Problem Set for "Intro to CFD" Notes

Consider the following differential equation

$$\frac{d^2u}{dx^2} - 2\,u^3 = 0; \quad 0 \le x \le 9; \quad u(0) = 1, \quad u(9) = 0.1$$

- Apply the finite-difference method to this equation to get a linearized difference equation at grid point i away from the boundary. Note that a second-order difference approximation for the second-derivative is

$$\left(\frac{d^2u}{dx^2}\right)_i = \frac{u_{i-1} - 2u_i + u_{i+1}}{\Delta x^2} + O\left(\Delta x^2\right)$$

- Assemble the discrete system of equations for a four-point grid into a matrix system of the form

$$[\mathbf{A}]\{\mathbf{u}\} = \{\mathbf{b}\}$$

  where

$$\{\mathbf{u}\} = \{u_1\ u_2\ u_3\ u_4\}^T$$

- Develop a MATLAB program to solve the finite-difference equations on a grid with N points. Apply this code to obtain the solution on a 4-point grid ($\Delta x = 3$). For the initial guess, use a linear variation between the two boundary values. Converge your solution until the residual is below 10^{-6}. Plot the residuals vs. iteration number.

  Hint: In MATLAB, initialize all elements of $[\mathbf{A}]$ to zero. For row i of $[\mathbf{A}]$ when $2 \le i \le N-1$, you need to set only the elements $A_{i,i-1}$, $A_{i,i}$ and $A_{i,i+1}$.

- Plot the finite-difference solution obtained on the 4-point grid and compare it with the exact solution

$$u_{exact} = \frac{1}{x+1}$$

- Use your MATLAB program to obtain the solution on a 7-point grid ($\Delta x = 1.5$). Plot the solution and compare it with the solution for the 4-point grid and the exact solution.