Homework - Associators

Linear associator

Imagine a neural network with two layers of neurons, one "input" layer f and one "output" layer g. Neurons in layer f can synapses onto neurons in layer g with modifiable synapses.



You are using this neural network to store the associations between to set of patterns:



**A1**.Assume that the neurons' activities can be either zero or positive. For the patterns drawn above, black dots mean +1, white dots 0. If we note f1, f2, etc the activities of neurons f1, f2 et in later f, then the activities associated with each pattern are:

A: f1=1, f2=1, f3=0, f4=0; g1=1, g2=0, g3=1, g4=0.

B: f1=0, f2=1, f3=1, f4=0; g1=0, g2=1, g3=0, g4=1.

Knowing that wij = gi\*fj for each combination of patterns and that initially, all w's are zero, calculate the changes in synaptic weights w(A) and w(B) necessary to store the associations A and B as well as the resulting weight matrix w= w(A)+w(B).

Hint:

PatternA: w11 = g1\*f1 = 1; w12 = g1\*f2 = 1; w13 = g1\*f3 = 0; w14 = g1\*f4 =0 etc!

PatternB: w11 = g1\*f1 = 0; w12 = g1\*f2 = 0; w13 = g1\*f3 = 0; w14 = g1\*f4 =0 etc!

Total: w11 = w11(A)+w11(B) = 1.. etc

**A2.** Using the resulting weight matrix w, calculate what the activities of the neurons in the output layer for each of the two input patterns f(A) and f(B), knowing that

gi= wij\*fj (g1=w11\*f1+w12\*f2+w13\*f3+w14\*f4 etc).

**B**. Assume all weights are zero again. Now assume that for each pattern, black dots mean +1 activity and white dots mean -1 activity.

A: f1=1, f2=1, f3=-1, f4=-1; g1=1, g2=-1, g3=1, g4=-1.

B: f1=-1, f2=1, f3=1, f4=-1; g1=-1, g2=1, g3=-1, g4=1.

**B1**. Using the same equations than before, calculate the resulting synaptic weight matrix for storage of these two associations.

**B2**. Using the resulting weight matrix w, calculate what the activities of the neurons in the output layer for each of the two input patterns f(A) and f(B), knowing that

gi= wij\*fj (g1=w11\*f1+w12\*f2+w13\*f3+w14\*f4 etc).

**C**. Comment on the comparison of the two results!

**D2.** Repeat both parts of the exercise using the following normalization when you calculate the synaptic weights: wij = 1/sqrt(Ng)\*gi\*1/sqrt(Nf)\*fj where N is the total number of neurons in each layer. Similarly, use the following normalization when you calculate the activation of the output layer neurons: gi=1/sqrt(Nf)\*  wij\*fj

Compare and comment.

2) You have a Hopfield network with 4 neurons. Each neuron can take the values -1 (inactive) or +1 (active). Outputs and weights are calculated according to the equations in your notes.

a) Calculate the weights needed to store the pattern (-1, 1, -1, 1).

b) Using the weights you calculated, determine if the pattern (-1, 1, -1, 1) is stable.

c) Does the state (-1, 1, -1, 1) have a basin of attraction?

d) Is there a second, equally stable state for the weights you have calculated?

3) Now use a Hopfield network with two neurons.

a) Calculate the weights to store the pattern (-1, 1). Calculate the energy for each possible state of the network using the weights you have determined. Which states are stable and which are not?

b. Then calculate the weights to store the pattern (-1, -1) and add them to the weights you have previously calculated. Calculate the energy for each possible state of the network with these new (summed up) weights. Comment!