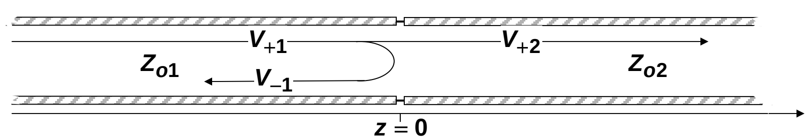
Lecture 23: Addition

Multilayer Structures – Non-Normal Incidence

In this lecture you will learn:

- **Multilayer structures: non-normal incidence**

Waves at Interfaces and Transmission Lines

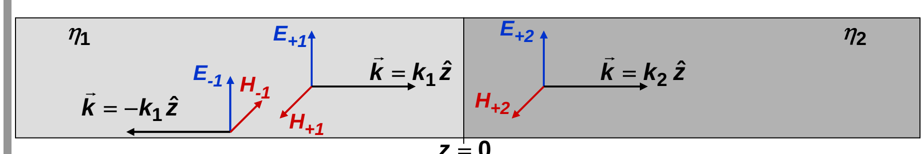


$$V(z)\big|_{z<0} = V_{+1}\,e^{-j k_1 z} + V_{-1}\,e^{+j k_1 z} \qquad V(z)\big|_{z>0} = V_{+2}\,e^{-j k_2 z}$$

Boundary conditions:

$$(1) \;\Rightarrow\; V_{+1} + V_{-1} = V_{+2}$$

$$(2) \;\Rightarrow\; \frac{V_{+1}}{Z_{o1}} - \frac{V_{-1}}{Z_{o1}} = \frac{V_{+2}}{Z_{o2}} \longrightarrow \Gamma = \frac{V_{-1}}{V_{+1}} = \frac{Z_{o2}/Z_{o1} - 1}{Z_{o2}/Z_{o1} + 1}$$



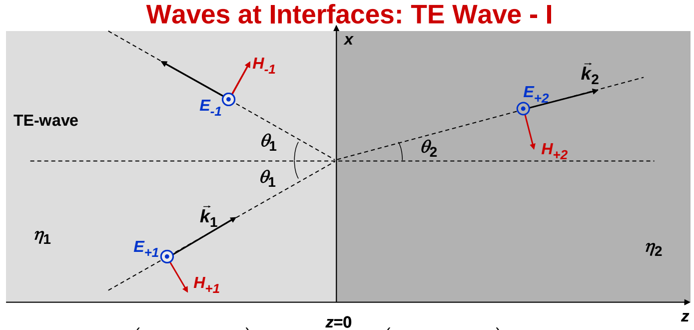
$$\vec{E}(z)\big|_{z<0} = \hat{x}\,E_{+1}\,e^{-j k_1 z} + \hat{x}\,E_{-1}\,e^{+j k_1 z} \qquad \vec{E}(z)\big|_{z>0} = \hat{x}\,E_{+2}\,e^{-j k_2 z}$$

Boundary conditions:

$$(1) \;\Rightarrow\; E_{+1} + E_{-1} = E_{+2}$$

$$(2) \;\Rightarrow\; \frac{E_{+1}}{\eta_1} - \frac{E_{-1}}{\eta_1} = \frac{E_{+2}}{\eta_2} \longrightarrow \Gamma = \frac{E_{-1}}{E_{+1}} = \frac{\eta_2/\eta_1 - 1}{\eta_2/\eta_1 + 1}$$

Waves at Interfaces: TE Wave - I



$$\vec{E}(\vec{r})\Big|_{z<0} = \hat{y}\ E_{+1}\, e^{-j\left(\vec{k}_{1x}\, x + \vec{k}_{1z}\, z\right)} + \hat{y}\ E_{-1}\, e^{-j\left(\vec{k}_{1x}\, x - \vec{k}_{1z}\, z\right)}$$

$$\vec{E}(\vec{r})\Big|_{z>0} = \hat{y}\ E_{+2}\, e^{-j\left(k_{2x}\, x + k_{2z}\, z\right)}$$

Boundary Conditions:

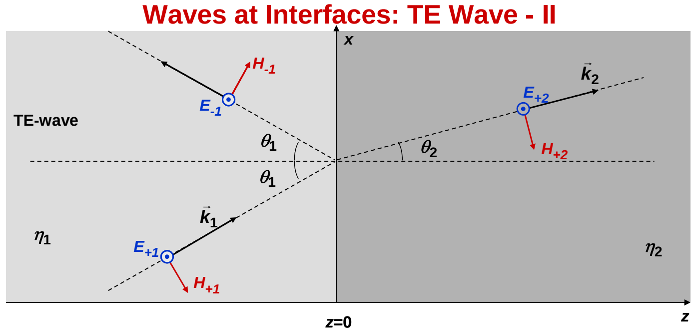
(1) $\Rightarrow \quad E_{+1} + E_{-1} = E_{+2}$

(2) $\Rightarrow \quad \left[\dfrac{E_{+1}}{\eta_1/\cos(\theta_1)} - \dfrac{E_{-1}}{\eta_1/\cos(\theta_1)}\right] = \dfrac{E_{+2}}{\eta_2/\cos(\theta_2)}$

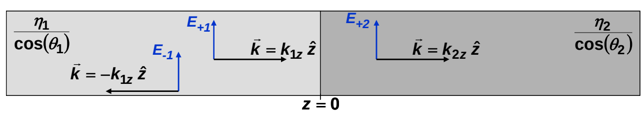
$$T = \frac{E_{+2}}{E_{+1}} = \frac{2\,\dfrac{\eta_2/\cos(\theta_2)}{\eta_1/\cos(\theta_1)}}{\dfrac{\eta_2/\cos(\theta_2)}{\eta_1/\cos(\theta_1)} + 1}$$

$$\Gamma = \frac{E_{-1}}{E_{+1}} = \frac{\dfrac{\eta_2/\cos(\theta_2)}{\eta_1/\cos(\theta_1)} - 1}{\dfrac{\eta_2/\cos(\theta_2)}{\eta_1/\cos(\theta_1)} + 1}$$

Waves at Interfaces: TE Wave - II



One may replace the above problem with a "dummy" normal incidence problem:



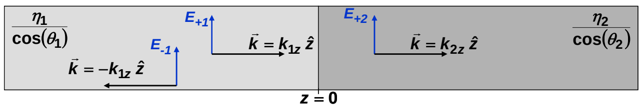
And then calculate the reflection and transmission coefficients for the E-field:

$$\Gamma = \frac{E_{-1}}{E_{+1}} = \frac{\dfrac{\eta_2/\cos(\theta_2)}{\eta_1/\cos(\theta_1)} - 1}{\dfrac{\eta_2/\cos(\theta_2)}{\eta_1/\cos(\theta_1)} + 1} \qquad T = \frac{E_{+2}}{E_{+1}} = \frac{2\,\dfrac{\eta_2/\cos(\theta_2)}{\eta_1/\cos(\theta_1)}}{\dfrac{\eta_2/\cos(\theta_2)}{\eta_1/\cos(\theta_1)} + 1}$$

Waves at Interfaces: TE Wave - III

In the “dummy” normal incidence problem:

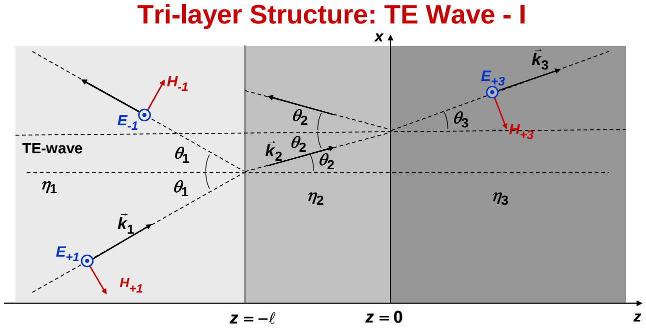
- **The wavevectors are taken to be the z-component of the actual wavevectors in each medium**
- **The impedances in each medium are taken to be the actual impedances divided by the cosines of the angles of propagation w.r.t. the z-axis**



$$\Gamma = \frac{E_{-1}}{E_{+1}} = \frac{\dfrac{\eta_2/\cos(\theta_2)}{\eta_1/\cos(\theta_1)} - 1}{\dfrac{\eta_2/\cos(\theta_2)}{\eta_1/\cos(\theta_1)} + 1} \qquad T = \frac{E_{+2}}{E_{+1}} = \frac{2\dfrac{\eta_2/\cos(\theta_2)}{\eta_1/\cos(\theta_1)}}{\dfrac{\eta_2/\cos(\theta_2)}{\eta_1/\cos(\theta_1)} + 1}$$

Warning: **The “dummy” normal incidence problem is only meant to be used to calculate reflection and transmission coefficients – don’t use this framework for anything else**

Tri-layer Structure: TE Wave - I



$$\vec{E}(\vec{r})\Big|_{z<-\ell} = \hat{y}\, E_{+1}\, e^{-j\left(\vec{k}_{1x}\,x + \vec{k}_{1z}\,(z+\ell)\right)} + \hat{y}\, E_{-1}\, e^{-j\left(\vec{k}_{1x}\,x - \vec{k}_{1z}\,(z+\ell)\right)}$$

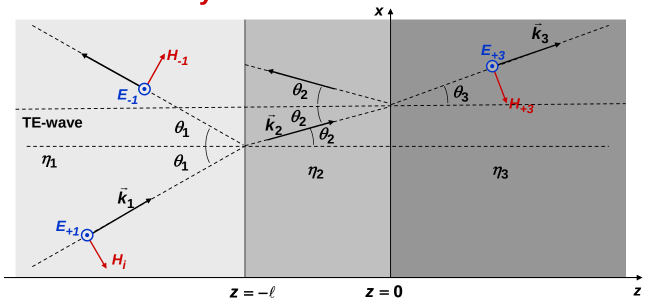
$$\vec{E}(\vec{r})\Big|_{-\ell<z<0} = \hat{y}\, E_{+2}\, e^{-j\left(\vec{k}_{2x}\,x + \vec{k}_{2z}\,z\right)} + \hat{y}\, E_{-2}\, e^{-j\left(\vec{k}_{2x}\,x - \vec{k}_{2z}\,z\right)}$$

$$\vec{E}(\vec{r})\Big|_{z>0} = \hat{y}\, E_{+3}\, e^{-j\left(k_{3x}\,x + k_{3z}\,z\right)}$$

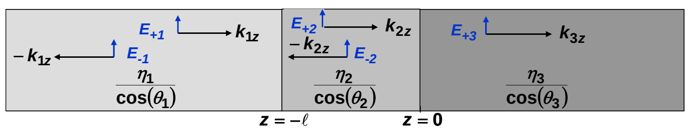
How does one solve a problem like this?

$$\Gamma = \frac{E_{-1}}{E_{+1}} = ?$$

Tri-layer Structure: TE Wave - II

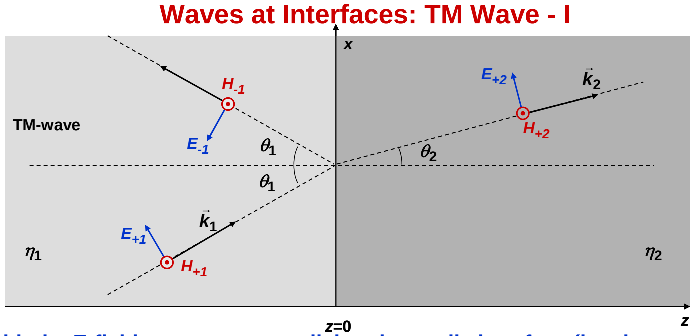


Replace the above problem with a "dummy" normal incidence problem:



And then use the usual methods to solve it (impedance transformations, Smith chart, etc)

Waves at Interfaces: TM Wave - I



Work with the E-field component parallel to the media interface (i.e. the x-component)

$$E_x(\vec{r})\big|_{z<0} = E_{+1x}\, e^{-j\left(\vec{k}_{1x}\,x + \vec{k}_{1z}\,z\right)} + E_{-1x}\, e^{-j\left(\vec{k}_{1x}\,x - \vec{k}_{1z}\,z\right)}$$

$$E_x(\vec{r})\big|_{z>0} = E_{+2x}\, e^{-j\left(k_{2x}\,x + k_{2z}\,z\right)}$$

Boundary Conditions:

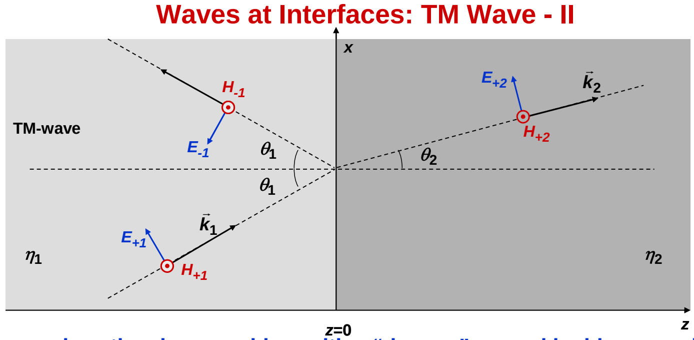
(1) $\Rightarrow \quad E_{+1x} + E_{-1x} = E_{+2x}$

(2) $\Rightarrow \quad \left[\dfrac{E_{+1x}}{\eta_1 \cos(\theta_1)} - \dfrac{E_{-1x}}{\eta_1 \cos(\theta_1)}\right] = \dfrac{E_{+2x}}{\eta_2 \cos(\theta_2)}$

$$\Gamma = \frac{E_{-1x}}{E_{+1x}} = \frac{\dfrac{\eta_2 \cos(\theta_2)}{\eta_1 \cos(\theta_1)} - 1}{\dfrac{\eta_2 \cos(\theta_2)}{\eta_1 \cos(\theta_1)} + 1}$$

$$T = \frac{E_{+2x}}{E_{+1x}} = \frac{2\,\dfrac{\eta_2 \cos(\theta_2)}{\eta_1 \cos(\theta_1)}}{\dfrac{\eta_2 \cos(\theta_2)}{\eta_1 \cos(\theta_1)} + 1}$$

Waves at Interfaces: TM Wave - II



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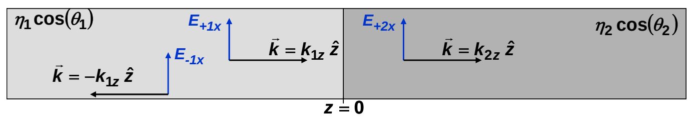
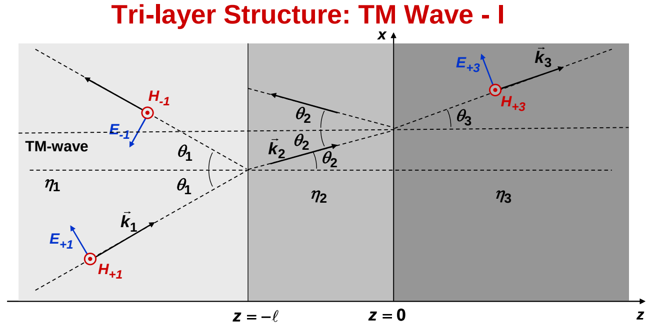


And then calculate the reflection and transmission coefficients for the x-component of the E-field:

$$\Gamma = \frac{E_{-1x}}{E_{+1x}} = \frac{\dfrac{\eta_2\cos(\theta_2)}{\eta_1\cos(\theta_1)} - 1}{\dfrac{\eta_2\cos(\theta_2)}{\eta_1\cos(\theta_1)} + 1} \qquad T = \frac{E_{+2x}}{E_{+1x}} = \frac{2\,\dfrac{\eta_2\cos(\theta_2)}{\eta_1\cos(\theta_1)}}{\dfrac{\eta_2\cos(\theta_2)}{\eta_1\cos(\theta_1)} + 1}$$

Tri-layer Structure: TM Wave - I



Work with the E-field component parallel to the media interface (i.e. the x-component)

$$E_x(\vec{r})\big|_{z<-\ell} = E_{+1x}\, e^{-j\left(\vec{k}_{1x}\, x + \vec{k}_{1z}\,(z+\ell)\right)} + E_{-1x}\, e^{-j\left(\vec{k}_{1x}\, x - \vec{k}_{1z}\,(z+\ell)\right)}$$

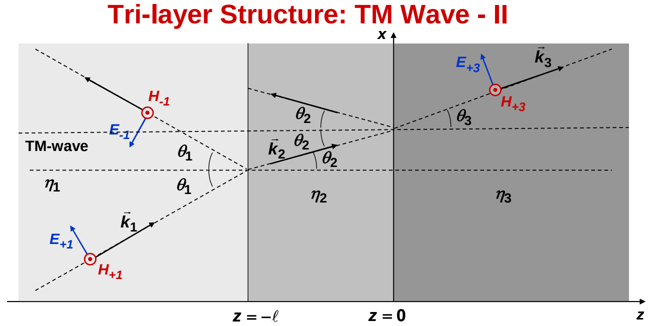
$$E_x(\vec{r})\big|_{-\ell<z<0} = E_{+2x}\, e^{-j\left(\vec{k}_{2x}\, x + \vec{k}_{2z}\, z\right)} + E_{-2x}\, e^{-j\left(\vec{k}_{2x}\, x - \vec{k}_{2z}\, z\right)}$$

$$E_x(\vec{r})\big|_{z>0} = E_{+3x}\, e^{-j\left(k_{3x}\, x + k_{3z}\, z\right)}$$

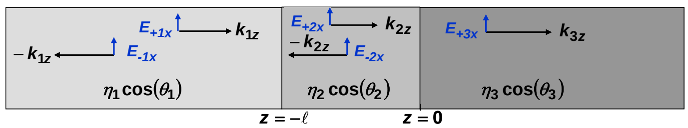
How does one solve a problem like this?

$$\Gamma = \frac{E_{-1x}}{E_{+1x}} = ?$$

Tri-layer Structure: TM Wave - II



Replace the above problem with a "dummy" normal incidence problem:



And then use the usual methods to solve it (impedance transformations, Smith chart, etc)