

Lecture 33

Antenna Arrays and Phase Arrays

In this lecture you will learn:

- Antenna arrays
- Gain and radiation pattern for antenna arrays
- Antenna beam steering with phase array antennas

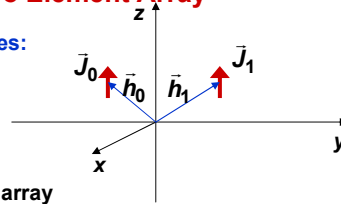
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Two Hertzian Dipoles – A Two Element Array

Consider first an array of just two Hertzian dipoles:

$$\bar{J}_0(\vec{r}) = \hat{z} I_0 d \delta^3(\vec{r} - \vec{h}_0)$$

$$\bar{J}_1(\vec{r}) = \hat{z} I_1 d \delta^3(\vec{r} - \vec{h}_1)$$



Each antenna in the array is an “element” of the array

One can write the E-field in the far-field as a superposition of the E-fields produced by all the elements in the array:

$$\begin{aligned} \bar{E}_{ff}(\vec{r}) &= \hat{\theta} \frac{j \eta_0 k d}{4\pi r} \sin(\theta) e^{-jk r} \left[I_0 e^{jk \hat{r} \cdot \vec{h}_0} + I_1 e^{jk \hat{r} \cdot \vec{h}_1} \right] \\ &= \underbrace{\hat{\theta} \frac{j \eta_0 k I_0 d}{4\pi r} \sin(\theta) e^{-jk r}}_{\text{Element Factor}} \underbrace{\left[\frac{I_0}{I_0} e^{jk \hat{r} \cdot \vec{h}_0} + \frac{I_1}{I_0} e^{jk \hat{r} \cdot \vec{h}_1} \right]}_{\text{Array Factor}} \\ &= \bar{E}(r, \theta, \phi) F(\theta, \phi) \end{aligned}$$

• The “element factor” is just the E-field produced by the first element if it were sitting at the origin

• The “array factor” captures all the interference effects

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An N-Element Antenna Array - I

Consider an N-element antenna array where the elements are:

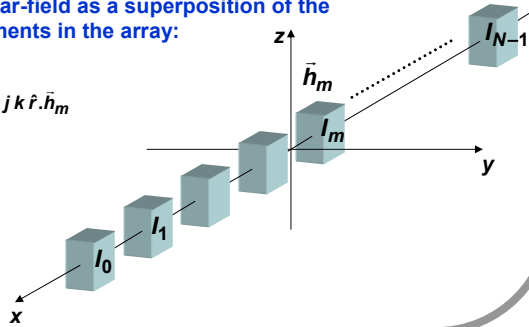
- all identical (all loops, or all Hertzian dipoles, or all Half-wave dipoles, etc)
- all oriented in the same way
- but with possibly different current phasors

Let the current phasor of the m -th antenna be I_m

Let the position vector of the m -th antenna be \vec{h}_m

One can write the E-field in the far-field as a superposition of the E-fields produced by all the elements in the array:

$$\begin{aligned}\vec{E}_{ff}(\vec{r}) &= \vec{E}(r, \theta, \phi) \sum_{m=0}^{N-1} \frac{I_m}{I_0} e^{jk\hat{r}\cdot\vec{h}_m} \\ &= \vec{E}(r, \theta, \phi) F(\theta, \phi)\end{aligned}$$



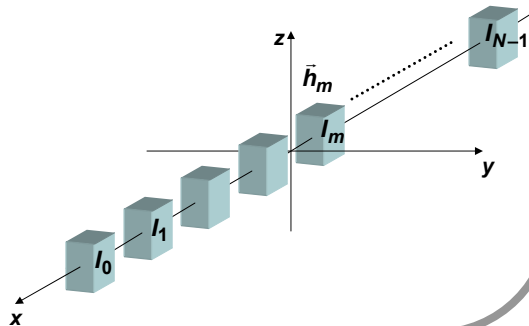
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An N-Element Antenna Array - II

$$\begin{aligned}\vec{E}_{ff}(\vec{r}) &= \vec{E}(\vec{r}) \sum_{m=0}^{N-1} \frac{I_m}{I_0} e^{jk\hat{r}\cdot\vec{h}_m} \\ &= \vec{E}(r, \theta, \phi) F(\theta, \phi)\end{aligned}$$

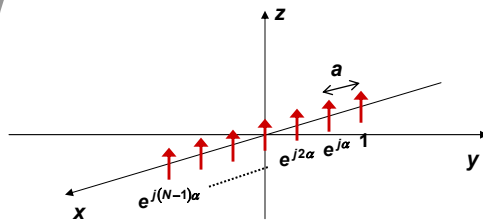
Gain: $G(\theta, \phi) = \frac{\langle \vec{S}(\vec{r}, t) \rangle \cdot \hat{r}}{P/4\pi r^2} \propto \left[\text{angular part of } |\vec{E}(r, \theta, \phi)|^2 \right] |F(\theta, \phi)|^2$

Pattern: $p(\theta, \phi) = \frac{G(\theta, \phi)}{G(\theta, \phi)_{\max}}$



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An N-Element Hertzian Dipole Phase Array



N-element Hertzian dipole array

- Current magnitude is the same for all the dipoles
- Current phase difference between successive dipoles is α

$$\frac{I_{m+1}}{I_m} = e^{j\alpha}$$

Element Factor:

$$\bar{E}(r, \theta, \phi) = \hat{\theta} \frac{j \eta_0 k I_0 d}{4\pi r} \sin(\theta) e^{-jk r}$$

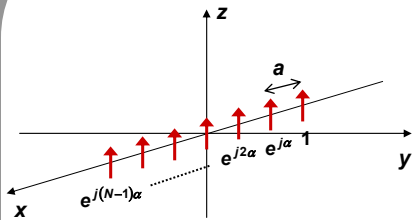
Array Factor:

$$F(\theta, \phi) = \sum_{m=0}^{N-1} \frac{I_m}{I_0} e^{jk \hat{r} \cdot \hat{h}_m} = e^{jk \hat{r} \cdot \hat{h}_0} \sum_{m=0}^{N-1} e^{j\alpha m} e^{jk a m \sin(\theta) \cos(\phi)}$$

$$\Rightarrow |F(\theta, \phi)|^2 = \left| \sum_{m=0}^{N-1} \left(e^{jk a \sin(\theta) \cos(\phi) + \alpha} \right)^m \right|^2 = \frac{\sin^2 \left[\frac{N}{2} (\alpha + ka \sin(\theta) \cos(\phi)) \right]}{\sin^2 \left[\frac{1}{2} (\alpha + ka \sin(\theta) \cos(\phi)) \right]}$$

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An N-Element Hertzian Dipole Phase Array



N-element Hertzian dipole phase array antenna

$$P = \frac{\eta_0}{12\pi} |k I_0 d|^2 N$$

$$G(\theta, \phi) = \frac{\langle \bar{S}(\vec{r}, t) \rangle \cdot \hat{r}}{P/4\pi r^2} = \frac{3}{2N} \sin^2(\theta) |F(\theta, \phi)|^2$$

Coming from the element factor array factor

Pattern:

$$\rho(\theta, \phi) = \frac{1}{N^2} \sin^2(\theta) |F(\theta, \phi)|^2$$

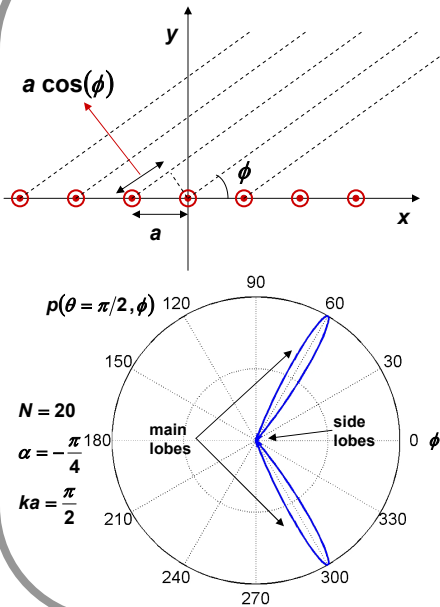
$$\rho(\theta, \phi) = \frac{1}{N^2} \sin^2(\theta) \frac{\sin^2 \left[\frac{N}{2} (\alpha + ka \sin(\theta) \cos(\phi)) \right]}{\sin^2 \left[\frac{1}{2} (\alpha + ka \sin(\theta) \cos(\phi)) \right]}$$

Coming from the element factor

array factor

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An N-Element Hertzian Dipole Phase Array: Maxima

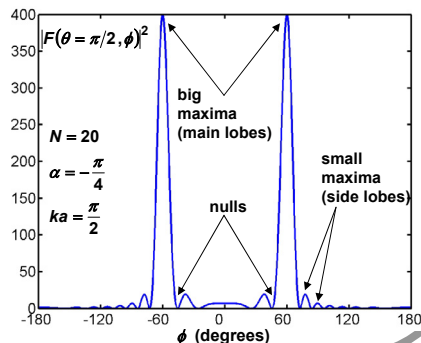


Lets look at radiation in the x-y plane:

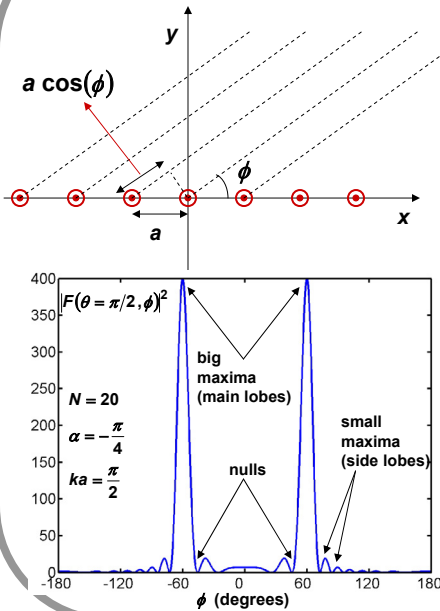
Radiation going in the ϕ direction from adjacent elements will add in-phase, and one will have a big maximum in the radiation pattern, provided:

$$k a \cos(\phi) + \alpha = \pm 2\pi n$$

$$\{ n = 0, 1, 2, 3, \dots \}$$



An N-Element Hertzian Dipole Phase Array: Maxima



Condition for a big maximum in the x-y plane:

$$k a \cos(\phi) + \alpha = \pm 2\pi n$$

$$\{ n = 0, 1, 2, 3, \dots \}$$

At a big maximum the value of the array factor is:

$$|F(\theta = \pi/2, \phi)|_{\max}^2 = \frac{\sin^2 \left[\frac{N}{2} (\alpha + k a \cos(\phi)) \right]}{\sin^2 \left[\frac{1}{2} (\alpha + k a \cos(\phi)) \right]}$$

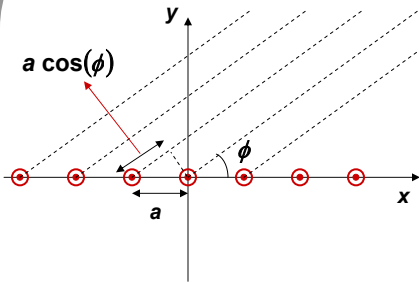
$$= N^2$$

And the value of the antenna gain is:

$$G(\theta, \phi) = \frac{3}{2} N \sin^2(\theta) |F(\theta, \phi)|^2$$

$$\Rightarrow G \left(\theta = \frac{\pi}{2}, \phi \right)_{\max} = \frac{3}{2} N$$

An N-Element Hertzian Dipole Phase Array: Nulls



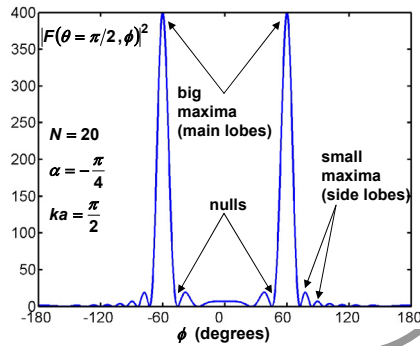
Radiation in the x-y plane:

The array factor will give a null in the radiation pattern provided:

$$k a \cos(\phi) + \alpha = \pm \frac{2\pi n}{N}$$

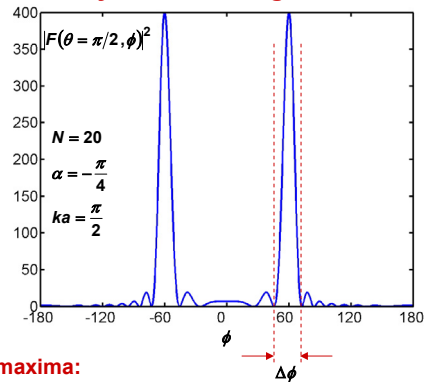
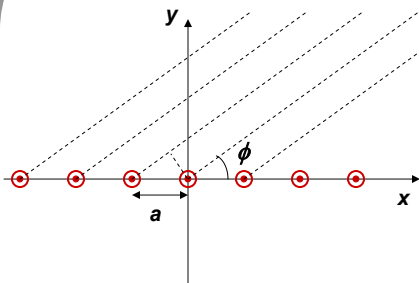
$$\{ n \neq 0, N, 2N, 3N, \dots \}$$

$$|F(\theta = \pi/2, \phi)|^2 = \frac{\sin^2 \left[\frac{N}{2} (\alpha + k a \cos(\phi)) \right]}{\sin^2 \left[\frac{1}{2} (\alpha + k a \cos(\phi)) \right]}$$



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An N-Element Hertzian Dipole Phase Array: Width of Big Maxima



Angular width of a lobe associated with a big maxima:

For a big maxima: $k a \cos(\phi) + \alpha = 2\pi n = 2\pi \frac{nN}{N}$ $\{ n = 0, \pm 1, \pm 2, \pm 3, \dots \}$

At the nulls nearest to the above maxima we must have:

$$k a \cos\left(\phi + \frac{\Delta\phi}{2}\right) + \alpha = 2\pi \frac{nN \pm 1}{N}$$

Which can be solved for $\Delta\phi$ - and for N large ($N \gg 1$) one gets approximately:

$$\Delta\phi \approx \left| \frac{4\pi}{N} \frac{1}{k a \sin(\phi)} \right|$$

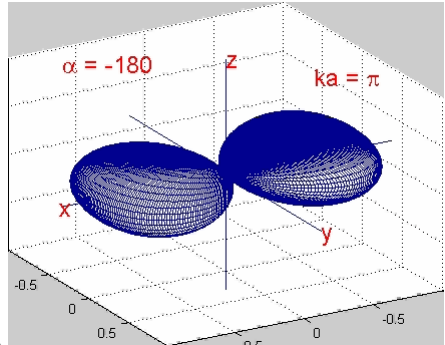
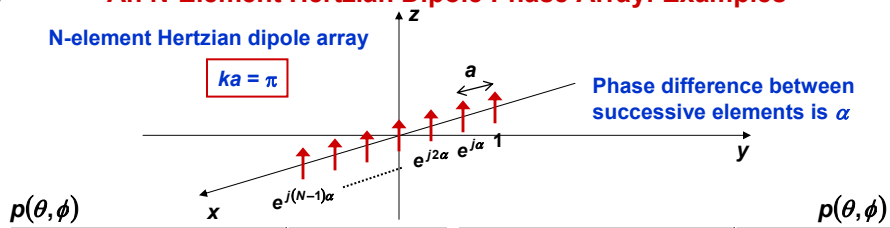
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An N-Element Hertzian Dipole Phase Array: Examples

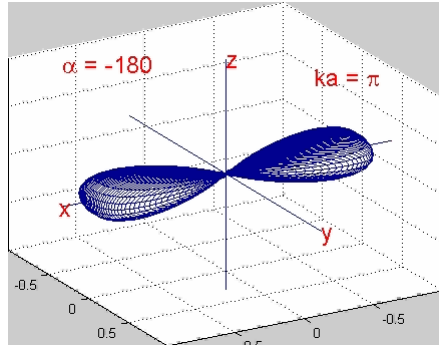
N-element Hertzian dipole array

$$ka = \pi$$

Phase difference between successive elements is α



$N = 2$



$N = 10$

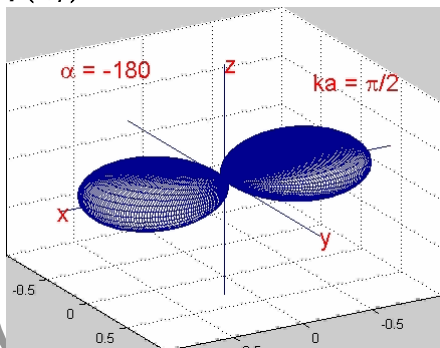
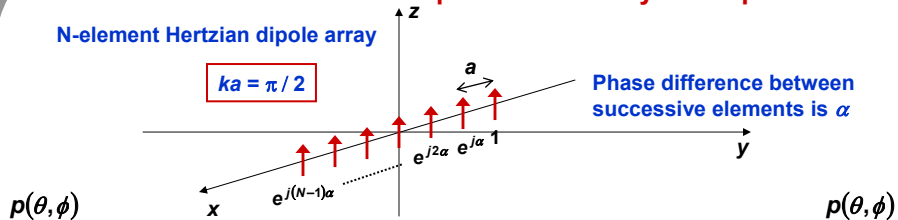
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An N-Element Hertzian Dipole Phase Array: Examples

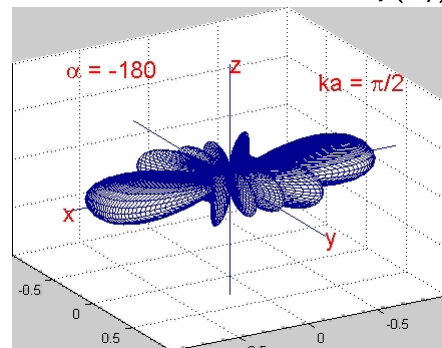
N-element Hertzian dipole array

$$ka = \pi/2$$

Phase difference between successive elements is α



$N = 2$



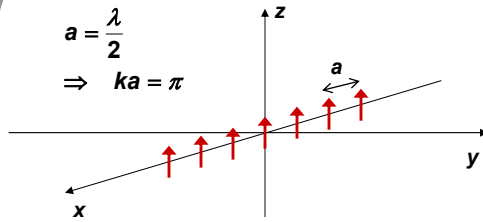
$N = 10$

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An (N+1)-Element Hertzian Dipole Binomial Array

$$a = \frac{\lambda}{2}$$

$$\Rightarrow ka = \pi$$



(N+1)-element Hertzian dipole array

- Current phase is the same for all the dipoles
- Currents magnitudes in the dipoles follow a binomial distribution

$$\frac{I_m}{I_0} = \frac{N!}{m!(N-m)!}$$

Element Factor:

$$\bar{E}(r, \theta, \phi) = \hat{\theta} \frac{j \eta_0 k I_0 d}{4\pi r} \sin(\theta) e^{-jk r}$$

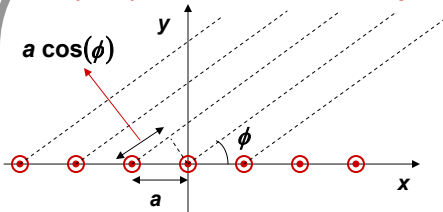
Array Factor:

$$F(\theta, \phi) = \sum_{m=0}^N \frac{I_m}{I_0} e^{jk r \cdot \hat{r}_m} = e^{jk r \cdot \hat{r}_0} \sum_{m=0}^N \frac{N!}{m!(N-m)!} e^{jk a m \sin(\theta) \cos(\phi)}$$

$$\Rightarrow |F(\theta, \phi)|^2 = \left| 1 + e^{jk a \sin(\theta) \cos(\phi)} \right|^{2N} = 2^{2N} \cos^{2N} \left[\frac{k}{2} a \sin(\theta) \cos(\phi) \right]$$

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An (N+1)-Element Hertzian Dipole Binomial Array: Maxima and Nulls



Lets look at radiation in the x-y plane:

Radiation going in the ϕ direction from adjacent elements will add in-phase, and one will have a big maximum in the radiation pattern, provided:

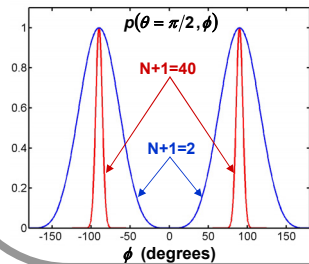
$$k a \cos(\phi) = \pm 2\pi n$$

$$\{ n = 0, 1, 2, 3, \dots \}$$

$$\Rightarrow \frac{\pi}{2} \cos(\phi) = \pm \pi n$$

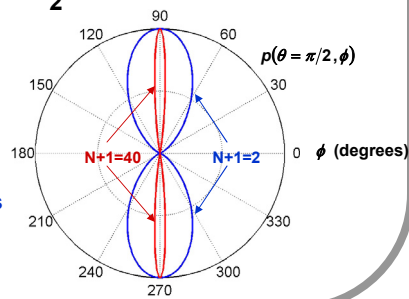
$$|F(\theta = \pi/2, \phi)|^2 = 2^{2N} \cos^{2N} \left[\frac{\pi}{2} \cos(\phi) \right]$$

$$p(\theta = \pi/2, \phi) = \cos^{2N} \left[\frac{\pi}{2} \cos(\phi) \right]$$



Note:

- No side lobes
- Two nulls at $\phi = 0, \pi$



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Applications of Antenna Arrays - I



Phased Array TRack to Intercept Of Target

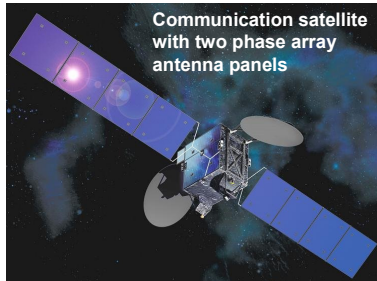


Alaska



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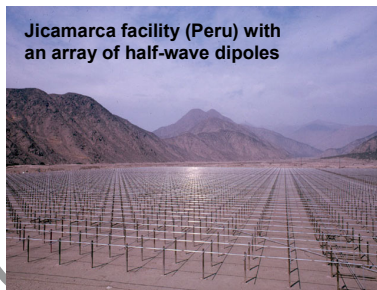
Applications of Antenna Arrays - II



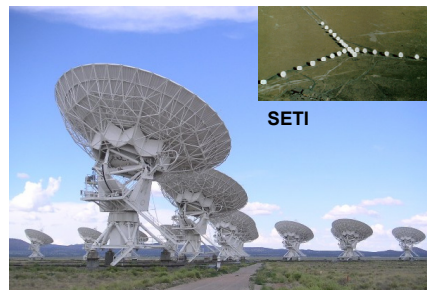
Communication satellite with two phase array antenna panels



MRI system with phase array RF receivers



Jicamarca facility (Peru) with an array of half-wave dipoles



SETI

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