Lecture 26

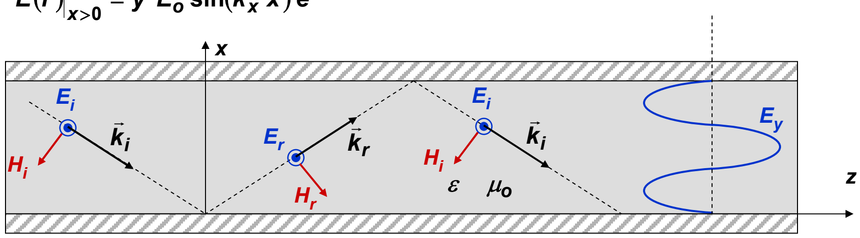
Dielectric Slab Waveguides

In this lecture you will learn:

- Dielectric slab waveguides
- TE and TM guided modes in dielectric slab waveguides

TE Guided Modes in Parallel-Plate Metal Waveguides

$$\vec{E}(\vec{r})\big|_{x>0} = \hat{y}\, E_o \sin(k_x\, x)\, e^{-j k_z z}$$



$$\vec{k}_i = -k_x \hat{x} + k_z \hat{z} \qquad \vec{k}_r = k_x \hat{x} + k_z \hat{z}$$

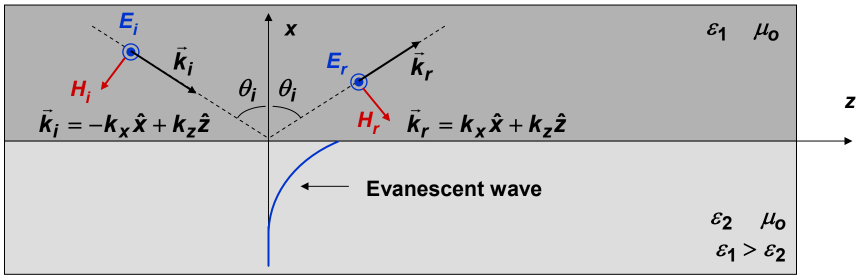
Guided TE modes are TE-waves bouncing back and fourth between two metal plates and propagating in the z-direction !

The x-component of the wavevector can have only discrete values – its quantized

$$k_x = \frac{m\pi}{d} \qquad \text{where : } m = 1, 2, 3, \ldots\ldots$$

Dielectric Waveguides - I

Consider TE-wave undergoing total internal reflection:



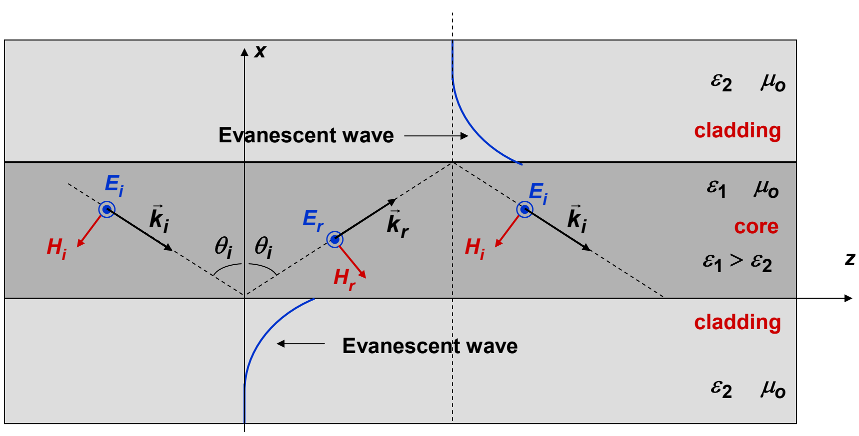
$$\left.\vec{E}(\vec{r})\right|_{x>0} = \hat{y}\ E_i\ e^{-j(-k_x x + k_z z)} + \hat{y}\ \Gamma E_i\ e^{-j(k_x x + k_z z)} \quad \Big\}\ k_z^2 + k_x^2 = \omega^2 \mu_o \varepsilon_1$$

$$|\Gamma| = 1 \quad \text{when} \quad \theta_i > \theta_c$$

When $\theta_i > \theta_c$:

$$\left.\vec{E}(\vec{r})\right|_{x<0} = \hat{y}\ T E_i\ e^{-j k_z z}\ e^{-\alpha_x |x|} \quad \Big\}\ \begin{array}{l} k_x = -j\alpha_x \\ k_z^2 - \alpha_x^2 = \omega^2 \mu_o \varepsilon_2 \end{array}$$

Dielectric Waveguides - II



One can have a guided wave that is bouncing between two dielectric interfaces due to total internal reflection and moving in the z-direction

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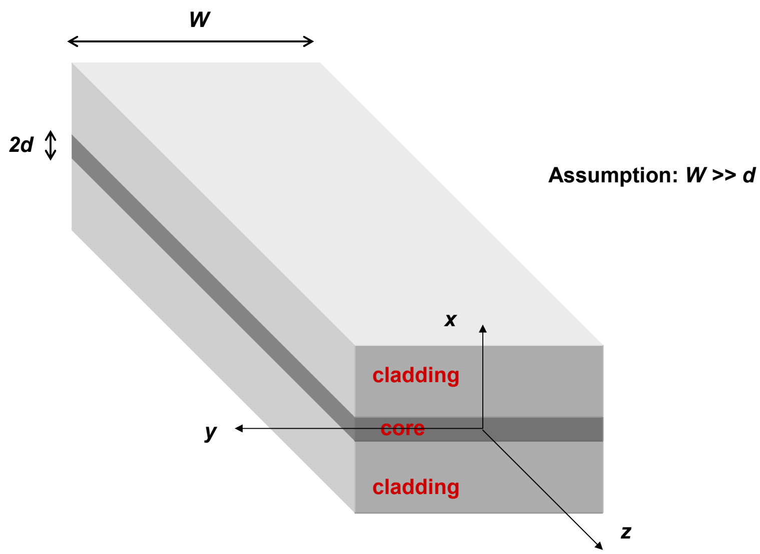
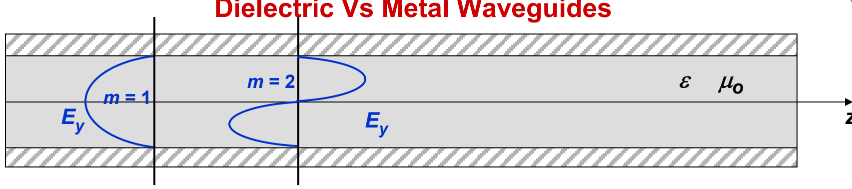
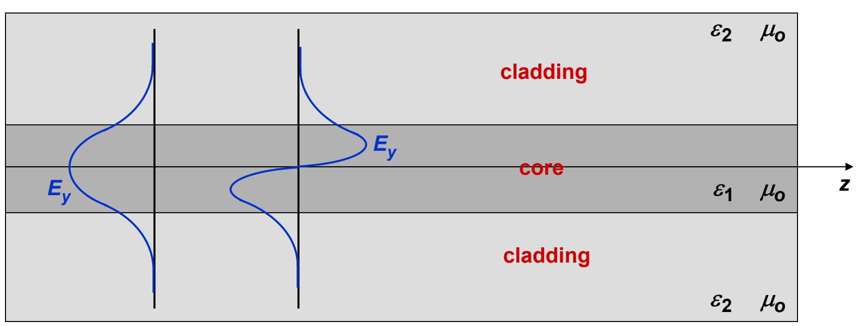
Dielectric Slab Waveguides



Dielectric Vs Metal Waveguides

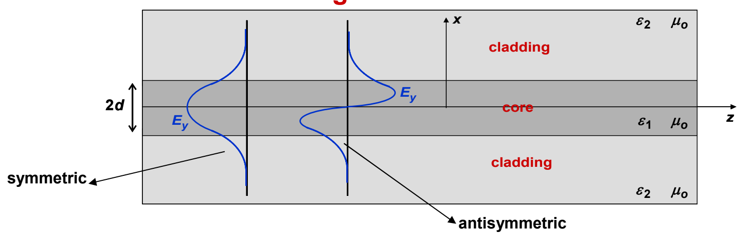


Metal Waveguides
(modes are tightly confined)



Dielectric Slab Waveguides
(modes are loosely confined)

Dielectric Slab Waveguides – TE Modes: Formal Solution



The TE solutions are of the form:

$$\vec{E}(\vec{r})\Big|_{|x|<d} = \hat{y}\, E_o \begin{Bmatrix} \cos(k_x x) \\ \sin(k_x x) \end{Bmatrix} e^{-j k_z z}$$

$$\vec{E}(\vec{r})\Big|_{x>d} = \hat{y}\, E_1\, e^{-\alpha_x (x-d)}\, e^{-j k_z z}$$

$$\vec{E}(\vec{r})\Big|_{x<-d} = \begin{Bmatrix} + \\ - \end{Bmatrix} \hat{y}\, E_1\, e^{+\alpha_x (x+d)}\, e^{-j k_z z}$$

The "sine" and "cosine" represent the symmetric and antisymmetric solutions w.r.t. the z-axis

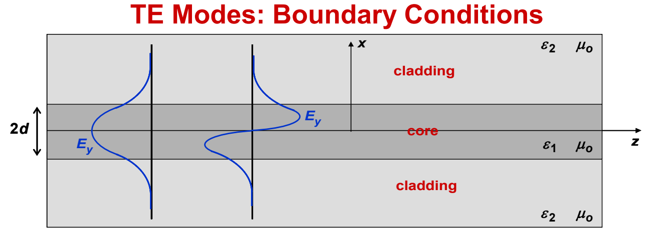
Where:

$$k_z^2 + k_x^2 = \omega^2 \mu_o \varepsilon_1$$

$$k_z^2 - \alpha_x^2 = \omega^2 \mu_o \varepsilon_2$$

Given a frequency ω, the values of k_z, k_x, and α_x are still not known

TE Modes: Boundary Conditions



Boundary conditions:

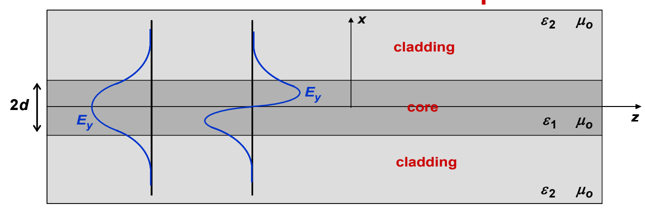
(1) At $x = \pm d$ the component of E-field parallel to the interface (i.e. the y-component) is continuous for all z

$$\Rightarrow \quad E_o \begin{Bmatrix} \cos(k_x d) \\ \sin(k_x d) \end{Bmatrix} = E_1 \qquad \text{——— (1)}$$

(2) At $x = \pm d$ the component of H-field parallel to the interface (i.e. the z-component) is continuous for all z

$$\Rightarrow \quad E_o \begin{Bmatrix} -k_x \sin(k_x d) \\ k_x \cos(k_x d) \end{Bmatrix} = -\alpha_x E_1 \qquad \text{——— (2)}$$

TE Modes: Transcendental Equation



Dividing (2) by (1) on the previous slide gives:

$$\left\{ \begin{array}{c} \tan(k_x d) \\ -\cot(k_x d) \end{array} \right\} = \frac{\alpha_x}{k_x}$$

But:

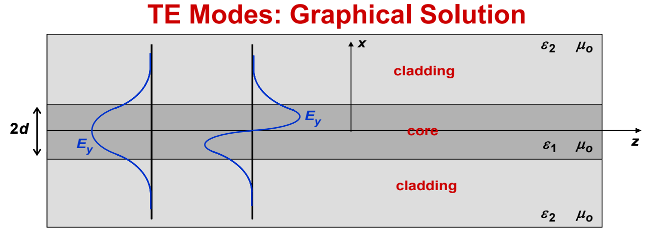
$$\left. \begin{array}{l} k_z^2 + k_x^2 = \omega^2 \mu_o \varepsilon_1 \\ k_z^2 - \alpha_x^2 = \omega^2 \mu_o \varepsilon_2 \end{array} \right\} \longrightarrow \alpha_x^2 + k_x^2 = \omega^2 \mu_o (\varepsilon_1 - \varepsilon_2)$$

So we finally get:

$$\left\{ \begin{array}{c} \tan(k_x d) \\ -\cot(k_x d) \end{array} \right\} = \sqrt{\frac{\omega^2 \mu_o (\varepsilon_1 - \varepsilon_2)}{k_x^2} - 1}$$

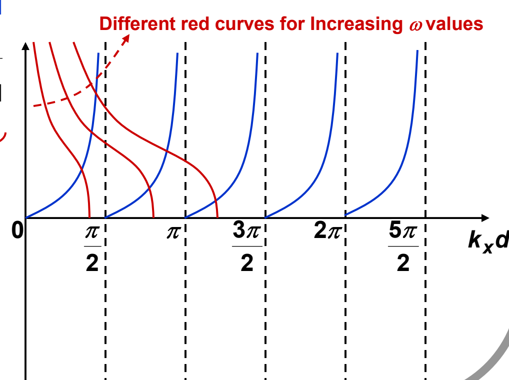
Transcendental equation that can be used to solve for k_x in terms of the frequency ω

TE Modes: Graphical Solution



Graphic solution of the transcendental equation

$$\underbrace{\left\{ \begin{array}{c} \tan(k_x d) \\ -\cot(k_x d) \end{array} \right\}}_{\text{LHS}} = \underbrace{\sqrt{\frac{\omega^2 \mu_o (\varepsilon_1 - \varepsilon_2) d^2}{(k_x d)^2} - 1}}_{\text{RHS}}$$

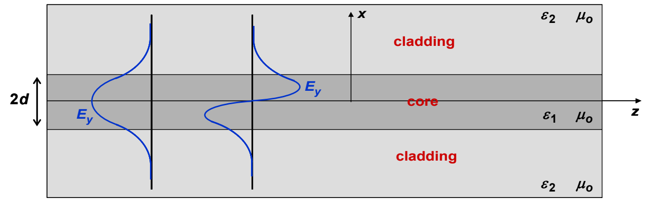


For the *m*-th TE mode (TE$_m$ mode) the value of k_x is in the range (depending on the frequency ω):

$$(m-1)\frac{\pi}{2} \le k_x d \le m\frac{\pi}{2}$$

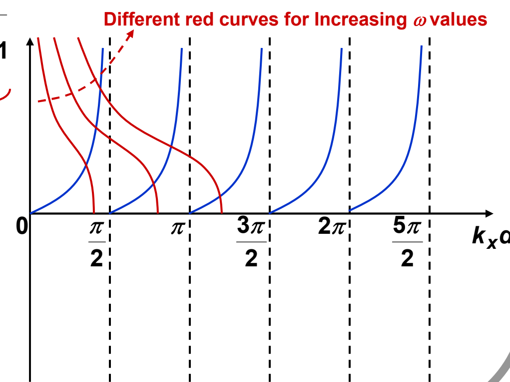
$$m = 1, 2, 3, \ldots\ldots$$

TE Modes: Cut-off Frequencies



$$\begin{Bmatrix} \tan(k_x d) \\ -\cot(k_x d) \end{Bmatrix} = \sqrt{\frac{\omega^2 \mu_o(\varepsilon_1 - \varepsilon_2) d^2}{(k_x d)^2} - 1}$$

LHS — RHS

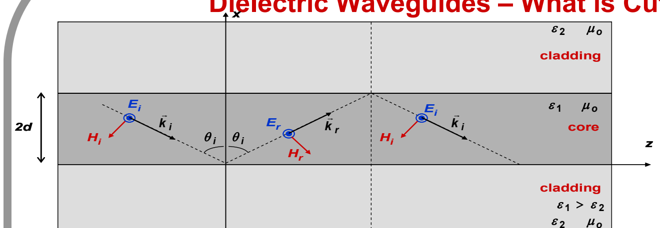


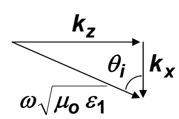
Cut-off frequency of the *m*-th TE mode can be obtained by setting the RHS equal to zero for $k_x d = (m-1)\pi/2$

$$\omega_m = \frac{(m-1)\pi}{2d} \frac{1}{\sqrt{\mu_o(\varepsilon_1 - \varepsilon_2)}}$$

$$m = 1, 2, 3, \ldots\ldots$$

Dielectric Waveguides – What is Cut-off ?





So what does "cut-off" really mean? It means that the wave is no longer being guided through total internal reflection since $\theta_i < \theta_c$

A wave will not be guided if:

$$\theta_i < \theta_c \quad \Rightarrow \quad \sin(\theta_i) < \sin(\theta_c) = \sqrt{\varepsilon_2/\varepsilon_1}$$

$$\Rightarrow \quad \frac{k_z}{\omega\sqrt{\mu_o \varepsilon_1}} < \sqrt{\varepsilon_2/\varepsilon_1}$$

$$\Rightarrow \quad k_z < \omega\sqrt{\mu_o \varepsilon_2}$$

$$\Rightarrow \quad \sqrt{\omega^2 \mu_o \varepsilon_1 - k_x^2} < \omega\sqrt{\mu_o \varepsilon_2}$$

$$\Rightarrow \quad \omega < \frac{k_x}{\sqrt{\mu_o(\varepsilon_1 - \varepsilon_2)}}$$

For the *m*-th TE mode (TE_m mode) the smallest value of k_x is:

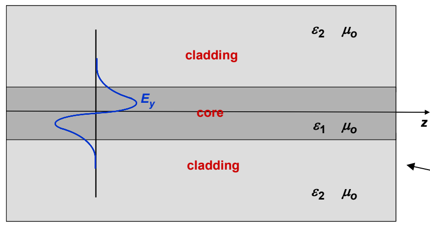
$$(m-1)\frac{\pi}{2d}$$

So TE_m mode will not be guided if:

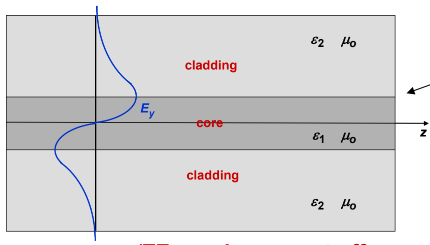
$$\omega < \left.\frac{k_x}{\sqrt{\mu_o(\varepsilon_1 - \varepsilon_2)}}\right|_{k_x = \frac{(m-1)\pi}{2d}}$$

$$\Rightarrow \quad \omega_m = \frac{(m-1)\pi}{2d} \frac{1}{\sqrt{\mu_o(\varepsilon_1 - \varepsilon_2)}}$$

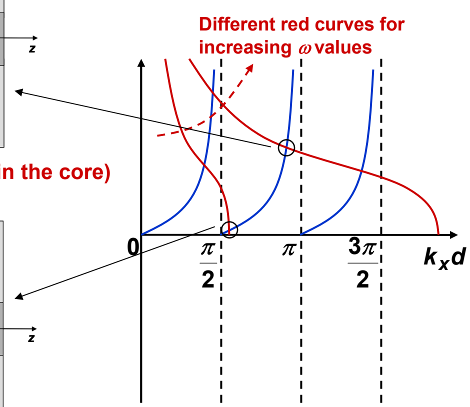
TE Modes: Near Cut-off Behavior



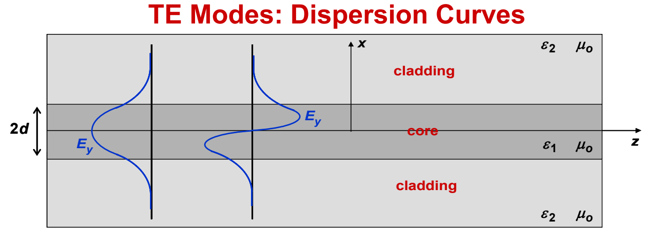
$\omega >> \omega_2$ (TE$_2$ mode well confined in the core)



$\omega >\approx \omega_2$ (TE$_2$ mode near cut-off – mode not well confined in the core)



TE Modes: Dispersion Curves



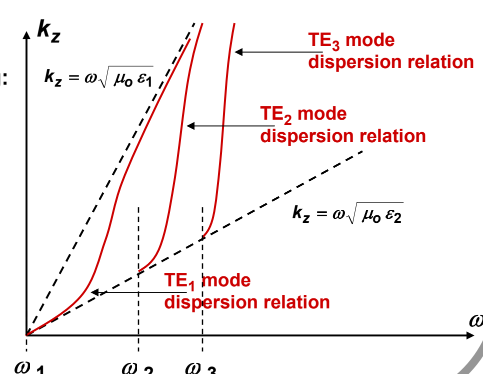
How does one obtain dispersion curves?

(1) For a given frequency ω find k_x using:

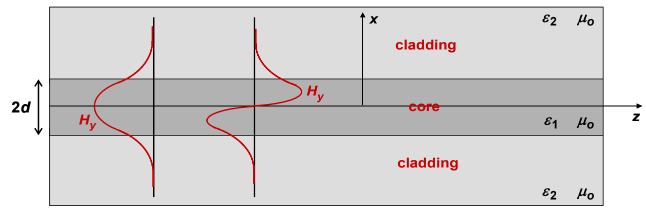
$$\left\{\begin{array}{c}\tan(k_x d)\\ -\cot(k_x d)\end{array}\right\} = \sqrt{\frac{\omega^2 \mu_o(\varepsilon_1-\varepsilon_2)d^2}{(k_x d)^2}-1}$$

(2) Then find k_z using:

$$k_z = \sqrt{\omega^2 \mu_o \varepsilon_1 - k_x^2}$$



TM Modes: Formal Solution



The TM solutions are of the form:

$$\vec{H}(\vec{r})\Big|_{|x|<d} = \hat{y}\, H_o \begin{Bmatrix} \cos(k_x x) \\ \sin(k_x x) \end{Bmatrix} e^{-j k_z z}$$

$$\vec{H}(\vec{r})\Big|_{x>d} = \hat{y}\, H_1\, e^{-\alpha_x (x-d)}\, e^{-j k_z z}$$

$$\vec{H}(\vec{r})\Big|_{x<-d} = \begin{Bmatrix} + \\ - \end{Bmatrix} \hat{y}\, H_1\, e^{+\alpha_x (x+d)}\, e^{-j k_z z}$$

The "sine" and "cosine" represent the symmetric and antisymmetric solutions w.r.t. the z-axis

Where:

$$k_z^2 + k_x^2 = \omega^2 \mu_o \varepsilon_1$$

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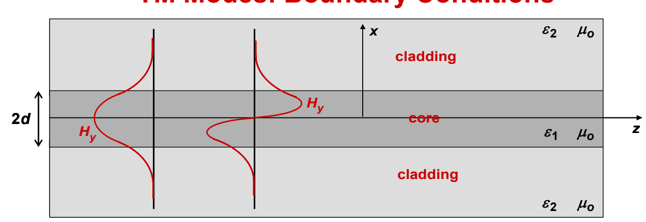
Given a frequency ω, the values of k_z, k_x, and α_x are still not known

TM Modes: Boundary Conditions



Boundary conditions:

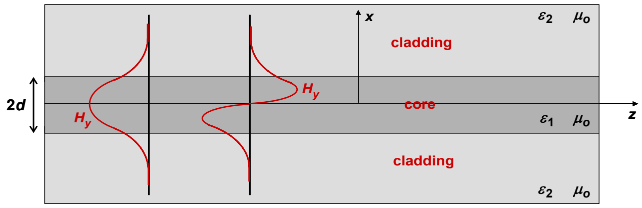
(1) At $x = \pm d$ the component of H-field parallel to the interface (i.e. the y-component) is continuous for all z

$$\Rightarrow \quad H_o \begin{Bmatrix} \cos(k_x d) \\ \sin(k_x d) \end{Bmatrix} = H_1 \qquad \text{——— (1)}$$

(2) At $x = \pm d$ the component of E-field parallel to the interface (i.e. the z-component) is continuous for all z

$$\Rightarrow \quad H_o \begin{Bmatrix} -\dfrac{k_x}{\varepsilon_1} \sin(k_x d) \\ \dfrac{k_x}{\varepsilon_1} \cos(k_x d) \end{Bmatrix} = -\frac{\alpha_x}{\varepsilon_2} H_1 \qquad \text{——— (2)}$$

TM Modes: Transcendental Equation



Dividing (2) by (1) on the previous slide gives:

$$\begin{Bmatrix} \tan(k_x d) \\ -\cot(k_x d) \end{Bmatrix} = \frac{\alpha_x \varepsilon_1}{k_x \varepsilon_2}$$

But:

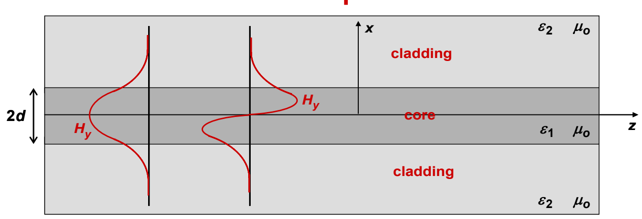
$$\left.\begin{aligned} k_z^2 + k_x^2 &= \omega^2 \mu_o \varepsilon_1 \\ k_z^2 - \alpha_x^2 &= \omega^2 \mu_o \varepsilon_2 \end{aligned}\right\} \longrightarrow \alpha_x^2 + k_x^2 = \omega^2 \mu_o (\varepsilon_1 - \varepsilon_2)$$

So we finally get:

$$\begin{Bmatrix} \tan(k_x d) \\ -\cot(k_x d) \end{Bmatrix} = \frac{\varepsilon_1}{\varepsilon_2} \sqrt{\frac{\omega^2 \mu_o (\varepsilon_1 - \varepsilon_2) d^2}{(k_x d)^2} - 1}$$

Transcendental equation that can be used to solve for k_x in terms of the frequency ω

TM Modes: Graphical Solution



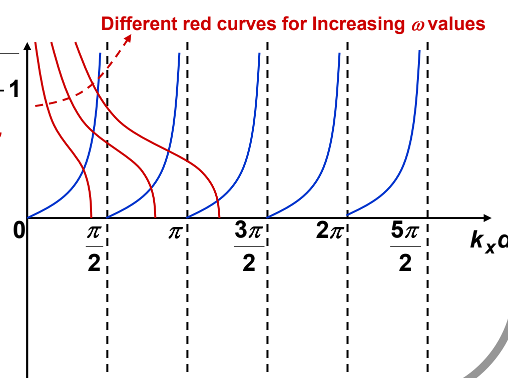
Graphic solution of the transcendental equation

$$\underbrace{\begin{Bmatrix} \tan(k_x d) \\ -\cot(k_x d) \end{Bmatrix}}_{\text{LHS}} = \underbrace{\frac{\varepsilon_1}{\varepsilon_2} \sqrt{\frac{\omega^2 \mu_o (\varepsilon_1 - \varepsilon_2) d^2}{(k_x d)^2} - 1}}_{\text{RHS}}$$

For the *m*-th TM mode (TM$_m$ mode) the value of k_x is in the range (depending on the frequency ω):

$$(m-1)\frac{\pi}{2} \le k_x d \le m\frac{\pi}{2}$$

$$m = 1, 2, 3, \ldots\ldots$$



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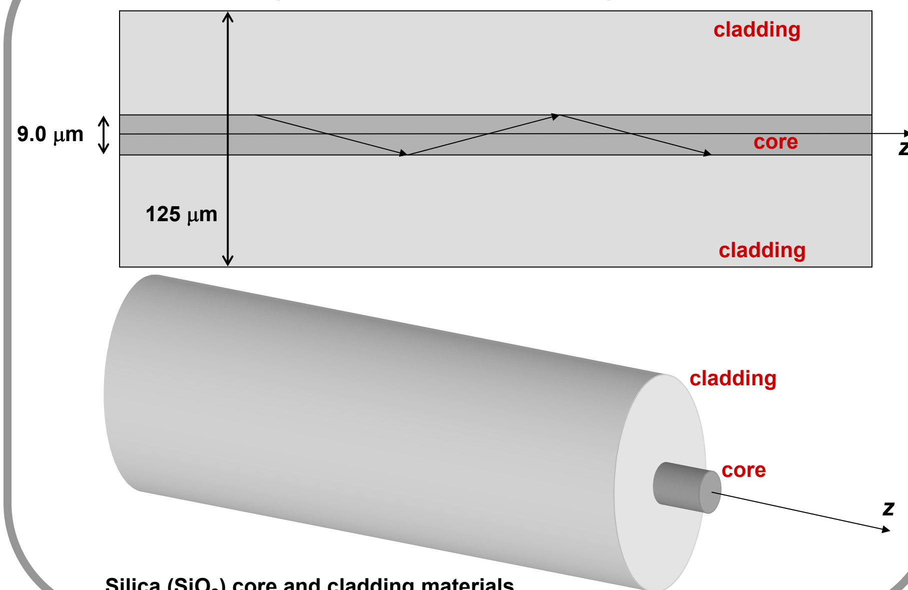
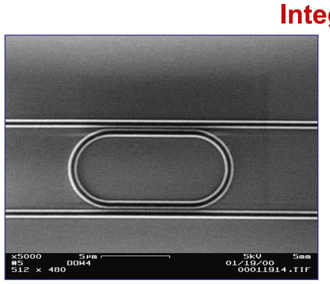
Fiber Optical Communications: Optical Fibers

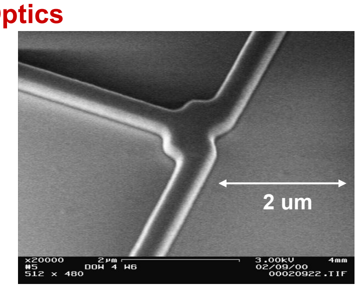


Silica (SiO_2) core and cladding materials

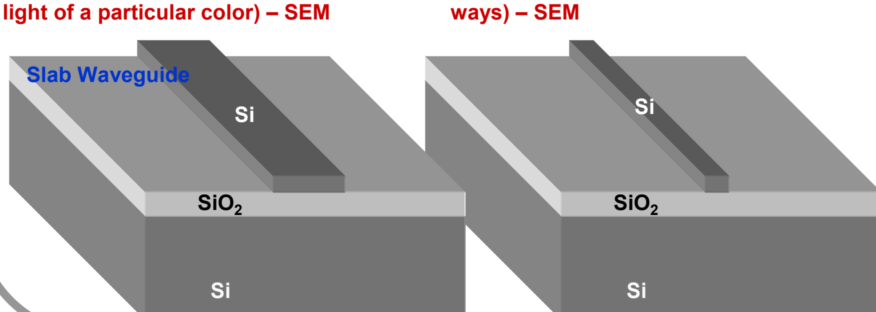
Integrated Optics



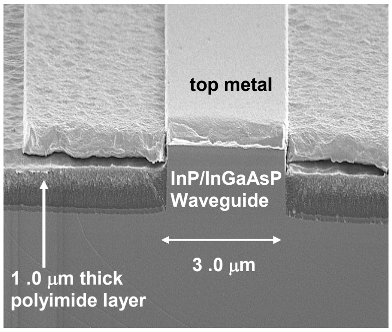
An optical micro-ring filter (separates out light of a particular color) – SEM



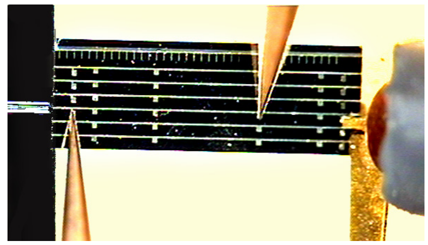
An optical micro-splitter (splits light two ways) – SEM



Integrated Optics: Semiconductor Quantum Well Lasers



The dielectric waveguide for a semiconductor quantum well laser with metal on top (for electrical connection) is shown



A microchip containing several semiconductor laser stripes running in parallel is shown.