Resting Potential

1. **Goldman equation.**

   The Nernst potential describes an equilibrium potential when only one ion is permeable. Things are more complicated in neurons because the membrane can be permeable to multiple ions and the permeabilities can change in response to inputs from other neurons and changes in membrane voltage.

   A. *Intuitively, if the permeability to an ion goes up, then we might expect that the membrane potential of a cell will move toward the equilibrium potential of that ion.*

   B. *This idea that the membrane potential \( V_m \) at any time is governed by the equilibrium potentials of the individual ions along with their relative permeabilities is described quantitatively by the Goldman equation.*

   \[
   V_m = 58 \log \left( \frac{P_{Kout}[K]_{out} + P_{Naout}[Na]_{out} + P_{Clin}[Cl]_{in}}{P_{Kin}[K]_{in} + P_{Naout}[Na]_{out} + P_{Clout}[Cl]_{out}} \right)
   \]

   This equation is commonly written with relative permeabilities, with the permeability of \( K^+ = 1 \) and \( b \) and \( c \) representing the permeabilities of the other ions relative to it.

   \[
   V_m = 58 \log \left( \frac{[K]_{out} + b[Na]_{out} + c[Cl]_{in}}{[K]_{in} + b[Na]_{in} + c[Cl]_{out}} \right)
   \]

   What is the membrane potential if the ratio of sodium permeability to potassium is .02, chloride is not permeable, and the concentrations of the ions are as in the earlier table in the notes?

   What happens if, suddenly, the permeability to sodium becomes very high relative to potassium?

2. **Passive properties:**

   Signaling in the nervous system depends upon changes from the resting membrane potential that are produced by currents that flow when channels in the membrane open or close at synaptic connections or during action potentials. We need to understand something about what influences how currents entering the neuron at one place affect the local membrane potential, as well more distant parts of the cell.

   A. *Synapses are often very localized on a neuron.* They open channels that allow current to flow into the cell. If, for example, the channels that open are permeable to sodium, sodium ions will enter and depolarize the cell as the membrane potential moves toward the sodium equilibrium potential.
B. The speed and magnitude of the depolarization and the distance along the neuron that is depolarized are determined by properties of the neuronal cell membrane. These properties are very important for the neuron’s ability to integrate information from different inputs.

C. This is best seen by analogy with electrical circuits.

Channels provide variable resistance - high when closed and low when opened. Lipid bilayer behaves as a capacitor. The concentration gradient provides a potential energy source – a battery.

Electrical equivalent circuit diagram

D. Time constant - If we inject a rectangular current into a neuron, the potential rises (or falls when the current is turned off) slowly because of the combination of the capacitance and resistance of the cell.
The rise is described by:

\[ V(t) = V_{\text{max}}(1-e^{-t/\tau}) \]

The fall is described by:

\[ V(t) = V_{\text{max}} e^{-t/\tau} \]

The membrane properties have the effect of slowing down potential changes, so they can sum with later ones…

\[ \tau = \text{time constant} = \text{membrane resistance} \times \text{membrane capacitance} = r_m c_m \]

The larger the time constant, the slower the rise or fall of the potential in response to a current injection.

Generally, smaller cells have longer time constants than larger cells, but it depends on the exact values for \( r_m \) and \( c_m \).

E. **Length constant** – Current coming in at one point on a cell will depolarize the nearby membrane more than the membrane farther away because:

Current leaks out through the membrane, so there is less and less as one moves farther from where it came into the neuron.

There is a resistance to current flow inside the neuron that makes it harder to depolarize sites far away.
The equation that defines the decay of the potential (V) with distance (x) is:

\[ V(x) = V_{\text{max}} e^{-x/\lambda} \]

\( \lambda = \text{length constant} = \sqrt{\frac{r_m}{(r_o + r_i)}} \)

If we ignore \( r_o \) because \( r_o << r_i \) this becomes

\( \text{length constant} = \sqrt{\frac{r_m}{r_i}} \)

The larger the length constant, the bigger the effect of a depolarization at one place on sites farther away.

This is analogous to conduction of heat through a rod. To improve the conduction, reduce \( r_i \) (make the rod out of a better conductor) and increase \( r_m \) (insulate the rod).

This decay is a real problem for nerve cells because they need to transmit information over long distances… the decay seems incompatible with that. One major solution is:

Action potentials, our next topic.