10.1)

a) 

b) \( g_m = \sqrt{2k_I_D} = 0.0057 \)  \( \mu \)  \( I_D = 8 \times 10^{-5} \)  \( \mu \)

\( g_m \) is \(~70\) times larger than \( g_0 \).

c) \( \frac{V_{out}}{V_{oc}} + (V_{oc} - V_a) g_{o2} - g_{m2} V_a + V_{out} jw C_{gd2} = 0 \)

\[ \Rightarrow \frac{V_{out}}{V_a} = \frac{g_{m2} + jw C_{gd2}}{\frac{1}{\tau_{oc}} + g_{o2} + jw C_{gd2}} \]

\[ \frac{g_{m2}}{\frac{1}{\tau_{oc}} + g_{o2} + jw C_{gd2}} = \frac{g_{m2} (\tau_{oc} || r_{o2})}{1 + jw C_{gd2} (\tau_{oc} || r_{o2})} = H_2(w) \]

d) see attached plot.

e) \( V_a g_{o1} + g_{m1} V_{in} + jw C_{gd1} (V_a - V_{in}) + V_a jw C_{gs2} + g_{m2} V_a + (V_a - V_{out}) g_{o2} = 0 \)

\[ \Rightarrow \frac{V_a}{V_{in}} = \frac{-g_{m1} + jw C_{gd1}}{g_{o1} + g_{o2} + g_{m2} + jw (C_{gd1} + C_{gs2}) - g_{o2} H_2(w)} = H_1(w) \]

f) see attached plot.
g) \[ \frac{V_i - V_s}{R_s} + V_i j\omega C_{gs1} + (V_i - V_a) j\omega C_{gd1} = 0. \]

\[ \frac{V_i}{V_s} = \frac{1}{1 + j\omega (C_{gs1} + C_{gd1}) R_s - j\omega C_{gd1} R_s H_1(\omega)} = H_0(\omega). \]

h) See the attached plot.

i) \[ |H_2(\omega)|^2 \text{ has a } 3\text{dB freq of } \approx 2.5 \text{ GHz}. \]
\[ |H_1(\omega)|^2 \text{ has a } 3\text{dB freq of } \approx 1.58 \text{ GHz}. \]
\[ |H_0(\omega)|^2 \text{ has a } 3\text{dB freq of } \approx 1.29 \text{ GHz}. \]

So the first and the second stages limit the freq. bandwidth of the amplifier.

j) \[ \frac{V_{out}}{V_s} = \frac{V_{out}}{V_a} \cdot \frac{V_a}{V_i} \cdot \frac{V_i}{V_s} = H_2(\omega) H_1(\omega) H_0(\omega). = |H_T(\omega)|^2 \]

See the attached plot. The 3dB frequency for \[ |H_T(\omega)|^2 \text{ is } \approx 0.80 \text{ GHz}. \]

b) See the attached plot.

The 3dB freq. of the Common Source amp is \( \approx 0.18 \text{ GHz}. \)

The cascade has \( \approx 4.4 \) times larger 3dB frequency, compared to the Common Source.
1) Common drain does not suffer from the Miller effect or there is no gain provided by this stage.

\{ Cascode? \}

See (i)(j) for the pole frequencies of each curve.
\( |H_T(\omega)|^2 \)

\[ \text{dB} \]

\[ \text{Frequency (Hz)} \]

(Common Source)
Problem 9.2: (RF Amplifier/Filter in one)

Suppose:

\[ W/L = 8 \quad C_{gs} = 0.5 \text{ fF} \quad C_{gd} = 0.1 \text{ fF} \]

\[ \mu_n C_{ox} = 200 \mu A/V^2 \quad R_S = 10 \text{ k\Omega} \]

\[ \lambda_n = 0.04 \text{ V} \quad L = 25 \text{ nH} \quad C = ? \text{ pF} \]

\[ V_{DD} = 3.5 \quad V_{BIAS} = 2.5 \quad V_TN = 0.5 \text{ V} \]

a) What ought to be the value of the capacitor \( C \) so that the maximum gain occurs at \( \approx 1 \text{ GHz} \)?

Impedance of the LC circuit on top is:

\[ Z(\omega) = \frac{j\omega L}{(j\omega)^2 LC + 1} \]

Impedance is maximum when:

\[ \omega = \frac{1}{\sqrt{LC}} \]

If one wants \( \frac{1}{\sqrt{LC}} = 1 \text{ GHz} \), then \( C \approx 1 \text{ pF} \).

b) What is the impedance \( Z(\omega) \) of the load sitting on top of the FET?

\[ Z(\omega) = \frac{j\omega L}{(j\omega)^2 LC + 1} \]

c) Draw a small signal model of the circuit. Use the high frequency small signal model for the FET. Represent the LC load by its impedance \( Z(\omega) \).

d) Find the small signal voltage gain of the amplifier:

\[ A_v(\omega) = \frac{v_{out}(\omega)}{v_s(\omega)} \]

Proceed as follows:

i) Do a KVL at the output node to get:
\[
\frac{v_{out}(\omega)}{v_{in}(\omega)} = \frac{-g_mZ - j\omega C_{gd}}{1 + g_mZ + j\omega C_{gd}} = -g_m(Z \parallel r_o) - j\omega C_{gd} g_m 1 - j\omega C_{gd} (Z \parallel r_o) = -h_1(\omega)
\]

ii) Then do a KCL at the input node to get:
\[
v_{in}(\omega) \approx \frac{1}{1 + j\omega (C_{gs} + C_{gd} (1 + h_1(\omega)))} R_s
\]
\[
A_v(\omega) = \frac{v_{out}(\omega)}{v_{in}(\omega)} = \frac{v_{out}(\omega)}{v_{in}(\omega) v_{in}(\omega)}
\]

e) Plot the gain (in dB units: i.e. \(20 \log_{10}|Av|\)) as a function of frequency from 10 MHz to 100 GHz. Use matlab or your favorite plotting program.

f) What is the maximum gain (in dB units) you see in your plot in part (e)?

Around 27.9 dB.

g) What is the full width half max (FWHM) gain bandwidth you see in your plot in part (e)?

Around 20 MHz.

(Part (h) has been made optional - Grader please note)

h) Derive an analytic expression for the FWHM gain bandwidth and show that it matches what you see in your plot.

Looking at the small signal model one can see that low frequencies when both \(C_{gd}\) and \(C_{gs}\) are open, the effective resistance that appears in parallel with the L and C is the output resistance \(r_o\) of the FET. Therefore, the amplifier gain is essentially (at these low frequencies),
\[
-g_m(Z \parallel r_o) = -g_m \frac{j\omega}{1} + \frac{1}{LC} + \frac{1}{r_o C} + (j\omega)^2
\]

The above looks like the expression for the response of a damped simple harmonic oscillator. The FWHM frequency bandwidth is \(\frac{1}{2\pi r_o C}\) (in Hz) and comes out to be very close to \(~20\) MHz.

i) A simple inductor loaded CS stage can also perform well at high frequencies compared to a resistively loaded CS stage in terms of gain and output voltage swing. Suppose in the problem, the LC circuit is replaced by just an inductor of inductance 25 nH. Plot the gain (in dB units: i.e. \(20 \log_{10}|Av|\)) as a function of frequency from 10 MHz to 100 GHz.

2
Now $Z(\omega) = j\omega L$. The plot is shown below.
Problem 10.3: (Folded Cascode Differential Amplifier)

a) 

The small signal circuit is shown above. Since each arm of the diff amp contains a cascode, the standard cascode analysis will apply. We first find $v_{o3}$ as a function of $-v_{id}/2$ input. Note that the left cascode is loaded on top with a very small resistance $1/g_{mn}$. So the current pulled in by the cascode (drain current of $M_3$) stage on the left side will be $-v_{id} g_{mn} /2$ and the voltage $v_{o3}$ will be $v_{id}/2$. Similarly, the drain current of $M_4$ will be $+v_{id} g_{mn} /2$. The cascode on the right side as well as the current source on the top on the right side pull current through the resistance $r_{on}$ (on the top right side).

$$v_o = v_{o4} \approx -g_{mn}v_{o3}r_{on} - g_{mn}r_{on} \frac{v_{id}}{2} = -g_{mn}r_{on}v_{id}$$

$$\Rightarrow A_{vd} = -g_{mn}r_{on}$$

b) 

Now the output is shorted to the ground. $v_{o3}$ will equal $v_{id}/2$, as in part (a) above. But on the right hand side, the current pulled in by the cascode and the current pulled in by the current source on the top come not through the resistance resistance $r_{on}$ (on the top right) but from the shorted output. So,
\[ i_{\text{out}} \approx -g_{mn}v_{o3} - g_{mn} \frac{v_{id}}{2} = -g_{mn}v_{id} \]

c) From parts (a) and (b), the output resistance (ratio of open circuit output voltage to the short circuit output current) is just resistance \( r_{on} \).