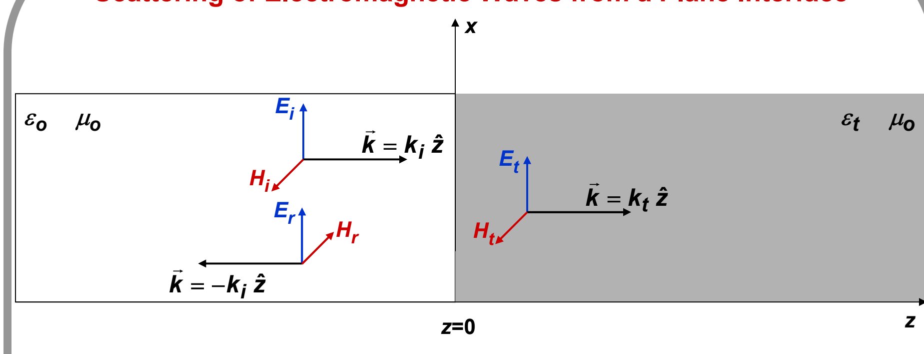
Lecture 34

Electromagnetic Scattering

In this lecture you will learn:

- Scattering of electromagnetic waves from objects
- Rayleigh Scattering
- Why the sky is blue
- Radar range equation

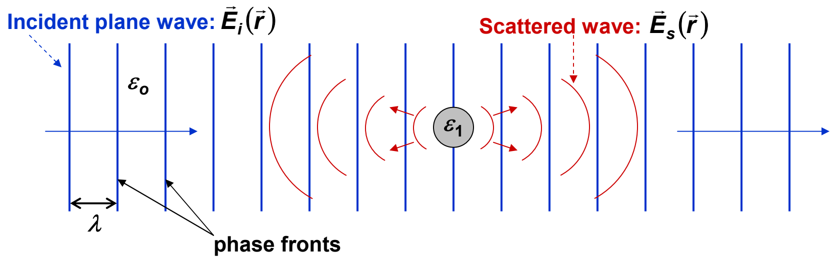
Scattering of Electromagnetic Waves from a Plane Interface



Incident, transmitted, and reflected waves are all plane waves

The reflected and transmitted waves can also be called the scattered waves

Scattering of Electromagnetic Waves from Objects



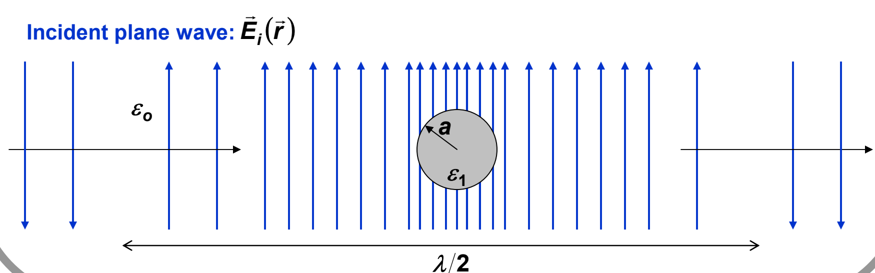
Questions:

- How does one find the scattered field?
- How much power from the incident field goes into the scattered field?
- In which direction(s) does the scattered power go?

Scattering of Electromagnetic Waves From Spherical Particles

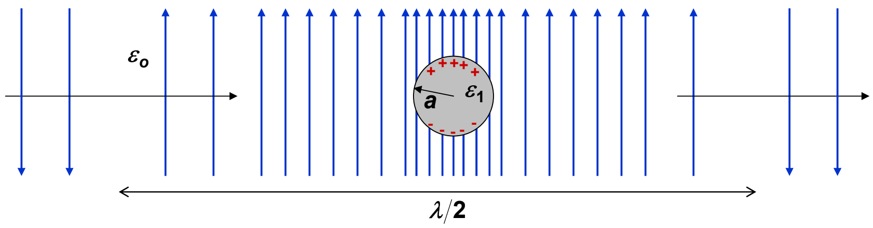
Assumption:

- Assume that the particle radius is much smaller than the wavelength of the incident wave, i.e.: $ka << 1$
  - When the above condition holds the scattering is called **"Rayleigh Scattering"**
  - When the above condition does not hold the scattering is called **"Mie Scattering"**
- When $ka << 1$, the particle sees a uniform E-field that is slowly oscillating in time



Rayleigh Scattering: Basic Mechanism

Incident plane wave: $\vec{E}_i(\vec{r})$



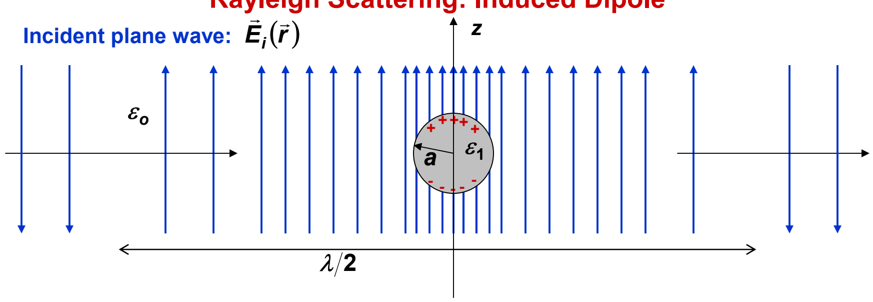
One way to understand scattering is as follows:

i) The incident E-field induces a time-varying dipole moment in the sphere (recall the electrostatics problem of a dielectric sphere in a uniform E-field from homework 3)

ii) The time-varying dipole radiates like a Hertzian dipole, and this radiation is the scattered radiation

Rayleigh Scattering: Induced Dipole

Incident plane wave: $\vec{E}_i(\vec{r})$



Suppose the z-directed E-field phasor for the incident plane wave at the location of the particle is: $\vec{E}(\vec{r}=0)=\hat{z}\,E_i$

From homework (3), the z-directed dipole moment p induced in a sphere in the presence of E-field E is:

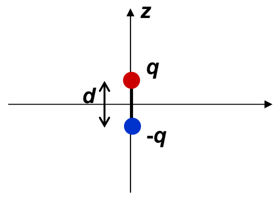
$$p = 4\pi\,\varepsilon_o a^3\left(\frac{\varepsilon_1-\varepsilon_o}{\varepsilon_1+2\,\varepsilon_o}\right)E$$

In the present case, the dipole moment phasor p induced in the sphere is therefore:

$$p = 4\pi\,\varepsilon_o a^3\left(\frac{\varepsilon_1-\varepsilon_o}{\varepsilon_1+2\,\varepsilon_o}\right)E_i$$

Rayleigh Scattering: Scattered Radiation

Hertzian Dipole



$$\vec{E}_{ff}(\vec{r}) = \hat{\theta}\, \frac{j\,\eta_o\, k\, Id}{4\pi\, r} \sin(\theta) e^{-jkr}$$

Dipole moment = $p = qd$

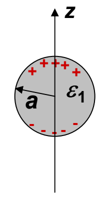
Current = $I = j\,\omega\, q$

$\Rightarrow Id = j\,\omega\, qd = j\,\omega\, p$

Therefore:

$$\vec{E}_{ff}(\vec{r}) = \hat{\theta}\, \frac{j\,\eta_o\, k\,(j\omega\, p)}{4\pi\, r} \sin(\theta) e^{-jkr}$$

Induced Dipole



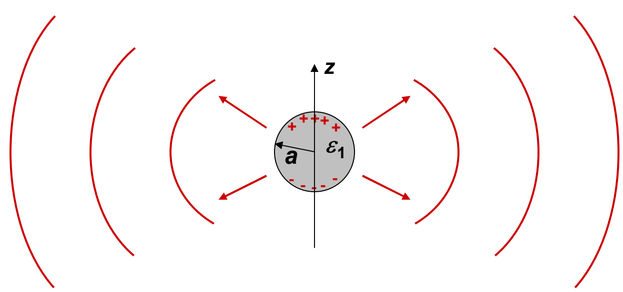
Dipole moment = $p = 4\pi\,\varepsilon_o a^3 \left(\frac{\varepsilon_1 - \varepsilon_o}{\varepsilon_1 + 2\,\varepsilon_o}\right) E_i$

The far-field scattered radiation is:

$$\vec{E}_{s-ff}(\vec{r}) = \hat{\theta}\, \frac{j\,\eta_o\, k\,(j\omega\, p)}{4\pi\, r} \sin(\theta) e^{-jkr}$$

$$= -\hat{\theta}\, \frac{k^2 a^3}{r} \left(\frac{\varepsilon_1 - \varepsilon_o}{\varepsilon_1 + 2\varepsilon_o}\right) E_i \sin(\theta) e^{-jkr}$$

Rayleigh Scattering: Total Scattered Power



$$P_s = \int_0^{2\pi}\int_0^{\pi} \frac{\left|\vec{E}_{s-ff}(\vec{r})\right|^2}{2\eta_o}\; r^2 \sin(\theta)\, d\theta\, d\phi$$

$$= \frac{4\pi}{3\eta_o} k^4 a^6 \left(\frac{\varepsilon_1 - \varepsilon_o}{\varepsilon_1 + 2\varepsilon_o}\right)^2 |E_i|^2$$

Rayleigh Scattering: Scattering Cross-Section

Total scattered power P_s from a dielectric sphere is:

$$P_s = \int_0^{2\pi}\int_0^{\pi} \frac{\left|\vec{E}_{s-ff}(\vec{r})\right|^2}{2\eta_o} r^2 \sin(\theta)\, d\theta\, d\phi = \frac{4\pi}{3\eta_o} k^4 a^6 \left(\frac{\varepsilon_1 - \varepsilon_o}{\varepsilon_1 + 2\varepsilon_o}\right)^2 |E_i|^2$$

The incident power per unit area was just the Poynting vector of the incident wave: $\dfrac{\left|\vec{E}_i(\vec{r})\right|^2}{2\eta_o}$

The scattering cross-section σ_s of a scatterer is defined as the area of a plane oriented perpendicular to the direction of incident wave that would intercept the same total incident power as the power P_s that the scatterer radiates

$$\sigma_s = \frac{P_s}{\left|\vec{E}_i(\vec{r})\right|^2 / 2\eta_o}$$

σ_s is also the ratio of the total scattered power to the power per unit area of the incident wave at the location of the scatterer

For the dielectric sphere:

$$\sigma_s = \frac{P_s}{\left|\vec{E}_i(\vec{r})\right|^2 / 2\eta_o} = \frac{8\pi}{3} k^4 a^6 \left(\frac{\varepsilon_1 - \varepsilon_o}{\varepsilon_1 + 2\varepsilon_o}\right)^2$$

Rayleigh Scattering: Why is the Sky Blue

The scattering cross-section of a dielectric sphere is:

$$\sigma_s = \frac{8\pi}{3} k^4 a^6 \left(\frac{\varepsilon_1 - \varepsilon_o}{\varepsilon_1 + 2\varepsilon_o}\right)^2$$

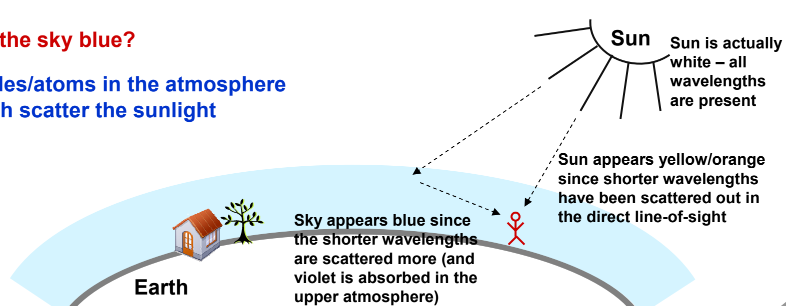
The scattered power is inversely proportional to the fourth power of the wavelength:

$$\sigma_s \propto k^4 \propto \frac{1}{\lambda^4}$$

Shorter wavelengths are scattered more than longer wavelengths in the Rayleigh limit

Why is the sky blue?

Molecules/atoms in the atmosphere Rayleigh scatter the sunlight



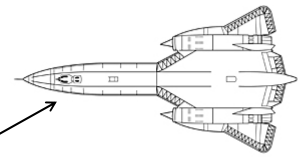
Example: Radar Range Equation

Power per unit area at the location of the target:

$$S_{target} = \frac{P_{in}}{4\pi r^2} G(\theta,\phi)$$

If the scattering cross-section of the target is σ_s then the total scattered power P_s is:

$$P_s = \sigma_s S_{target} = \sigma_s \frac{P_{in}}{4\pi r^2} G(\theta,\phi)$$



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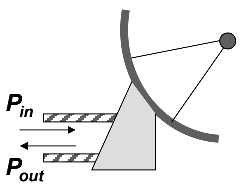
If the target scatters isotropically (equally in all directions) then the power P_{out} received by the matched antenna is:



Transmitting-receiving antenna (matched)

$$P_{out} = \eta_p \frac{P_s}{4\pi r^2} A(\theta,\phi)$$

$$= \eta_p \frac{P_{in}}{\left(4\pi r^2\right)^2} \sigma_s G(\theta,\phi) A(\theta,\phi)$$

Radar range equation

$$\boxed{\Rightarrow \frac{P_{out}}{P_{in}} = \eta_p \frac{\sigma_s G^2(\theta,\phi)}{\left(4\pi r^2\right)^2}\left(\frac{\lambda^2}{4\pi}\right) = \eta_p \frac{\sigma_s G^2(\theta,\phi)\lambda^2}{(4\pi)^3 r^4}}$$