1} = v_{ic} + \frac{v_{id}}{2}$$

$$\Rightarrow v_{i2} = v_{ic} - \frac{v_{id}}{2}$$

Differential Amplifier with Single-Ended Output



$$v_{out} = A_{vd}(v_{i1} - v_{i2}) + A_{vc}\left(\frac{v_{i1} + v_{i2}}{2}\right)$$
$$= A_{vd}v_{id} + A_{vc}v_{ic}$$
$$= \left(A_{vd} + \frac{A_{vc}}{2}\right)v_{i1} + \left(-A_{vd} + \frac{A_{vc}}{2}\right)v_{i2}$$

Difference-Mode Gain: A_{vd}

Common-Mode Gain: A_{vc}

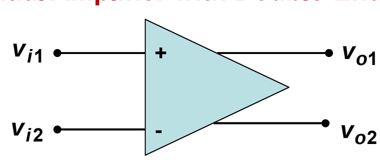
One always wants the difference-mode gain to be much much larger than the common-mode gain (ideally one would want the common mode gain to be zero!)

Common-Mode Rejection Ratio (CMRR):

$$CMRR = \frac{A_{vd}}{A_{vc}}$$

Differential Amplifier with Double-Ended Output

$$\left.\begin{aligned} v_{id} &= v_{i1} - v_{i2} \\ v_{ic} &= \frac{v_{i1} + v_{i2}}{2} \end{aligned}\right\}$$



$$\left\{\begin{aligned} v_{od} &= v_{o1} - v_{o2} \\ v_{oc} &= \frac{v_{o1} + v_{o2}}{2} \end{aligned}\right.$$

Difference-Mode Output:

$$v_{od} = v_{o1} - v_{o2} = A_{vd}v_{id}$$

Common-Mode Output:

$$v_{oc} = \frac{v_{o1} + v_{o2}}{2} = A_{vc}v_{ic}$$

One always wants the difference-mode gain to be much much larger than the common-mode gain

Common-Mode Rejection Ratio (CMRR):

$$CMRR = \frac{A_{vd}}{A_{vc}}$$

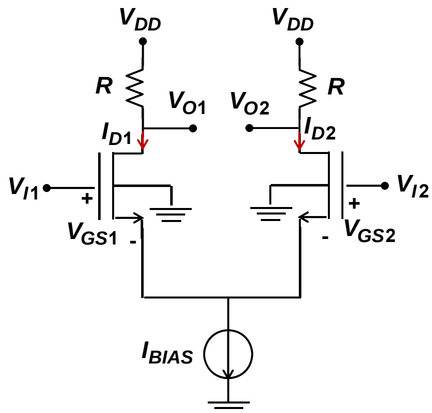
One can also right:

$$v_{o2} = -\frac{A_{vd}}{2}(v_{i1} - v_{i2}) + A_{vc}\left(\frac{v_{i1} + v_{i2}}{2}\right)$$
$$= -A_{vd}\frac{v_{id}}{2} + A_{vc}v_{ic}$$

$$v_{o1} = \frac{A_{vd}}{2}(v_{i1} - v_{i2}) + A_{vc}\left(\frac{v_{i1} + v_{i2}}{2}\right)$$
$$= A_{vd}\frac{v_{id}}{2} + A_{vc}v_{ic}$$

FET Differential Amplifiers

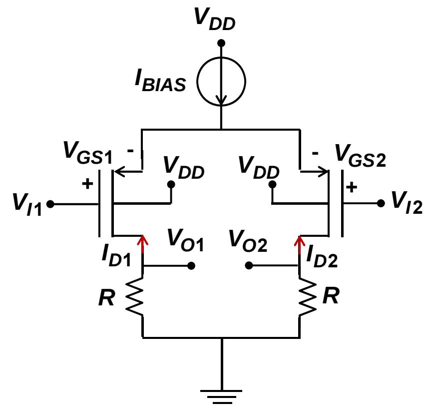
Matched NFETs



Comments:

$$I_{D1} + I_{D2} = I_{BIAS}$$

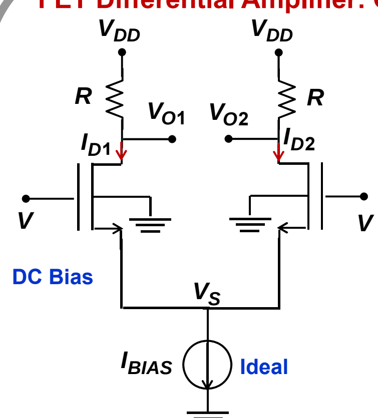
Matched PFETs



Comments:

$$I_{D1} + I_{D2} = -I_{BIAS}$$

FET Differential Amplifier: Common Mode Input (Rough Analysis)

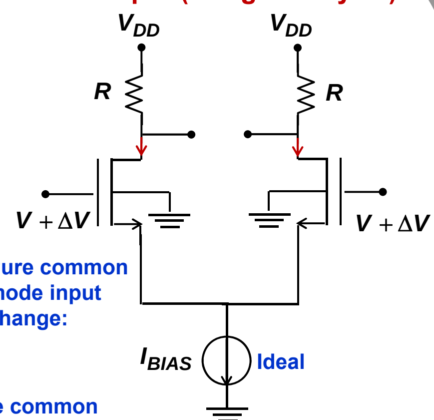


DC Bias

$$I_{D1} = I_{D2} = \frac{I_{BIAS}}{2}$$

$$V_{I1} = V_{I2} = V$$

$$V_{O1} = V_{O2} = V_{DD} - \frac{I_{BIAS}}{2} R$$



Pure common mode input change:

Pure common mode input change:

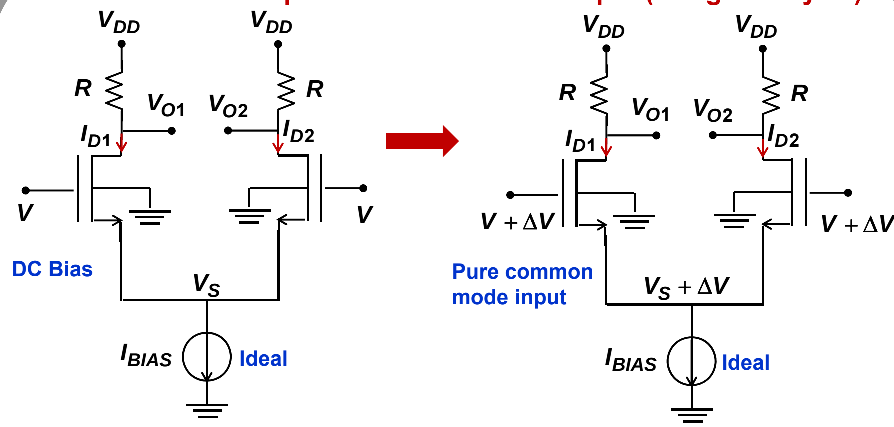
$$\Delta V_{i1} = \Delta V$$

$$\Delta V_{i2} = \Delta V$$

$$\Rightarrow \Delta V_{IC} = \frac{\Delta V_{i1} + \Delta V_{i2}}{2} = \Delta V$$

$$\Rightarrow \Delta V_{ID} = \Delta V_{i1} - \Delta V_{i2} = 0$$

FET Differential Amplifier: Common Mode Input (Rough Analysis)



$$I_{D1}=I_{D2}=\frac{I_{BIAS}}{2}$$

$$V_{I1}=V_{I2}=V$$

$$V_{O1}=V_{O2}=V_{DD}-\frac{I_{BIAS}}{2}R$$

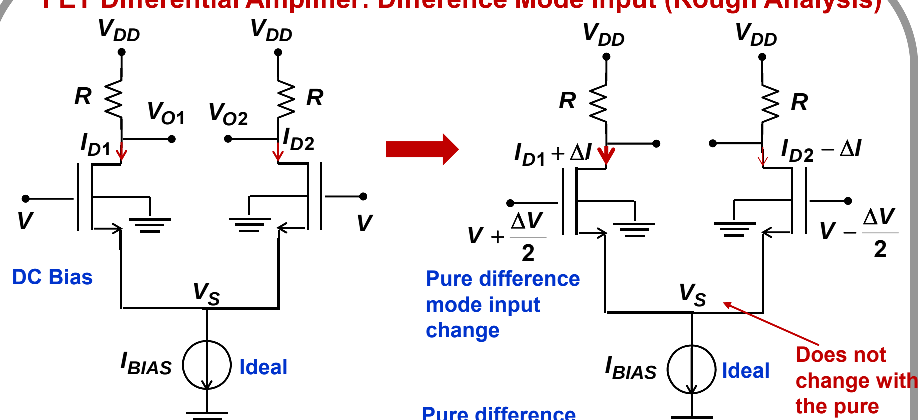
Still: $I_{D1}=I_{D2}=\frac{I_{BIAS}}{2}$

=> $\Delta V_{O1}=0$

$\Delta V_{O2}=0$

$$\Rightarrow A_{VC}=\frac{\Delta V_{OC}}{\Delta V_{IC}}=\frac{\frac{\Delta V_{O1}+\Delta V_{O2}}{2}}{\Delta V}=0$$

FET Differential Amplifier: Difference Mode Input (Rough Analysis)



$$I_{D1}=I_{D2}=\frac{I_{BIAS}}{2}$$

$$V_{I1}=V_{I2}=V$$

$$V_{O1}=V_{O2}=V_{DD}-\frac{I_{BIAS}}{2}R$$

Pure difference mode input change:

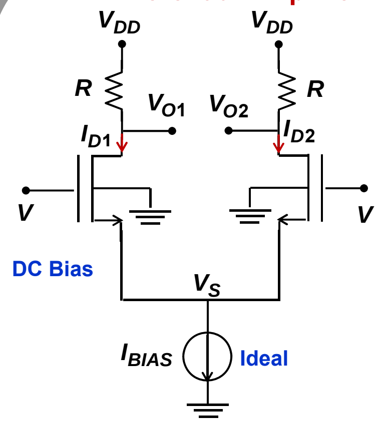
$\Delta V_{i1}=\Delta V/2$

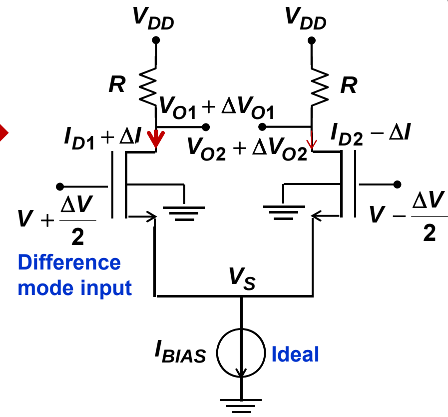
$\Delta V_{i2}=-\Delta V/2$

$$\Rightarrow \Delta V_{IC}=\frac{\Delta V_{i1}+\Delta V_{i2}}{2}=0$$

$$\Rightarrow \Delta V_{ID}=\Delta V_{i1}-\Delta V_{i2}=\Delta V$$

FET Differential Amplifier: Difference Mode Input (Rough Analysis)





$$I_{D1} = I_{D2} = \frac{I_{BIAS}}{2}$$

$$V_{I1} = V_{I2} = V$$

$$V_{O1} = V_{O2} = V_{DD} - \frac{I_{BIAS}}{2} R$$

Now: $(I_{D1} + \Delta I) + (I_{D2} - \Delta I) = I_{BIAS}$

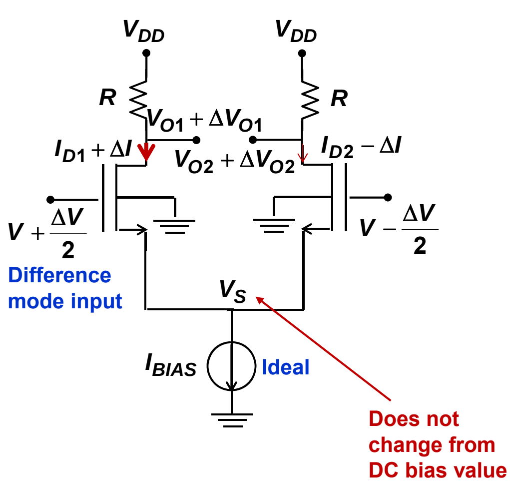
=> $\Delta V_{O1} = -\Delta I\, R$

$\Delta V_{O2} = \Delta I\, R$

$$\Rightarrow A_{VD} = \frac{\Delta V_{OD}}{\Delta V_{ID}} = \frac{\Delta V_{O1} - \Delta V_{O2}}{\Delta V} = -2\frac{\Delta I\, R}{\Delta V}$$

Large

FET Differential Amplifier: Difference Mode Input (Rough Analysis)



$$\Delta V_{GS1} = \frac{\Delta V}{2}$$

$$\Delta V_{GS2} = -\frac{\Delta V}{2}$$

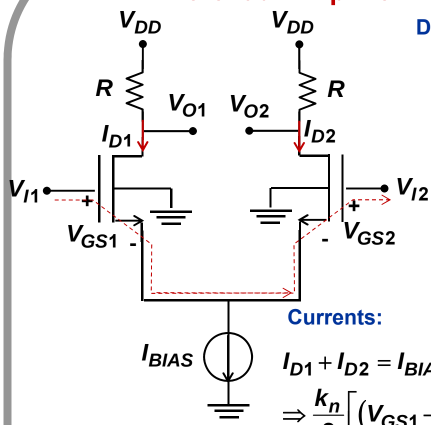
$$\Rightarrow \Delta I = g_m \Delta V_{GS1} = g_m \frac{\Delta V}{2}$$

$$\Rightarrow A_{VD} = -2\frac{\Delta I\, R}{\Delta V} = -g_m R$$

Large

From previous slide

A FET Differential Amplifier: Large Signal Analysis (Assume λ_n=0)



Difference-Mode and Common-Mode inputs:

$$V_{ID} = V_{I1} - V_{I2}$$

$$V_{IC} = \frac{V_{I1} + V_{I2}}{2}$$

$$V_{I1} - V_{GS1} + V_{GS2} = V_{I2} \quad \text{KVL}$$

$$\Rightarrow V_{ID} = V_{I1} - V_{I2} = V_{GS1} - V_{GS2}$$

Currents:

$$I_{D1} + I_{D2} = I_{BIAS}$$

$$\Rightarrow \frac{k_n}{2}\left[(V_{GS1} - V_{TN})^2 + (V_{GS2} - V_{TN})^2\right] = I_{BIAS}$$

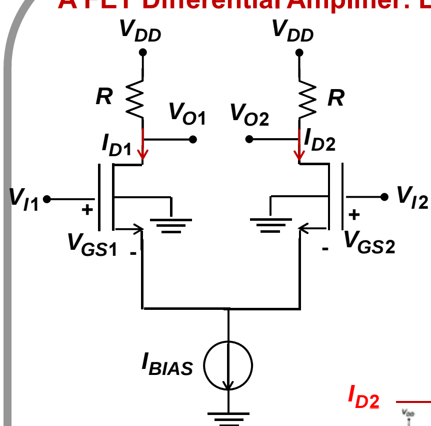
$$-\sqrt{2\frac{I_{BIAS}}{k_n}} \le V_{ID} \le \sqrt{2\frac{I_{BIAS}}{k_n}}$$

$$\Rightarrow I_{D1} = \frac{k_n}{2}\left[(V_{GS1} - V_{TN})^2\right] = \frac{I_{BIAS}}{2} + k_n \frac{V_{ID}}{4}\sqrt{4\frac{I_{BIAS}}{k_n} - V_{ID}^2}$$

$$\Rightarrow I_{D2} = \frac{k_n}{2}\left[(V_{GS2} - V_{TN})^2\right] = \frac{I_{BIAS}}{2} - k_n \frac{V_{ID}}{4}\sqrt{4\frac{I_{BIAS}}{k_n} - V_{ID}^2}$$

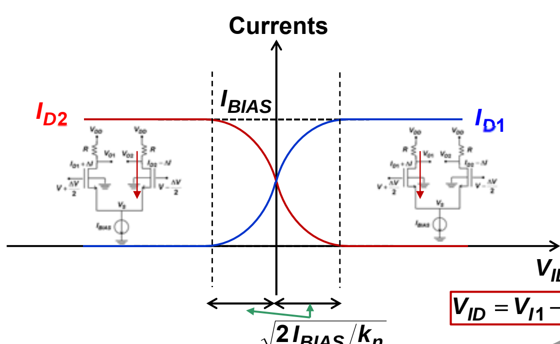
Drain currents depend only on the difference-mode input signal

A FET Differential Amplifier: Large Signal Analysis (Assume λ_n=0)



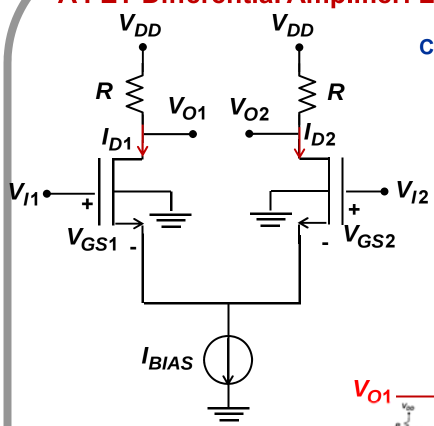
$$\Rightarrow I_{D1} = \frac{I_{BIAS}}{2} + k_n \frac{V_{ID}}{4}\sqrt{4\frac{I_{BIAS}}{k_n} - V_{ID}^2}$$

$$\Rightarrow I_{D2} = \frac{I_{BIAS}}{2} - k_n \frac{V_{ID}}{4}\sqrt{4\frac{I_{BIAS}}{k_n} - V_{ID}^2}$$



$V_{ID} = V_{I1} - V_{I2}$

A FET Differential Amplifier: Large Signal Analysis (Assume λ_n=0)



Currents:

$$I_{D1} = \frac{I_{BIAS}}{2} + k_n \frac{V_{ID}}{4}\sqrt{4\frac{I_{BIAS}}{k_n} - V_{ID}^2}$$

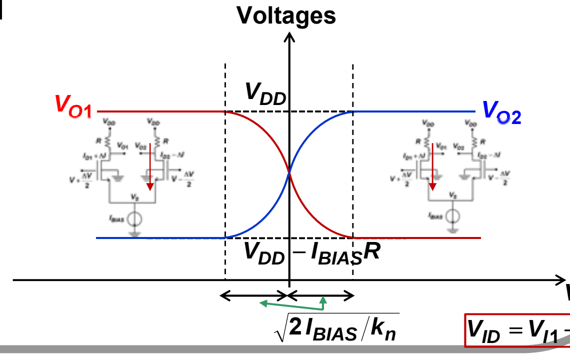
$$I_{D2} = \frac{I_{BIAS}}{2} - k_n \frac{V_{ID}}{4}\sqrt{4\frac{I_{BIAS}}{k_n} - V_{ID}^2}$$

$$\left[-\sqrt{2\frac{I_{BIAS}}{k_n}} \le V_{ID} \le \sqrt{2\frac{I_{BIAS}}{k_n}}\right]$$

Voltages:

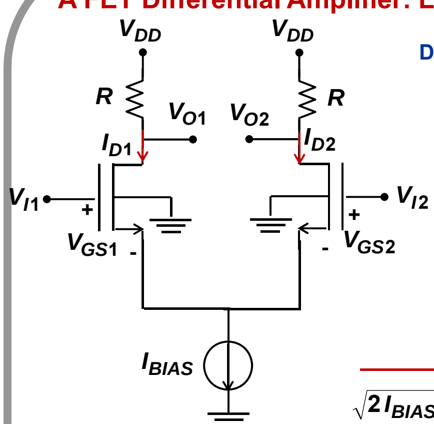
$$V_{O1} = V_{DD} - I_{D1}R$$

$$V_{O2} = V_{DD} - I_{D2}R$$



$V_{ID} = V_{I1} - V_{I2}$

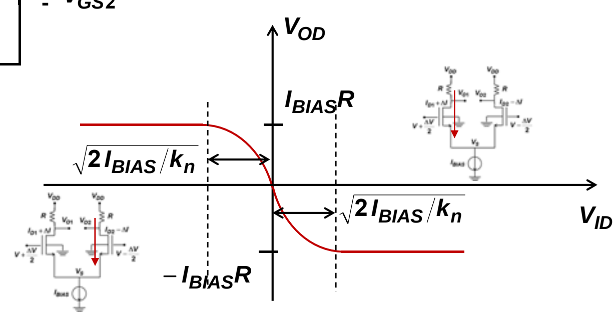
A FET Differential Amplifier: Large Signal Analysis (Assume λ_n=0)



Difference-Mode Output Voltage:

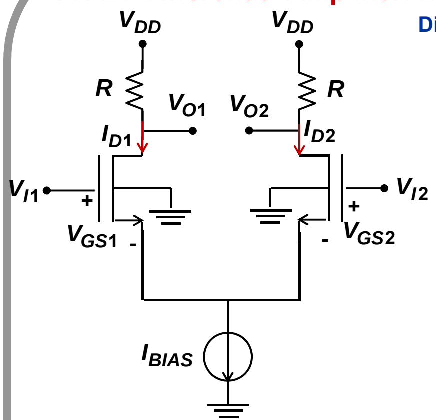
$$V_{OD} = V_{O1} - V_{O2} = -Rk_n \frac{V_{ID}}{2}\sqrt{4\frac{I_{BIAS}}{k_n} - V_{ID}^2}$$

$$\left[-\sqrt{2\frac{I_{BIAS}}{k_n}} \le V_{Id} \le \sqrt{2\frac{I_{BIAS}}{k_n}}\right]$$



The difference-mode output is sensitive to only the difference-mode input and not to the common-mode input

A FET Differential Amplifier: Large Signal Analysis (Assume λ_n=0)



Difference-Mode Output Voltage:

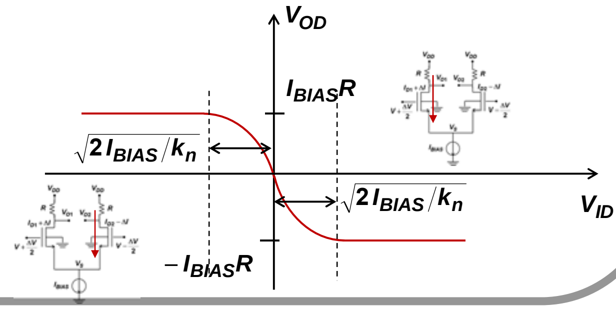
$$V_{OD} = -Rk_n \frac{V_{ID}}{2}\sqrt{4\frac{I_{BIAS}}{k_n} - V_{ID}^2}$$

small

$$\approx -Rk_n \frac{V_{ID}}{2}\sqrt{4\frac{I_{BIAS}}{k_n}}$$

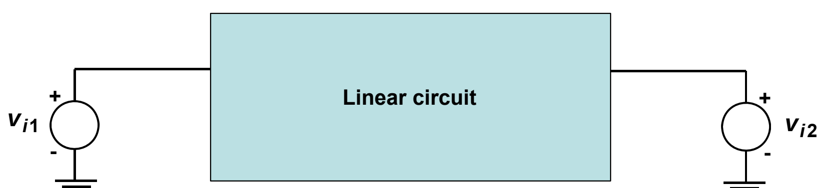
$$\approx -(g_m R)V_{ID}$$

$$\Rightarrow \frac{V_{OD}}{V_{ID}} \approx -g_m R$$

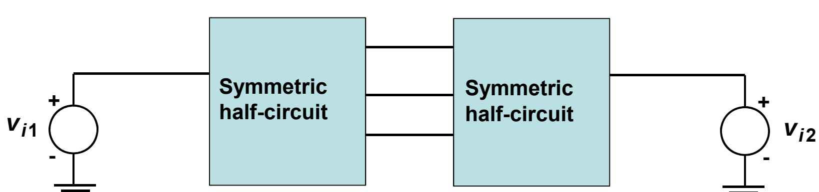


Half-Circuit Techniques

Consider the following linear circuit



Suppose the circuit consists of identical parts that can be separated into two symmetric half-circuits as shown:



Half-Circuit Techniques

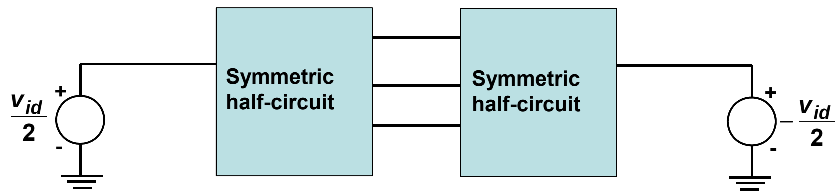
Since the circuit is linear, and superposition will hold, one can decompose the input signals into difference-mode and common-mode signals and then separately consider the circuit response to each signal

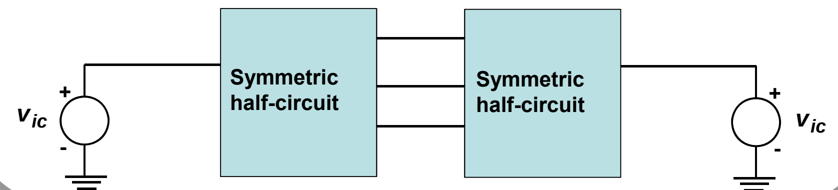
$$v_{id} = v_{i1} - v_{i2}$$

$$v_{ic} = \frac{v_{i1} + v_{i2}}{2}$$

$$v_{i1} = v_{ic} + \frac{v_{id}}{2}$$

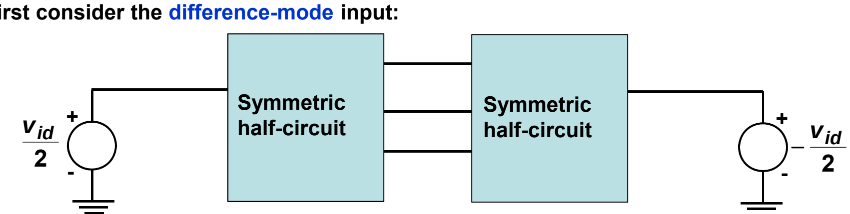
$$v_{i2} = v_{ic} - \frac{v_{id}}{2}$$





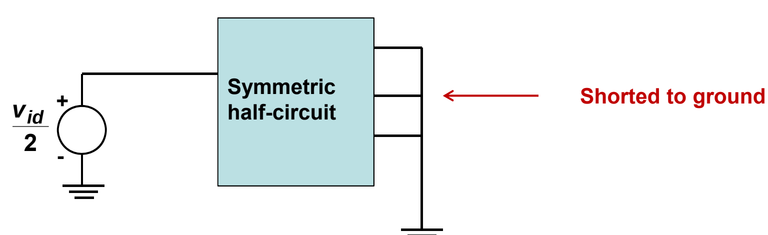
Half-Circuit Techniques: Difference-Mode Input

First consider the difference-mode input:



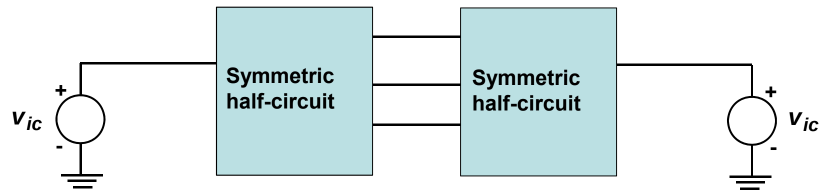
Because of symmetry, the nodes in the center will be at zero potential

Therefore, one can use the following half-circuit to perform the analysis:



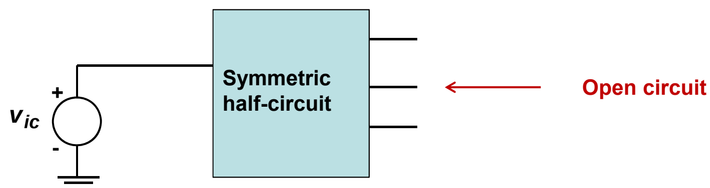
Half-Circuit Techniques: Difference-Mode Input

Now consider the **common-mode** input:

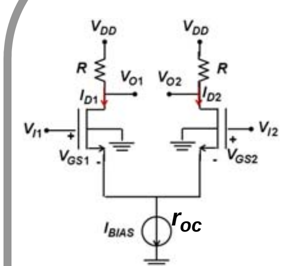


Because of symmetry, the wires in the center will carry no current

Therefore, one can use the following half-circuit to perform the analysis:



A FET Differential Amplifier: Small Signal Analysis



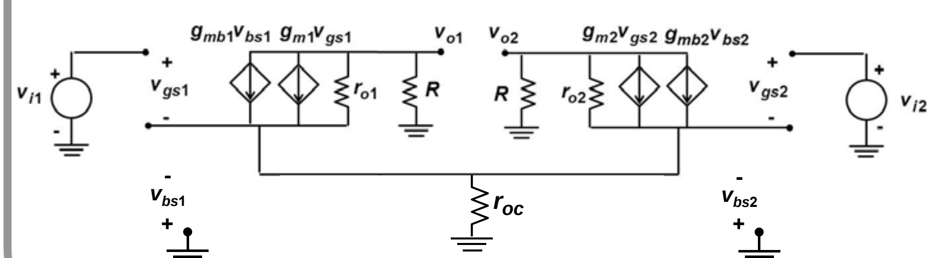
DC Bias:

$$V_{I1} = V_{I2}$$

$$I_{D1} = I_{D2} = \frac{I_{BIAS}}{2}$$

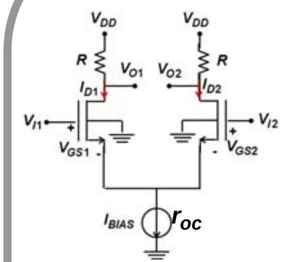
The small signal model can be built using the standard techniques

The small signal circuit models are always **linear**



Does this circuit consist of two identical and symmetric halves??

A FET Differential Amplifier: Small Signal Analysis



DC Bias:

$$V_{I1} = V_{I2}$$

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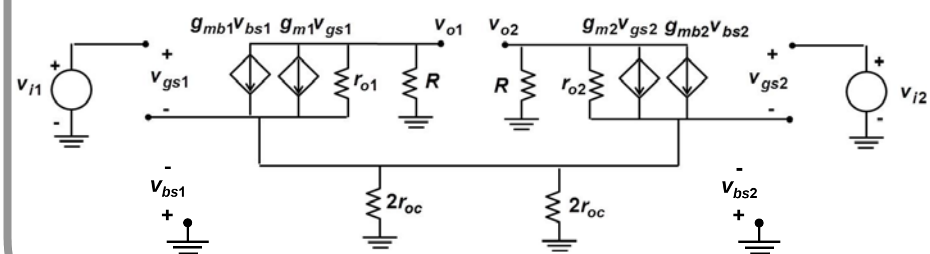
The small signal model can be built using the standard techniques

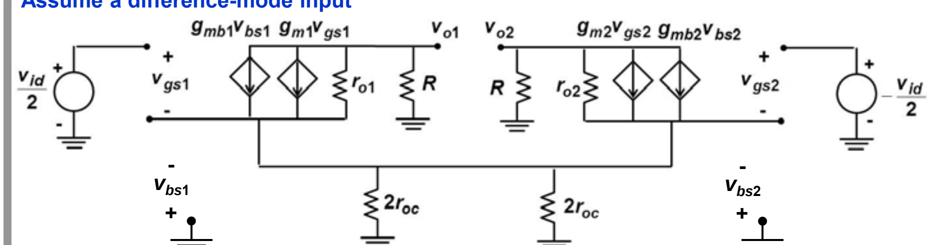
The small signal circuit models are always linear



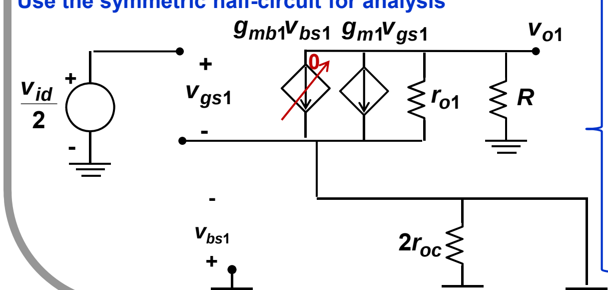
Now it does have two identical and symmetric halves!

A FET Differential Amplifier: Small Signal Analysis for Difference-Mode Input

Assume a difference-mode input



Use the symmetric half-circuit for analysis



$$v_{o1} = -g_{m1}(r_{o1} \,||\, R)\frac{v_{id}}{2}$$

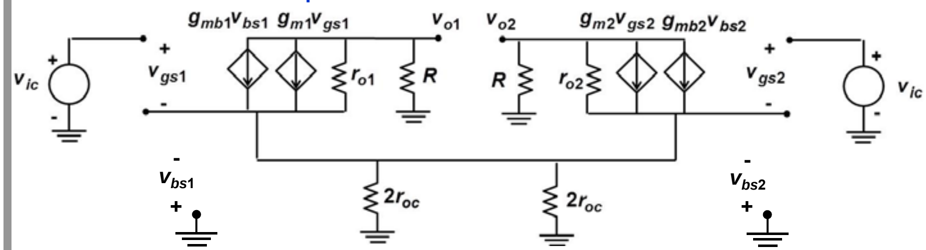
$$\Rightarrow v_{o2} = g_{m1}(r_{o1} \,||\, R)\frac{v_{id}}{2}$$

$$v_{od} = v_{o1} - v_{o2}$$
$$= -g_{m1}(r_{o1} \,||\, R)v_{id}$$

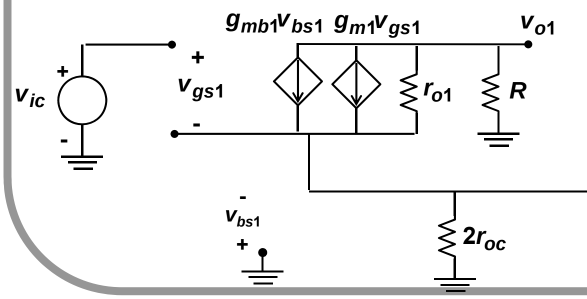
$$\Rightarrow A_{vd} = \frac{v_{od}}{v_{id}} = -g_{m1}(r_{o1} \,||\, R)$$

A FET Differential Amplifier: Small Signal Analysis for Common-Mode Input

Assume a common-mode input



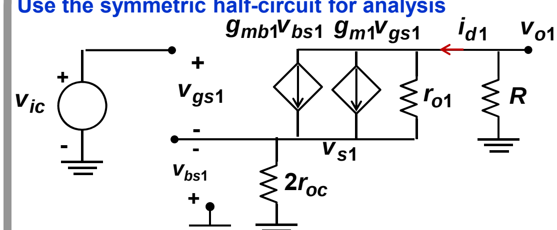
Use the symmetric half-circuit for analysis



This is like the small signal circuit of a CS amplifier with source degeneration.....!

A FET Differential Amplifier: Small Signal Analysis for Common-Mode Input

Use the symmetric half-circuit for analysis



$$v_{oc} = \frac{v_{o1} + v_{o2}}{2}$$

$$v_{o2} = v_{o1}$$

$$\Rightarrow v_{oc} = v_{o1}$$

$$\Rightarrow A_{vc} = \frac{v_{oc}}{v_{ic}} = \frac{v_{o1}}{v_{ic}}$$

$$v_{ic} = v_{gs1} + v_{s1} = v_{gs1} + i_{d1}(2r_{oc})$$

$$i_{d1} = g_{m1}v_{gs1} - g_{mb1}v_{s1} + \frac{v_{o1} - v_{s1}}{r_{o1}} =$$

$$= g_{m1}(v_{ic} - i_{d1}(2r_{oc})) - g_{mb1}i_{d1}(2r_{oc}) - \frac{i_{d1}R + i_{d1}(2r_{oc})}{r_{o1}}$$

$$\Rightarrow i_{d1} = \frac{g_{m1}r_{o1}}{r_{o1} + R + 2r_{oc} + (g_{m1} + g_{mb1})r_{o1}(2r_{oc})} v_{ic}$$

$$\Rightarrow A_{vc} = \frac{v_{o1}}{v_{ic}} = \frac{-i_{d1}R}{v_{ic}} = -\frac{g_{m1}r_{o1}R}{r_{o1} + R + 2r_{oc} + (g_{m1} + g_{mb1})r_{o1}(2r_{oc})} \longrightarrow 0 \text{ if } r_{oc} = \infty$$

A FET Differential Amplifier: CMRR

Difference-Mode Gain:

$$A_{vd} = \frac{v_{od}}{v_{id}} = -g_{m1}(r_{o1} \,||\, R)$$

Common-Mode Gain:

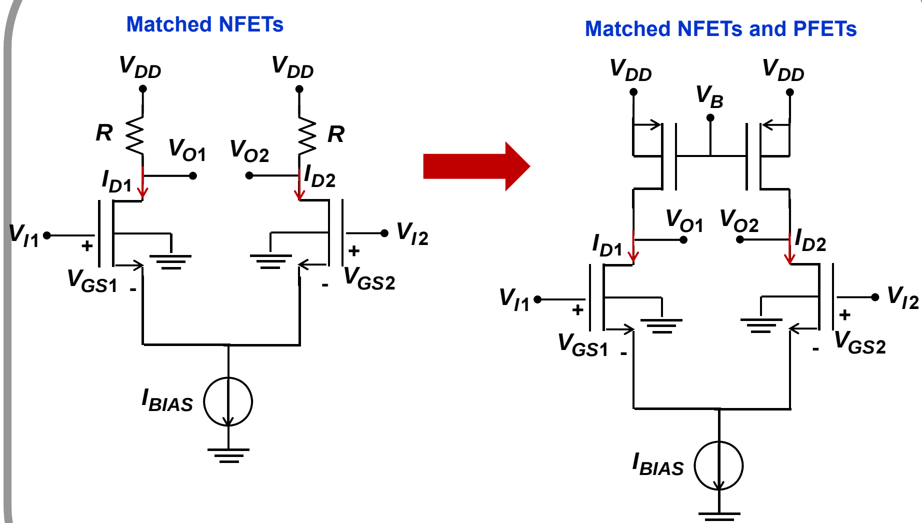
$$A_{vc} = \frac{v_{oc}}{v_{ic}} = -\frac{g_{m1} r_{o1} R}{r_{o1} + R + 2r_{oc} + (g_{m1} + g_{mb1}) r_{o1} (2r_{oc})} = -\frac{g_{m1}(r_{o1} \,||\, R)}{1 + \frac{2r_{oc}}{r_{o1}+R}\left[1 + (g_{m1} + g_{mb1}) r_{o1}\right]}$$

Common-Mode Rejection Ratio (CMRR):

$$CMRR = \frac{A_{vd}}{A_{vc}} = 1 + \frac{2r_{oc}}{r_{o1}+R}\left[1 + (g_{m1} + g_{mb1}) r_{o1}\right]$$

Large if r_{oc} is large

A Better FET Differential Amplifier: PFET Loads



A Better FET Differential Amplifier: PFET Loads