Homework 5
Due on Sep. 28, 2007 by 5:00 PM

Reading Assignments:

i) Review the lecture notes.
ii) Review sections 1.1-1.6 of the paperback book *Electromagnetic Waves*. We have officially started the paperback textbook and will follow it till the end of the semester.

Problem 5.1: (Vector Calculus and Plane Waves)

Consider a plane wave whose E-field vector phasor is given by the following expression:

\[ \mathbf{E}(\mathbf{r}) = \hat{n} \mathbf{E}_o \, e^{-j \mathbf{k} \cdot \mathbf{r}} \]

Where \( \mathbf{k} \) and the unit vector \( \hat{n} \) can be decomposed into components as follows:

\[ \mathbf{k} = k_x \hat{x} + k_y \hat{y} + k_z \hat{z} \]
\[ \hat{n} = n_x \hat{x} + n_y \hat{y} + n_z \hat{z} \]

Prove the following three relations:

a) \( \nabla \times \mathbf{E}(\mathbf{r}) = \left( -j \mathbf{k} \times \hat{n} \right) \mathbf{E}_o \, e^{-j \mathbf{k} \cdot \mathbf{r}} \)

b) \( \nabla^2 \mathbf{E}(\mathbf{r}) = \left( -k^2 \right) \hat{n} \mathbf{E}_o \, e^{-j \mathbf{k} \cdot \mathbf{r}} \)

c) \( \nabla \cdot \mathbf{E}(\mathbf{r}) = \left( -j \mathbf{k} \cdot \hat{n} \right) \mathbf{E}_o \, e^{-j \mathbf{k} \cdot \mathbf{r}} \)

Problem 5.2: (Polarization of Plane Waves)

a) Find the Polarization (i.e., linear, circular, or elliptical, and left-handed or right-handed) of the following plane waves:

i) \( \mathbf{E}(\mathbf{r}) = (j \hat{y} + \hat{z}) \mathbf{E}_o \, e^{-jk \hat{x}} \)

ii) \( \mathbf{E}(\mathbf{r}) = \left[ \hat{x} (2 + j) + \hat{z} (3j + 1) \right] \mathbf{E}_o \, e^{jk \hat{y}} \)

iii) \( \mathbf{H}(\mathbf{r}) = \left[ \hat{x} - j \hat{y} \right] \mathbf{H}_o \, e^{jk \hat{z}} \)

iv) \( \mathbf{E}(\mathbf{r}) = \left( - \hat{x} + \hat{y} \right) \mathbf{E}_o \, e^{-jk \frac{x+y}{\sqrt{2}}} \)
b) Find the magnetic field (or the electric field - whichever is not given) for the following plane waves:

i) \( \vec{E}(r) = (j\hat{y} + \hat{z})E_o e^{-jkx} \)

ii) \( \vec{E}(r) = [\hat{x}(2 + j) + \hat{z}(3j + 1)]E_o e^{jky} \)

iii) \( \vec{H}(r) = [\hat{x} - j\hat{y}]H_o e^{jkz} \)

iv) \( \vec{E}(r) = (-\hat{x} + \hat{y})E_o e^{-jk(x+y) \sqrt{2}} \)

c) Find the time-average power flow per unit area (magnitude and direction) carried by the following plane waves:

i) \( \vec{E}(r) = (j\hat{y} + \hat{z})E_o e^{-jkx} \)

ii) \( \vec{E}(r) = [\hat{x}(2 + j) + \hat{z}(3j + 1)]E_o e^{jky} \)

iii) \( \vec{H}(r) = [\hat{x} - j\hat{y}]H_o e^{jkz} \)

iv) \( \vec{E}(r) = (-\hat{x} + \hat{y})E_o e^{-jk(x+y) \sqrt{2}} \)