In this lecture you will learn:

- General amplifier concepts (in terms of the two-port models)
- Common source amplifier (CS)
- Small signal models of amplifiers

Reminder: Thevenin and Norton Equivalent Circuits

Consider an arbitrary linear circuit:

It can be represented by any two of the following equivalent circuits:

\[ V_{TH} = I_{TH}R_{TH} \]
Two-Port Amplifier Models: A Voltage Amplifier

A Voltage Amplifier:

\[
\frac{v_{out}}{v_{in}} = A_v \left( \frac{R_{in}}{R_{in} + R_S} \right) \left( \frac{R_L}{R_{out} + R_L} \right)
\]

Requirements:
- Large input resistance \( R_{in} \)
- Small output resistance \( R_{out} \)

Open circuit output voltage gain (i.e. when \( R_L = \infty \)):

\[
\frac{v_{out}}{v_{in}} = A_v = \text{Voltage gain}
\]

Two-Port Amplifier Models: A Current Amplifier

A Current Amplifier:

\[
\frac{i_{out}}{i_{in}} = A_i \left( \frac{R_S}{R_S + R_{in}} \right) \left( \frac{R_{out}}{R_{out} + R_L} \right)
\]

Requirements:
- Small input resistance \( R_{in} \)
- Large output resistance \( R_{out} \)

Short circuit output current gain (i.e. when \( R_L = 0 \)):

\[
\frac{i_{out}}{i_{in}} = A_i = \text{Current gain}
\]
Two-Port Amplifier Models: A Transconductance Amplifier

A Transconductance Amplifier (or a Voltage-to-Current Amplifier):

\[
\frac{i_{out}}{v_s} = G_m \left( \frac{R_{in}}{R_S + R_{in}} \right) \left( \frac{R_{out}}{R_{out} + R_L} \right)
\]

**Requirements:**
- Large input resistance \( R_{in} \)
- Large output resistance \( R_{out} \)

Short circuit output current and transconductance gain (i.e. when \( R_L = 0 \)):

\[
\frac{i_{out}}{v_{in}} = G_m = \text{Transconductance gain}
\]

Two-Port Amplifier Models: A Transimpedance Amplifier

A Transimpedance (or a Transresistance) Amplifier (or a Current-to-Voltage Amplifier):

\[
\frac{v_{out}}{i_s} = R_m \left( \frac{R_S}{R_S + R_{in}} \right) \left( \frac{R_L}{R_{out} + R_L} \right)
\]

**Requirements:**
- Small input resistance \( R_{in} \)
- Small output resistance \( R_{out} \)

Open circuit output voltage and transimpedance gain (i.e. when \( R_L = \infty \)):

\[
\frac{v_{out}}{i_{in}} = R_m = \text{Transimpedance gain}
\]
Two-Port Amplifier Models: General Concepts

The two-port models are equivalent (inter-convertible)

The designation of an amplifier as a voltage, current, transconductance, or transimpedance amplifier depends on the values of the input and output resistances

Need to find the input resistance, output resistance, open circuit voltage gain, and short circuit current gain to characterize an amplifier

Unilateral Networks and Two-Port Amplifier Models

For many circuits and amplifiers, the kind of two-port models described here are not strictly valid

Reasons:
The input resistance $R_{in}$ can depend on the load resistance $R_L$
The output resistance $R_{out}$ can depend on the source resistance $R_s$

Circuits in which the above does not happen, and for which the two-port models described here are strictly valid, are unilateral

In many cases, even for non-unilateral networks, two-port models described here tend to be good approximations for hand-calculations
Example: A Two-Port Model for a Non-Unilateral Network

Consider the two-port model shown below:

One can write:

$$\begin{bmatrix} i_{in} \\ i_{out} \end{bmatrix} = \begin{bmatrix} 1 & \frac{A_a}{R_i} \\ \frac{A_b}{R_b} & 1 \end{bmatrix} \begin{bmatrix} v_{in} \\ v_{out} \end{bmatrix}$$

It is not difficult to show that the input resistance, calculated as,

$$R_{in} = \frac{v_{in}}{i_{in}}$$

will depend on what load is connected at the output terminals.

The Common Source Amplifier

The source terminal is “common” between the input and the output.
The Common Source Amplifier

DC Bias Analysis (Large Signal Analysis):
Make sure the output load resistance $R_L$ is included in the DC bias analysis

Start by assuming the FET is in saturation (and then later verify):

$$V_{DD} - (I_{OUT} + I_D)R = V_{OUT}$$
$$\Rightarrow V_{DD} - \left(\frac{V_{OUT}}{R_L} + I_D\right)R = V_{OUT}$$
$$\Rightarrow V_{OUT} = (V_{DD} - I_D R) \frac{R_L}{R + R_L}$$

$$I_D = \frac{k_n}{2} (V_{BIAS} - V_{TN})^2 (1 + \lambda_n V_{OUT})$$

The Common Source Amplifier: Small Signal Model

Compare with the standard voltage amplifier model:
The Common Source Amplifier: Small Signal Model

Compare with the standard transconductance amplifier model:

The Common Source Amplifier: Open Circuit Voltage Gain

Open circuit voltage gain and transimpedance gain:
To find the open circuit voltage gain or the transimpedance gain one must:

i) Remove the load resistance $R_L$ at the output that the circuit will drive
ii) Then apply a test voltage source at the input
iii) Then find the resulting open circuit output voltage
iv) Take the ratio of the output voltage and the input voltage to find the open circuit voltage gain:
\[ A_v = \frac{V_{out}}{V_{in}} = -\frac{i_d R}{V_{in}} = -g_m (r_o || R) \]
v) Or take the ratio of the output voltage and the input current to find the transimpedance gain:
\[ R_m = \frac{V_{out}}{I_{in}} = -\infty \] This result is somewhat artificial since at non-zero frequencies there will be a finite input current due to capacitances.
The Common Source Amplifier: Short Circuit Current Gain

Short circuit current gain and transconductance gain:
To find the short circuit current gain or the transconductance gain one must:

i) Short the load resistance $R_L$ at the output that the circuit will drive

ii) Then apply a test voltage source at the input

iii) Then find the resulting current at the shorted output

iv) Take the ratio of the output and the input currents to find the short circuit current gain:

$$ A_j = \frac{i_{out}}{i_{g}} = -\frac{g_m v_{gs}}{0} = \infty $$

This result is somewhat artificial since at non-zero frequencies there will be a finite input current due to capacitances

v) Or take the ratio of the output current and the input voltage to find the transconductance gain:

$$ G_m = \frac{i_{out}}{v_{in}} = -\frac{g_m v_{gs}}{v_{in}} = -g_m $$

The Common Source Amplifier: Input Resistance

Input resistance:
To find the input resistance one must:

i) Make sure the load resistance $R_L$ that the circuit will drive is in place at the output

ii) Then apply a test voltage source at the input

iii) Then find the resulting current at the input

iv) Then take the ratio of the input voltage and the input current

$$ R_{in} = \frac{v_{in}}{i_{g}} = \infty $$

This result is somewhat artificial since at non-zero frequencies there will be a finite input current due to capacitances
Analysis shows that CS (like CE) is a good transconductance amplifier!

Resistance looking into the drain end of a FET:

\[ R_{D} = r_{o} \]

Now we can use the standard expression:

\[ \frac{v_{out}}{v_{s}} = A_{v}\left(\frac{R_{in}}{R_{in}+R_{S}}\right)\left(\frac{R_{L}}{R_{out}+R_{L}}\right) = -g_{m}\left(r_{o} \parallel R\right)\left(\frac{R_{L}}{r_{o} \parallel R\parallel R_{L}}\right) \]
The Common Source Amplifier: Complete Two-Port Model

Compare with the standard transconductance amplifier model:

\[ R_{in} = \infty \]
\[ G_m = -g_m \]
\[ R_{out} = (r_o \parallel R) \]

Now we can use the standard expression:

\[ \frac{i_{out}}{v_s} = G_m \left( \frac{R_{in}}{R_{in} + R_S} \right) \left( \frac{R_{out}}{R_{out} + R_L} \right) = -g_m \left( \frac{(r_o \parallel R)}{(r_o \parallel R) + R_L} \right) \]

The Common Source Amplifier with Source Degeneration

\[ V_{DD} - (I_{OUT} + I_D)R = V_{OUT} \]
\[ \Rightarrow V_{DD} - \left( \frac{V_{OUT}}{R_L} + I_D \right)R = V_{OUT} \]
\[ \Rightarrow V_{OUT} = \left( V_{DD} - I_D R \right) \frac{R_L}{R + R_L} \]

\[ I_D = \frac{k_n}{2} (V_{GS} - V_{TN})^2 \left( 1 + \lambda_n V_{OUT} \right) \]
\[ I_D = \frac{k_p}{2} (V_{GS} - V_{TN})^2 \]
\[ V_{GS} = V_{BIAS} - I_D R_E \]
CS Amplifier with Source Degeneration: Open Circuit Voltage Gain

Open circuit voltage gain:

\[ i_d = g_m v_{gs} + \frac{v_{out} - v_s}{r_o} = g_m (v_{in} - v_s) + \frac{v_{out} - v_s}{r_o} \]

\[ \Rightarrow i_d = g_m (v_{in} - i_d R_E) + \frac{-i_d R - i_d R_E}{r_o} = g_m v_{in} - i_d g_m R_E - i_d \frac{R + R_E}{r_o} \]

\[ \Rightarrow i_d = \frac{g_m}{1 + \frac{R + R_E}{r_o} + g_m R_E} v_{in} \]

\[ \Rightarrow A_v = \frac{v_{out}}{v_{in}} = \frac{-i_d R}{v_{in}} = \frac{g_m R r_o}{(r_o + R + R_E (1 + g_m r_o))} = \frac{g_m (r_o || R)}{1 + \frac{r_o}{(r_o + R)(1 + g_m r_o)}} \]

\[ \Rightarrow A_v \text{ is reduced when } R_E \text{ is present} \]

CS Amplifier with Source Degeneration: Transconductance Gain

Transconductance gain:

\[ i_d = g_m v_{gs} + \frac{v_{out} - v_s}{r_o} = g_m (v_{in} - v_s) + \frac{v_{out} - v_s}{r_o} \]

\[ \Rightarrow i_d = g_m (v_{in} - i_d R_E) + \frac{0 - i_d R_E}{r_o} = g_m v_{in} - i_d g_m R_E - i_d \frac{R_E}{r_o} \]

\[ \Rightarrow i_d = \frac{g_m}{1 + \frac{R_E}{r_o} + g_m R_E} v_{in} \]

\[ \Rightarrow G_m = \frac{i_{out}}{v_{in}} = \frac{-i_d g_m R_o}{v_{in}} = -\frac{g_m R_o}{(r_o + R_E (1 + g_m R_o))} = \frac{g_m}{1 + \frac{R_E}{r_o} (1 + g_m R_o)} \]

\[ \Rightarrow G_m \text{ is reduced when } R_E \text{ is present} \]
CS Amplifier with Source Degeneration: Output Resistance

Output resistance:

\[ i_d = g_m v_{gs} + \frac{v_{test} - v_s}{r_o} = g_m (v_{in} - v_s) + \frac{v_{test} - v_s}{r_o} \]

\[ \Rightarrow i_d = g_m \left( i_g R_s - i_d R_E \right) + \frac{v_{test} - i_d R_E}{r_o} = -i_d \left( g_m + \frac{1}{r_o} \right) R_E + \frac{v_{test}}{r_o} \]

\[ \Rightarrow i_d = \frac{v_{test}}{r_o + R_E (1 + g_m r_o)} \]

\[ \Rightarrow R_D = \frac{v_{test}}{i_d} = R_o + R_E (1 + g_m r_o) \quad \text{(} R_g \text{ is increased when } R_E \text{ is present)} \]

\[ \Rightarrow R_{out} = \frac{v_{test}}{i_{test}} = \frac{R \parallel R_D}{r_o + R + R_E (1 + g_m r_o)} \]

Relations to Remember

For any small signal amplifier model, the following always hold:

(Transconductance) X (Output resistance) = (Open circuit voltage gain)

(Transimpedance) / (Output resistance) = (Short circuit current gain)

The above follows from the equivalent Thévenin and Norton models of the amplifier

***All quantities must be calculated assuming the same value of \( R_s \) (typically zero)***