Learning objectives:

To understand how neurons compute
To understand simplified neuron models
How the brain works.
Exercise:

(a) Consider the following scenario: A child sees a balloon for the first time. The balloon pops and makes a loud noise. The child is frightened. Two days later, the child sees another balloon and starts crying.

(b) Think through all the computations that may happen in the brain of this child. What information has to be processed? What information has to be stored?

(c) Pretend to write a computer program that would reproduce this scenario. Write a flow diagram for this program.

(d) Look up brain structures and how they connect to each other. Make a diagram of the information flow through such a diagram.
Dendrites

Terminal branches

Axon

Dendrites

Terminal branches
Arriving Action Potentials

EPSPs

IPSP

Filtered EPSPs at cell body

Filtered IPSP at cell body

Output Axon

Summed potential converted to outgoing action potentials

Summed potential
Some notations: (tell me if I change these during the semester)

$x(t)$:

g(t):

$I$

$i(t)$:

$v(t)$:

$V_{\text{rest}}$

$V_{N}$
McCulloch Pitts Neuron

\[ x(t) = F(I(t), \theta) \]

\[ I(t) = \sum_{j=1}^{N} i_j(t) \text{ total input} \]

\[ \text{i1(t), i2(t) .. inputs} \]

\[ \Sigma \]

\[ x(t) = F(I(t), \theta) \text{ output} \]

\[ 1.0 \]

\[ \Theta \]

\[ I(t) \]

\[ x(t) = F(I(t), \theta) \]
**Exercise**: Use networks of McCulloch Pitts neurons to create a NOT, AND, OR and XOR device. Connections or synaptic weights between neurons can be positive or negative. Define threshold and synaptic weights and assume that a neuron’s output can be either 0 or 1.

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Integrate and Fire Neuron

\[ I(t) = \sum i(t) \] .. Total input

\[ \frac{dv}{dt} = I(t) \] ... Internal variable

\[ x(t) = F(v(t), \Theta) \] .. Output

\[ v(t) \]

\[ \Theta \]

\[ x(t) = F(v(t), \theta) \]

\[ v(t) \]

\[ 1.0 \]

\[ \int_{t=0}^{t} I(t) dt \]

\[ f(I) = \frac{I}{CmVth} \]
\[ I_c(t) = \frac{q}{V(t)} = C \frac{dV}{dt} \quad C = \frac{Q}{V} \]
Leaky Integrate and Fire Neuron

\[ I(t) = \sum i_i(t) \] .. Total input
\[ \frac{dv}{dt} = -\gamma v(t) + I(t) \] ... Internal variable

\[ x(t) = F(v(t), \Theta) \] .. Output

\[ x(t) = F(v(t), \theta) \]

\[ v(t) \]

Leaky Integrate and Fire Neuron

\[ \tau \frac{dv}{dt} = -v(t) + RI(t) \]
\[ I(t) = I_R(t) + I_C(t) = \frac{v(t)}{R} + \frac{q}{v(t)} = \frac{v(t)}{R} + C \frac{dV}{dt} \]

\[ \tau_m \frac{dv(t)}{dt} = -v(t) + RI \quad \text{with} \quad \tau_m = RC \quad \text{(time constant)} \]
Exercise: Using firing rate as the variable that defines ON and OFF, create an NOT, AND, OR and XOR gate using leaky integrate and fire neurons. What happens if you lower or raise the firing threshold?
Exercise: make a list of characteristics of biological neurons that are captured by the leaky integrate and fire neuron and a list of characteristics that are not captured by it. For these latter ones, suggest changes to make that would capture these.
Exercise: (a) You want to create a sensory neuron that responds to sensory stimuli ABOVE a certain threshold with a close to linear input-output function. The response should be in spikes/second. You can assume that the input changes on a very slow time scale (minutes). Choose one of the three types of neurons above to implement this sensory neuron and defend your choices. (b) You want to create a sensory neuron that responds to ANY sensory input and has a linear input-output function. The response should be expressed in spikes/seconds. You can assume that the input changes on a very slow time scale (minutes). Choose one of the three types of neurons above to implement this sensory neuron and defend your choices.
Synapses

\[ ini = \sum_{i=1}^{n} wij \times xj \]
Exercise: You have three McCulloch Pitts neurons. All three have the possible states 0 and 1, and their thresholds are 1.0. Two neurons receive outside inputs (in1, in2), and these two make synapses with the third who is considered the output (o). You want the output to be 1 when the sum of the inputs $> 4$, 0 otherwise. How do you choose your synaptic weights?
\[ v(t) \]

\[ x(t) = F(v(t), \theta) \]

\[ I(t) \]
Internal variable $v(t)$

$$I(t) = F[\Sigma i_i(t)]$$

$$v(t) = F[I(t), v(t), \tau]$$

$$x(t) = F(v(t), \Theta, ..]$$
**Heavyside function**

\[ x = 0 \text{ if } u < \theta \]
\[ x = 1 \text{ if } u \geq \theta \]

**Linear threshold function**

\[ x = 0 \text{ if } u < \theta \]
\[ x = u \text{ if } u \geq \theta \]

**Sigmoid function**

\[ x = \frac{1}{1 + \exp(-u/\text{slope} + \theta)} \]