Problem: Suppose an urn has 10 balls. First Robert picks 6 balls, and returns them. Then Susan picks 4 balls then returns them.

a) What is the probability that they choose no balls in common?

b) What is the probability that they choose exactly two balls in common?

(Hint: it's almost as easy to consider the general case of an urn with N balls, from which Robert chooses r balls, replaces them, and Susan chooses s balls, and ask what is the probability they've chosen exactly m in common (for m less than r and s). Show that the number of ways of doing this is given by the number of ways of choosing three sets of objects from the total of N, of size m, $r-m$, and $s-m$. For the probability, divide this by the total number of ways that Robert can choose r balls times the total number of ways that Susan can choose s balls.)

Solution: In the general case, the number of ways of choosing three sets of objects from the total of N, of size m, $r-m$, and $s-m$ is $N!/\big(m!(r-m)!(s-m)!(N-(r+s-m))!\big)$. The total number of ways that Robert can choose r balls times the total number of ways that Susan can choose s balls is $\binom{N}{r}\cdot\binom{N}{s} = \big(N!/(N-r)!r!\big)\big(N!/(N-s)!s!\big)$, so the probability is

$$\frac{N!/\big(m!(r-m)!(s-m)!(N-(r+s-m))!\big)}{\big(N!/(N-r)!r!\big)\big(N!/(N-s)!s!\big)} = \frac{(N-r)!r!(N-s)!s!}{N!m!(r-m)!(s-m)!(N-(r+s-m))!}$$

a) For $m=0$ (no balls in common) and $r+s=N$ this reduces to $(N-r)!(N-s)!/N!$, and for $r=6$ and $s=4$ the probability is $6!4!/10! \approx .0048$

b) For $m=2$, $6!4!4!6!/10!2!4!2!2! \approx .429$